PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z^5+z^4+1, z^2+z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z^5+z^4+1, z^2+z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^5+z^4+1, z^2+z+1)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{13}+z^{10}+z^8+z^7+z^5+z+1,\\
p_1(z)&=z^{18}+z^{17}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^7+z^6+z,\\
p_2(z)&=z^{20}+z^{14}+z^{12}+z^{10}+z^6+z^2+1,\\
p_3(z)&=z^{18}+z^{17}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^7+z^6+z,\\
p_4(z)&=z^{16}+z^{14}+z^{13}+z^8+z^7+z^6+z^5+z^4+z^2+z+1,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{13}+z^{12}+z^{10}+z^8+z^7+z^5+z^4+z+1,\\
p_1(z)&=z^{18}+z^{17}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^7+z^6+z,\\
p_2(z)&=z^{20}+z^{14}+z^{12}+z^{10}+z^6+z^2+1,\\
p_3(z)&=z^{18}+z^{17}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^7+z^6+z,\\
p_4(z)&=z^{16}+z^{14}+z^{13}+z^{12}+z^8+z^7+z^6+z^5+z^2+z+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{5u}M^e[:2u] + x^{3u}M^e[:7u] \\ & + x^{6u}M^e[:4u] + x^{7u}M^e[:3u] + x^{8u}M^e[:2u] + x^{4u}M^e[:7u] \\ & + x^{7u}M^e[:4u] + x^{8u}M^e[:3u] + x^{9u}M^e[:2u] + x^{5u}M^e[:7u] \\ & + x^{8u}M^e[:4u] + x^{9u}M^e[:3u] + x^{10u}M^e[:2u] + x^{6u}M^e[:7u] \\ & + x^{9u}M^e[:4u] + x^{10u}M^e[:3u] + x^{11u}M^e[:2u] + x^{9u}M^e[:5u] \\ & + x^{10u}M^e[:4u] + x^{11u}M^e[:3u] \end{aligned}$$

$$
\begin{aligned}
&+ (x^{7u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e,\\
W^e[:14u] = {}& W^e[:7u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u] + x^{3u}W^e[:7u]\\
&+ x^{6u}W^e[:4u] + x^{7u}W^e[:3u] + x^{8u}W^e[:2u] + x^{4u}W^e[:7u]\\
&+ x^{7u}W^e[:4u] + x^{8u}W^e[:3u] + x^{9u}W^e[:2u] + x^{5u}W^e[:7u]\\
&+ x^{8u}W^e[:4u] + x^{9u}W^e[:3u] + x^{10u}W^e[:2u] + x^{6u}W^e[:7u]\\
&+ x^{9u}W^e[:4u] + x^{10u}W^e[:3u] + x^{11u}W^e[:2u] + x^{9u}W^e[:5u]\\
&+ x^{10u}W^e[:4u] + x^{11u}W^e[:3u]\\
&+ (x^{7u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u] + x^{3u}M^o[:7u]\\
&+ x^{6u}M^o[:4u] + x^{7u}M^o[:3u] + x^{8u}M^o[:2u] + x^{4u}M^o[:7u]\\
&+ x^{7u}M^o[:4u] + x^{8u}M^o[:3u] + x^{9u}M^o[:2u] + x^{5u}M^o[:7u]\\
&+ x^{8u}M^o[:4u] + x^{9u}M^o[:3u] + x^{10u}M^o[:2u] + x^{6u}M^o[:7u]\\
&+ x^{9u}M^o[:4u] + x^{10u}M^o[:3u] + x^{11u}M^o[:2u] + x^{9u}M^o[:5u]\\
&+ x^{10u}M^o[:4u] + x^{11u}M^o[:3u]\\
&+ (x^{7u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o,\\
W^o[:14u] = {}& W^o[:7u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u] + x^{3u}W^o[:7u]\\
&+ x^{6u}W^o[:4u] + x^{7u}W^o[:3u] + x^{8u}W^o[:2u] + x^{4u}W^o[:7u]\\
&+ x^{7u}W^o[:4u] + x^{8u}W^o[:3u] + x^{9u}W^o[:2u] + x^{5u}W^o[:7u]\\
&+ x^{8u}W^o[:4u] + x^{9u}W^o[:3u] + x^{10u}W^o[:2u] + x^{6u}W^o[:7u]\\
&+ x^{9u}W^o[:4u] + x^{10u}W^o[:3u] + x^{11u}W^o[:2u] + x^{9u}W^o[:5u]\\
&+ x^{10u}W^o[:4u] + x^{11u}W^o[:3u]\\
&+ (x^{7u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$
\begin{aligned}
T[:14u] = {}& T[:7u] + x^{3u}T[:4u] + x^{4u}T[:3u] + x^{5u}T[:2u] + x^{3u}T[:7u] + x^{6u}T[:4u]\\
&+ x^{7u}T[:3u] + x^{8u}T[:2u] + x^{4u}T[:7u] + x^{7u}T[:4u] + x^{8u}T[:3u]\\
&+ x^{9u}T[:2u] + x^{5u}T[:7u] + x^{8u}T[:4u] + x^{9u}T[:3u] + x^{10u}T[:2u]\\
&+ x^{6u}T[:7u] + x^{9u}T[:4u] + x^{10u}T[:3u] + x^{11u}T[:2u] + x^{9u}T[:5u]\\
&+ x^{10u}T[:4u] + x^{11u}T[:3u] + (x^{7u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}).
\end{aligned}
$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$
\begin{aligned}
x^7u &= x^{5u}T[2u:3u] + x^{4u}T[3u:4u] + x^{3u}T[4u:5u] + T[7u:8u],\\
0 &= x^{8u}T[:u] + x^{5u}T[3u:4u] + x^{4u}T[4u:5u] + x^{3u}T[5u:6u]
\end{aligned}
$$

$$
\begin{aligned}
&+ T[8u\!:\!9u],\\
0 &= x^{8u}T[u\!:\!2u] + x^{5u}T[4u\!:\!5u] + x^{4u}T[5u\!:\!6u] + x^{3u}T[6u\!:\!7u]\\
&\quad + T[9u\!:\!10u],\\
x^10u &= x^{10u}T[:u] + x^{7u}T[3u\!:\!4u] + x^{6u}T[4u\!:\!5u] + x^{5u}T[5u\!:\!6u]\\
&\quad + x^{4u}T[6u\!:\!7u] + T[10u\!:\!11u],\\
x^11u &= x^{10u}T[u\!:\!2u] + x^{9u}T[2u\!:\!3u] + x^{8u}T[3u\!:\!4u] + x^{6u}T[5u\!:\!6u]\\
&\quad + x^{5u}T[6u\!:\!7u] + T[11u\!:\!12u],\\
x^12u &= x^{6u}T[6u\!:\!7u] + T[12u\!:\!13u],\\
x^13u &= x^{11u}T[2u\!:\!3u] + x^{10u}T[3u\!:\!4u] + x^{9u}T[4u\!:\!5u] + T[13u\!:\!14u],
\end{aligned}
$$

which can be rewritten as

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.7}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.8}$$

$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.9}$$

$$\begin{aligned} 0 = {} & T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ & + T[[1010w]_2], \end{aligned} \tag{2.10}$$

$$\begin{aligned} 0 = {} & T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\ & + T[[1011w]_2], \end{aligned} \tag{2.11}$$

$$0 = T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$

$$\begin{aligned} 1 = {} & T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ & + T[[1010w]_2], \end{aligned} \tag{2.17}$$

$$\begin{aligned} 1 = {} & T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\ & + T[[1011w]_2], \end{aligned} \tag{2.18}$$

$$1 = T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$

$$1 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2015 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.21}$$

$$\begin{aligned}
0 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,1000)), && (2.22)\\
0 &= \tau(A(s,1)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1001)), && (2.23)\\
0 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1010)), && (2.24)\\
0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1011)), && (2.25)\\
0 &= \tau(A(s,110)) + \tau(A(s,1100)), && (2.26)\\
0 &= \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,1101)), && (2.27)
\end{aligned}$$

for all $s \in E_{2k}$, and

$$\begin{aligned}
1 &= \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,111)), && (2.28)\\
0 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,1000)), && (2.29)\\
0 &= \tau(A(s,1)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1001)), && (2.30)\\
1 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1010)), && (2.31)\\
1 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1011)), && (2.32)\\
1 &= \tau(A(s,110)) + \tau(A(s,1100)), && (2.33)\\
1 &= \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,1101)), && (2.34)
\end{aligned}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{44}), E_{44}) = (A(i, 0^{14}), E_{14})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 22$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:7u] + x^{3u} M^e[:4u] + x^{4u} M^e[:3u] + x^{5u} M^e[:2u] + x^{7u} R^e,$$
$$W_{2k} = W^e[:7u] + x^{3u} W^e[:4u] + x^{4u} W^e[:3u] + x^{5u} W^e[:2u] + x^{7u} R^e.$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:7u] + x^{3u} M^o[:4u] + x^{4u} M^o[:3u] + x^{5u} M^o[:2u] + x^{7u} R^o,$$
$$W_{2k+1} = W^o[:7u] + x^{3u} W^o[:4u] + x^{4u} W^o[:3u] + x^{5u} W^o[:2u] + x^{7u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:7v] + x^{3v}W^e[:4v] + x^{4v}W^e[:3v] + x^{5v}W^e[:2v] + x^{7u}R^e \\ &\big) \times \big(M^e[:7v] + x^{3v}M^e[:4v] + x^{4v}M^e[:3v] + x^{5v}M^e[:2v] + x^{7u}R^e \\ &\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} W^e[:14v]M^e[:14v] + x^{6v}W^e[:8v]M^e[:8v] + x^{8v}W^e[:6v]M^e[:6v] \\ + x^{10v}W^e[:4v]M^e[:4v]. \end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j, k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t \left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{20} + x^{19} + x^{15} + x^{13} + x^{12} + x^{10} + x^7,\\
q_1(x) &= x^{19} + x^{14} + x^{13} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^3 + x^2,\\
q_2(x) &= x^{20} + x^{18} + x^{14} + x^{10} + x^8 + x^6 + 1,\\
q_3(x) &= x^{19} + x^{14} + x^{13} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^3 + x^2,\\
q_4(x) &= x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^7 + x^6 + x^4.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{98} + x^{92} + x^{90} + x^{84} + x^{74} + x^{72} + x^{68} + x^{60} + x^{56} + x^{50}\\
&\quad + x^{46} + x^{42} + x^{40} + x^{24},\\
p_3(x) &= x^{100} + x^{92} + x^{86} + x^{84} + x^{82} + x^{74} + x^{70} + x^{66} + x^{64} + x^{58} + x^{54}\\
&\quad + x^{48} + x^{40} + x^{30} + x^{22} + x^{20} + x^{18} + x^{16},\\
p_6(x) &= x^{100} + x^{98} + x^{92} + x^{90} + x^{88} + x^{86} + x^{78} + x^{76} + x^{68} + x^{66} + x^{56}\\
&\quad + x^{54} + x^{50} + x^{44} + x^{40} + x^{38} + x^{36} + x^{32} + x^{26} + x^{24} + x^{14} + x^{12}\\
&\quad + x^8 + 1,\\
p_9(x) &= x^{100} + x^{96} + x^{86} + x^{84} + x^{80} + x^{78} + x^{74} + x^{68} + x^{62} + x^{56} + x^{46}\\
&\quad + x^{38} + x^{34} + x^{30} + x^{24} + x^{20} + x^{18} + x^{12} + x^6 + x^4,\\
p_{12}(x) &= x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{88} + x^{78} + x^{76} + x^{72} + x^{70} + x^{68}\\
&\quad + x^{64} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} + x^{40} + x^{30} + x^{28} + x^{24} + x^{14}\\
&\quad + x^{12} + x^8 + x^6 + x^4 + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(M^e,0,1)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{94}\\
&\quad + x^{93} + x^{88} + x^{86} + x^{85} + x^{84} + x^{83} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{51}\\
&+ x^{50} + x^{47} + x^{44} + x^{43} + x^{38} + x^{36} + x^{33},\\
p_3(x) &= x^{102} + x^{101} + x^{99} + x^{98} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79}\\
&+ x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{62}\\
&+ x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38}\\
&+ x^{32} + x^{31} + x^{29} + x^{28} + x^{20} + x^{19} + x^{18},\\
p_6(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91}\\
&+ x^{89} + x^{86} + x^{81} + x^{80} + x^{79} + x^{76} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68}\\
&+ x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} + x^{46}\\
&+ x^{44} + x^{41} + x^{40} + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{26}\\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12},\\
p_9(x) &= x^{108} + x^{107} + x^{105} + x^{104} + x^{98} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{86}\\
&+ x^{85} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{66}\\
&+ x^{65} + x^{63} + x^{61} + x^{59} + x^{57} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45}\\
&+ x^{44} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{18}\\
&+ x^{10} + x^{9} + x^{7} + x^{6},\\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{91}\\
&+ x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76}\\
&+ x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54}\\
&+ x^{52} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} + x^{27}\\
&+ x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12}\\
&+ x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0].

For $\phi(M^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{94}\\
&+ x^{93} + x^{88} + x^{86} + x^{85} + x^{84} + x^{83} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73}\\
&+ x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{51}\\
&+ x^{50} + x^{47} + x^{44} + x^{43} + x^{38} + x^{36} + x^{33},\\
p_3(x) &= x^{102} + x^{101} + x^{99} + x^{98} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79}\\
&+ x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{62}\\
&+ x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38}\\
&+ x^{32} + x^{31} + x^{29} + x^{28} + x^{20} + x^{19} + x^{18},\\
p_6(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91}\\
&+ x^{89} + x^{86} + x^{81} + x^{80} + x^{79} + x^{76} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68}\\
&+ x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} + x^{46}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{44} + x^{41} + x^{40} + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{26}\\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12},\\
p_9(x) &= x^{108} + x^{107} + x^{105} + x^{104} + x^{98} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{86}\\
&+ x^{85} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{66}\\
&+ x^{65} + x^{63} + x^{61} + x^{59} + x^{57} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45}\\
&+ x^{44} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{18}\\
&+ x^{10} + x^{9} + x^{7} + x^{6},\\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{91}\\
&+ x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76}\\
&+ x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54}\\
&+ x^{52} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} + x^{27}\\
&+ x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12}\\
&+ x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0].

For $\phi(M^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{118} + x^{116} + x^{114} + x^{110} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96}\\
&+ x^{94} + x^{92} + x^{90} + x^{88} + x^{84} + x^{82} + x^{78} + x^{76} + x^{70} + x^{66} + x^{64}\\
&+ x^{62} + x^{60} + x^{58} + x^{52} + x^{48} + x^{42},\\
p_3(x) &= x^{118} + x^{116} + x^{110} + x^{104} + x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{86}\\
&+ x^{82} + x^{80} + x^{76} + x^{74} + x^{72} + x^{66} + x^{64} + x^{62} + x^{60} + x^{54} + x^{52}\\
&+ x^{50} + x^{38} + x^{34} + x^{30} + x^{26} + x^{14} + x^{12},\\
p_6(x) &= x^{118} + x^{116} + x^{114} + x^{110} + x^{108} + x^{106} + x^{104} + x^{100} + x^{96} + x^{94}\\
&+ x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{76} + x^{70} + x^{62} + x^{58}\\
&+ x^{56} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44} + x^{38} + x^{26} + x^{24} + x^{22} + x^{20}\\
&+ x^{8} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{118} + x^{116} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{96} + x^{94}\\
&+ x^{90} + x^{88} + x^{82} + x^{80} + x^{78} + x^{74} + x^{66} + x^{62} + x^{52} + x^{46} + x^{42}\\
&+ x^{38} + x^{36} + x^{30} + x^{28} + x^{26} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12} + x^{8}\\
&+ x^{6} + x^{4},\\
p_{12}(x) &= x^{120} + x^{104} + x^{88} + x^{80} + x^{64} + x^{56} + x^{32} + x^{24} + x^{16} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{118} + x^{114} + x^{100} + x^{90} + x^{84} + x^{80} + x^{76} + x^{68} + x^{64} + x^{62}\\
&+ x^{54} + x^{52} + x^{50} + x^{44} + x^{38} + x^{30} + x^{28} + x^{24},\\
p_3(x) &= x^{118} + x^{116} + x^{114} + x^{106} + x^{98} + x^{96} + x^{92} + x^{88} + x^{86} + x^{84} + x^{82}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{74} + x^{68} + x^{54} + x^{52} + x^{46} + x^{42} + x^{40} + x^{36} + x^{26} + x^{24} + x^{20}\\
&+ x^{18} + x^{16},\\
p_6(x) &= x^{118} + x^{108} + x^{102} + x^{98} + x^{90} + x^{86} + x^{82} + x^{80} + x^{70} + x^{66} + x^{58}\\
&+ x^{54} + x^{52} + x^{48} + x^{44} + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28}\\
&+ x^{26} + x^{22} + x^{20} + x^{18} + x^{14} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{118} + x^{116} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{96} + x^{94}\\
&+ x^{90} + x^{88} + x^{82} + x^{80} + x^{78} + x^{74} + x^{66} + x^{62} + x^{52} + x^{46} + x^{42}\\
&+ x^{38} + x^{36} + x^{30} + x^{28} + x^{26} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12} + x^{8}\\
&+ x^{6} + x^{4},\\
p_{12}(x) &= x^{120} + x^{104} + x^{88} + x^{80} + x^{64} + x^{56} + x^{32} + x^{24} + x^{16} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{107} + x^{106} + x^{105} + x^{103} + x^{99} + x^{98} + x^{97} + x^{95}\\
&+ x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{85} + x^{81} + x^{80} + x^{78} + x^{77}\\
&+ x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{64} + x^{62} + x^{59} + x^{58} + x^{57}\\
&+ x^{55} + x^{53} + x^{51} + x^{47} + x^{42} + x^{41} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33},\\
p_3(x) &= x^{102} + x^{101} + x^{99} + x^{98} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79}\\
&+ x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{62}\\
&+ x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38}\\
&+ x^{32} + x^{31} + x^{29} + x^{28} + x^{20} + x^{19} + x^{18},\\
p_6(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91}\\
&+ x^{89} + x^{86} + x^{81} + x^{80} + x^{79} + x^{76} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68}\\
&+ x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} + x^{46}\\
&+ x^{44} + x^{41} + x^{40} + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{26}\\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12},\\
p_9(x) &= x^{108} + x^{107} + x^{105} + x^{104} + x^{98} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{86}\\
&+ x^{85} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{66}\\
&+ x^{65} + x^{63} + x^{61} + x^{59} + x^{57} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45}\\
&+ x^{44} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{18}\\
&+ x^{10} + x^{9} + x^{7} + x^{6},\\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{91}\\
&+ x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76}\\
&+ x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54}\\
&+ x^{52} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} + x^{27}\\
&+ x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12}
\end{aligned}
$$

$$+ x^3 + x^2 + 1,$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{107} + x^{106} + x^{105} + x^{103} + x^{99} + x^{98} + x^{97} + x^{95} \\
&\quad + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{85} + x^{81} + x^{80} + x^{78} + x^{77} \\
&\quad + x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{64} + x^{62} + x^{59} + x^{58} + x^{57} \\
&\quad + x^{55} + x^{53} + x^{51} + x^{47} + x^{42} + x^{41} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33}, \\
p_3(x) &= x^{102} + x^{101} + x^{99} + x^{98} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} \\
&\quad + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{62} \\
&\quad + x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} \\
&\quad + x^{32} + x^{31} + x^{29} + x^{28} + x^{20} + x^{19} + x^{18}, \\
p_6(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} \\
&\quad + x^{89} + x^{86} + x^{81} + x^{80} + x^{79} + x^{76} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} \\
&\quad + x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} + x^{46} \\
&\quad + x^{44} + x^{41} + x^{40} + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{26} \\
&\quad + x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12}, \\
p_9(x) &= x^{108} + x^{107} + x^{105} + x^{104} + x^{98} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{86} \\
&\quad + x^{85} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{66} \\
&\quad + x^{65} + x^{63} + x^{61} + x^{59} + x^{57} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45} \\
&\quad + x^{44} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{18} \\
&\quad + x^{10} + x^{9} + x^{7} + x^{6}, \\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{91} \\
&\quad + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76} \\
&\quad + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54} \\
&\quad + x^{52} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} + x^{27} \\
&\quad + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} \\
&\quad + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{80} + x^{64} + x^{60} + x^{58} + x^{52} \\
&\quad + x^{44} + x^{42}, \\
p_3(x) &= x^{100} + x^{96} + x^{94} + x^{86} + x^{80} + x^{76} + x^{68} + x^{66} + x^{64} + x^{58} + x^{56} \\
&\quad + x^{52} + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{26} + x^{24} + x^{22} + x^{16} \\
&\quad + x^{14} + x^{12}, \\
p_6(x) &= x^{100} + x^{94} + x^{92} + x^{86} + x^{66} + x^{64} + x^{58} + x^{56} + x^{48} + x^{44} + x^{42}
\end{aligned}$$

$$\begin{aligned}
&+ x^{40} + x^{38} + x^{34} + x^{28} + x^{26} + x^{22} + x^{20} + x^{14} + 1,\\
p_9(x) &= x^{100} + x^{96} + x^{86} + x^{84} + x^{80} + x^{78} + x^{74} + x^{68} + x^{62} + x^{56} + x^{46}\\
&\quad + x^{38} + x^{34} + x^{30} + x^{24} + x^{20} + x^{18} + x^{12} + x^{6} + x^{4},\\
p_{12}(x) &= x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{88} + x^{78} + x^{76} + x^{72} + x^{70} + x^{68}\\
&\quad + x^{64} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} + x^{40} + x^{30} + x^{28} + x^{24} + x^{14}\\
&\quad + x^{12} + x^{8} + x^{6} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{109} + x^{104} + x^{103} + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{82}\\
&\quad + x^{81} + x^{80} + x^{78} + x^{77} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{64} + x^{63}\\
&\quad + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} + x^{45} + x^{44} + x^{43} + x^{40}\\
&\quad + x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{27} + x^{26} + x^{24},\\
p_3(x) &= x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95}\\
&\quad + x^{94} + x^{93} + x^{91} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{72}\\
&\quad + x^{68} + x^{65} + x^{64} + x^{61} + x^{60} + x^{59} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47}\\
&\quad + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{31} + x^{28} + x^{27}\\
&\quad + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16},\\
p_6(x) &= x^{109} + x^{108} + x^{107} + x^{106} + x^{102} + x^{100} + x^{99} + x^{95} + x^{94} + x^{93}\\
&\quad + x^{92} + x^{83} + x^{79} + x^{77} + x^{74} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{67}\\
&\quad + x^{66} + x^{63} + x^{60} + x^{58} + x^{52} + x^{46} + x^{43} + x^{42} + x^{40} + x^{39} + x^{36}\\
&\quad + x^{34} + x^{33} + x^{31} + x^{28} + x^{26} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{14}\\
&\quad + x^{12} + x^{11} + x^{10} + x^{8} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} + x^{92} + x^{87}\\
&\quad + x^{86} + x^{84} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{59} + x^{58} + x^{56}\\
&\quad + x^{55} + x^{54} + x^{52} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30}\\
&\quad + x^{28} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82}\\
&\quad + x^{80} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{47}\\
&\quad + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28}\\
&\quad + x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{11} + x^{10} + x^{8} + x^{3} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{120} + x^{117} + x^{110} + x^{106} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99}\\
&\quad + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{83} + x^{82} + x^{79}\\
&\quad + x^{78} + x^{75} + x^{74} + x^{72} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{60} + x^{56} + x^{54} + x^{52} + x^{51} + x^{49} + x^{44} + x^{42} + x^{39} + x^{35} + x^{33},\\
p_3(x) &= x^{111} + x^{109} + x^{107} + x^{106} + x^{105} + x^{104} + x^{101} + x^{99} + x^{97} + x^{96}\\
&+ x^{94} + x^{91} + x^{90} + x^{89} + x^{87} + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76}\\
&+ x^{75} + x^{73} + x^{70} + x^{69} + x^{68} + x^{66} + x^{62} + x^{58} + x^{57} + x^{54} + x^{52}\\
&+ x^{50} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} + x^{42} + x^{40} + x^{36} + x^{35} + x^{32}\\
&+ x^{31} + x^{30} + x^{29} + x^{28} + x^{23} + x^{19} + x^{18},\\
p_6(x) &= x^{114} + x^{104} + x^{102} + x^{98} + x^{96} + x^{92} + x^{84} + x^{82} + x^{78} + x^{76} + x^{74}\\
&+ x^{72} + x^{58} + x^{50} + x^{48} + x^{34} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12},\\
p_9(x) &= x^{117} + x^{115} + x^{112} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{97} + x^{95}\\
&+ x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{83} + x^{81} + x^{80} + x^{79}\\
&+ x^{78} + x^{77} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{64} + x^{62}\\
&+ x^{61} + x^{60} + x^{59} + x^{58} + x^{49} + x^{47} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39}\\
&+ x^{38} + x^{33} + x^{31} + x^{28} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{19}\\
&+ x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_{12}(x) &= x^{120} + x^{116} + x^{108} + x^{104} + x^{96} + x^{84} + x^{76} + x^{72} + x^{68} + x^{64} + x^{60}\\
&+ x^{56} + x^{48} + x^{40} + x^{20} + x^{12} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{94} + x^{92} + x^{91}\\
&+ x^{89} + x^{87} + x^{85} + x^{84} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75}\\
&+ x^{73} + x^{72} + x^{68} + x^{65} + x^{64} + x^{62} + x^{61} + x^{57} + x^{54} + x^{53} + x^{50}\\
&+ x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{40} + x^{39} + x^{37} + x^{34}\\
&+ x^{33},\\
p_3(x) &= x^{99} + x^{97} + x^{94} + x^{92} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{82} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{63}\\
&+ x^{62} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43}\\
&+ x^{41} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27}\\
&+ x^{26} + x^{23} + x^{19} + x^{18},\\
p_6(x) &= x^{102} + x^{98} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{78} + x^{76} + x^{70} + x^{66}\\
&+ x^{60} + x^{58} + x^{56} + x^{52} + x^{44} + x^{38} + x^{36} + x^{34} + x^{30} + x^{24} + x^{20}\\
&+ x^{18} + x^{16} + x^{14} + x^{12},\\
p_9(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{93} + x^{91} + x^{88} + x^{86} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{77} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{57} + x^{55}\\
&+ x^{53} + x^{52} + x^{51} + x^{50} + x^{45} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35}\\
&+ x^{34} + x^{29} + x^{27} + x^{24} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{13} + x^{11}
\end{aligned}
$$

$$\begin{aligned}
&+ x^9 + x^8 + x^7 + x^6,\\
p_{12}(x) &= x^{108} + x^{92} + x^{88} + x^{80} + x^{72} + x^{64} + x^{60} + x^{44} + x^{40} + x^{32} + x^{28}\\
&+ x^{24} + x^{16} + x^8 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{104} + x^{100} + x^{98} + x^{97}\\
&+ x^{96} + x^{95} + x^{94} + x^{90} + x^{89} + x^{86} + x^{82} + x^{81} + x^{79} + x^{76} + x^{75}\\
&+ x^{74} + x^{72} + x^{69} + x^{67} + x^{65} + x^{58} + x^{55} + x^{54} + x^{52} + x^{51} + x^{50}\\
&+ x^{48} + x^{46} + x^{45} + x^{43} + x^{42},\\
p_3(x) &= x^{107} + x^{106} + x^{105} + x^{100} + x^{95} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{87}\\
&+ x^{86} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{66} + x^{62} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{45} + x^{42}\\
&+ x^{40} + x^{39} + x^{38} + x^{36} + x^{32} + x^{31} + x^{28} + x^{23} + x^{22},\\
p_6(x) &= x^{109} + x^{108} + x^{105} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{92} + x^{91}\\
&+ x^{89} + x^{86} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{71} + x^{70}\\
&+ x^{68} + x^{67} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54} + x^{51}\\
&+ x^{50} + x^{48} + x^{47} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34}\\
&+ x^{27} + x^{24} + x^{22} + x^{16} + x^{12} + x^3 + x^2 + 1,\\
p_9(x) &= x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} + x^{92} + x^{87}\\
&+ x^{86} + x^{84} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{59} + x^{58} + x^{56}\\
&+ x^{55} + x^{54} + x^{52} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30}\\
&+ x^{28} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^8,\\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82}\\
&+ x^{80} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{47}\\
&+ x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28}\\
&+ x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{11} + x^{10} + x^8 + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{110} + x^{109} + x^{104} + x^{103} + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{82}\\
&+ x^{81} + x^{80} + x^{78} + x^{77} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{64} + x^{63}\\
&+ x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} + x^{45} + x^{44} + x^{43} + x^{40}\\
&+ x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{27} + x^{26} + x^{24},\\
p_3(x) &= x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95}\\
&+ x^{94} + x^{93} + x^{91} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{72}\\
&+ x^{68} + x^{65} + x^{64} + x^{61} + x^{60} + x^{59} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{31} + x^{28} + x^{27} \\
&+ x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16}, \\
p_6(x) &= x^{109} + x^{108} + x^{107} + x^{106} + x^{102} + x^{100} + x^{99} + x^{95} + x^{94} + x^{93} \\
&+ x^{92} + x^{83} + x^{79} + x^{77} + x^{74} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{67} \\
&+ x^{66} + x^{63} + x^{60} + x^{58} + x^{52} + x^{46} + x^{43} + x^{42} + x^{40} + x^{39} + x^{36} \\
&+ x^{34} + x^{33} + x^{31} + x^{28} + x^{26} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{14} \\
&+ x^{12} + x^{11} + x^{10} + x^{8} + x^{3} + x^{2} + 1, \\
p_9(x) &= x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} + x^{92} + x^{87} \\
&+ x^{86} + x^{84} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{59} + x^{58} + x^{56} \\
&+ x^{55} + x^{54} + x^{52} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} \\
&+ x^{28} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^{8}, \\
p_{12}(x) &= x^{111} + x^{110} + x^{108} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82} \\
&+ x^{80} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{47} \\
&+ x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28} \\
&+ x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{11} + x^{10} + x^{8} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{94} + x^{92} + x^{91} \\
&+ x^{89} + x^{87} + x^{85} + x^{84} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} \\
&+ x^{73} + x^{72} + x^{68} + x^{65} + x^{64} + x^{62} + x^{61} + x^{57} + x^{54} + x^{53} + x^{50} \\
&+ x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{40} + x^{39} + x^{37} + x^{34} \\
&+ x^{33}, \\
p_3(x) &= x^{99} + x^{97} + x^{94} + x^{92} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{82} + x^{81} \\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{63} \\
&+ x^{62} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} \\
&+ x^{41} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} \\
&+ x^{26} + x^{23} + x^{19} + x^{18}, \\
p_6(x) &= x^{102} + x^{98} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{78} + x^{76} + x^{70} + x^{66} \\
&+ x^{60} + x^{58} + x^{56} + x^{52} + x^{44} + x^{38} + x^{36} + x^{34} + x^{30} + x^{24} + x^{20} \\
&+ x^{18} + x^{16} + x^{14} + x^{12}, \\
p_9(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{93} + x^{91} + x^{88} + x^{86} + x^{85} \\
&+ x^{84} + x^{83} + x^{82} + x^{77} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{57} + x^{55} \\
&+ x^{53} + x^{52} + x^{51} + x^{50} + x^{45} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} \\
&+ x^{34} + x^{29} + x^{27} + x^{24} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{13} + x^{11} \\
&+ x^{9} + x^{8} + x^{7} + x^{6},
\end{aligned}
$$

$$\begin{aligned}p_{12}(x) &= x^{108}+x^{92}+x^{88}+x^{80}+x^{72}+x^{64}+x^{60}+x^{44}+x^{40}+x^{32}+x^{28}\\&\quad+x^{24}+x^{16}+x^{8}+1,\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}p_0(x) &= x^{120}+x^{117}+x^{110}+x^{106}+x^{105}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}\\&\quad+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{89}+x^{88}+x^{87}+x^{83}+x^{82}+x^{79}\\&\quad+x^{78}+x^{75}+x^{74}+x^{72}+x^{71}+x^{69}+x^{67}+x^{66}+x^{65}+x^{62}+x^{61}\\&\quad+x^{60}+x^{56}+x^{54}+x^{52}+x^{51}+x^{49}+x^{44}+x^{42}+x^{39}+x^{35}+x^{33},\\p_3(x) &= x^{111}+x^{109}+x^{107}+x^{106}+x^{105}+x^{104}+x^{101}+x^{99}+x^{97}+x^{96}\\&\quad+x^{94}+x^{91}+x^{90}+x^{89}+x^{87}+x^{84}+x^{83}+x^{82}+x^{80}+x^{77}+x^{76}\\&\quad+x^{75}+x^{73}+x^{70}+x^{69}+x^{68}+x^{66}+x^{62}+x^{58}+x^{57}+x^{54}+x^{52}\\&\quad+x^{50}+x^{49}+x^{48}+x^{46}+x^{45}+x^{43}+x^{42}+x^{40}+x^{36}+x^{35}+x^{32}\\&\quad+x^{31}+x^{30}+x^{29}+x^{28}+x^{23}+x^{19}+x^{18},\\p_6(x) &= x^{114}+x^{104}+x^{102}+x^{98}+x^{96}+x^{92}+x^{84}+x^{82}+x^{78}+x^{76}+x^{74}\\&\quad+x^{72}+x^{58}+x^{50}+x^{48}+x^{34}+x^{24}+x^{22}+x^{18}+x^{16}+x^{14}+x^{12},\\p_9(x) &= x^{117}+x^{115}+x^{112}+x^{110}+x^{109}+x^{108}+x^{107}+x^{106}+x^{97}+x^{95}\\&\quad+x^{92}+x^{90}+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+x^{83}+x^{81}+x^{80}+x^{79}\\&\quad+x^{78}+x^{77}+x^{75}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{67}+x^{64}+x^{62}\\&\quad+x^{61}+x^{60}+x^{59}+x^{58}+x^{49}+x^{47}+x^{44}+x^{42}+x^{41}+x^{40}+x^{39}\\&\quad+x^{38}+x^{33}+x^{31}+x^{28}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{19}\\&\quad+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{11}+x^{9}+x^{8}+x^{7}+x^{6},\\p_{12}(x) &= x^{120}+x^{116}+x^{108}+x^{104}+x^{96}+x^{84}+x^{76}+x^{72}+x^{68}+x^{64}+x^{60}\\&\quad+x^{56}+x^{48}+x^{40}+x^{20}+x^{12}+1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}p_0(x) &= x^{111}+x^{110}+x^{109}+x^{107}+x^{106}+x^{105}+x^{104}+x^{100}+x^{98}+x^{97}\\&\quad+x^{96}+x^{95}+x^{94}+x^{90}+x^{89}+x^{86}+x^{82}+x^{81}+x^{79}+x^{76}+x^{75}\\&\quad+x^{74}+x^{72}+x^{69}+x^{67}+x^{65}+x^{58}+x^{55}+x^{54}+x^{52}+x^{51}+x^{50}\\&\quad+x^{48}+x^{46}+x^{45}+x^{43}+x^{42},\\p_3(x) &= x^{107}+x^{106}+x^{105}+x^{100}+x^{95}+x^{94}+x^{93}+x^{91}+x^{89}+x^{88}+x^{87}\\&\quad+x^{86}+x^{83}+x^{82}+x^{81}+x^{80}+x^{78}+x^{77}+x^{74}+x^{72}+x^{71}+x^{70}\\&\quad+x^{69}+x^{68}+x^{66}+x^{62}+x^{59}+x^{58}+x^{57}+x^{56}+x^{54}+x^{45}+x^{42}\\&\quad+x^{40}+x^{39}+x^{38}+x^{36}+x^{32}+x^{31}+x^{28}+x^{23}+x^{22},\\p_6(x) &= x^{109}+x^{108}+x^{105}+x^{101}+x^{99}+x^{98}+x^{97}+x^{96}+x^{94}+x^{92}+x^{91}\\&\quad+x^{89}+x^{86}+x^{82}+x^{81}+x^{79}+x^{78}+x^{77}+x^{75}+x^{72}+x^{71}+x^{70}\end{aligned}$$

$$
\begin{aligned}
&+ x^{68} + x^{67} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54} + x^{51} \\
&+ x^{50} + x^{48} + x^{47} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} \\
&+ x^{27} + x^{24} + x^{22} + x^{16} + x^{12} + x^{3} + x^{2} + 1, \\
p_9(x) = {}& x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} + x^{92} + x^{87} \\
&+ x^{86} + x^{84} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{59} + x^{58} + x^{56} \\
&+ x^{55} + x^{54} + x^{52} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} \\
&+ x^{28} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^{8}, \\
p_{12}(x) = {}& x^{111} + x^{110} + x^{108} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82} \\
&+ x^{80} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{47} \\
&+ x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28} \\
&+ x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{11} + x^{10} + x^{8} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	504	[885, 886]	1008	[1368, 1510]	1512	[1875, 1876]
1	[3, 4]	505	[887, 888]	1009	[488, 524]	1513	[1877, 114]
2	[5, 6]	506	[403, 889]	1010	[1511, 1512]	1514	[1844, 1878]
3	[7, 8]	507	[890, 891]	1011	[654, 237]	1515	[1031, 1879]
4	[9, 10]	508	[892, 893]	1012	[1513, 1514]	1516	[1083, 400]
5	[11, 12]	509	[894, 895]	1013	[1515, 1516]	1517	[432, 1722]
6	[13, 14]	510	[896, 864]	1014	[479, 1517]	1518	[1832, 1112]
7	[15, 16]	511	[897, 25]	1015	[1451, 1518]	1519	[405, 1794]
8	[17, 18]	512	[898, 899]	1016	[1519, 1384]	1520	[895, 1880]
9	[19, 20]	513	[900, 901]	1017	[510, 1520]	1521	[86, 912]
10	[21, 22]	514	[902, 903]	1018	[1521, 1522]	1522	[1881, 1074]
11	[23, 24]	515	[904, 905]	1019	[1507, 168]	1523	[322, 1882]
12	[25, 26]	516	[906, 254]	1020	[758, 1523]	1524	[1113, 994]
13	[27, 28]	517	[259, 907]	1021	[1524, 1525]	1525	[1115, 1883]
14	[29, 30]	518	[908, 909]	1022	[1526, 1527]	1526	[1029, 85]
15	[31, 32]	519	[910, 911]	1023	[1528, 130]	1527	[1884, 1885]
16	[33, 34]	520	[912, 913]	1024	[1462, 1529]	1528	[1073, 1886]
17	[35, 36]	521	[255, 914]	1025	[1530, 1531]	1529	[356, 1887]
18	[37, 38]	522	[915, 367]	1026	[1532, 1533]	1530	[1843, 1770]
19	[39, 40]	523	[916, 917]	1027	[36, 581]	1531	[1808, 1829]
20	[41, 42]	524	[113, 308]	1028	[1534, 230]	1532	[1822, 1725]
21	[43, 44]	525	[918, 919]	1029	[644, 1535]	1533	[1266, 1870]
22	[45, 46]	526	[920, 921]	1030	[1380, 1536]	1534	[74, 986]
23	[47, 48]	527	[922, 923]	1031	[1292, 1477]	1535	[1888, 324]
24	[49, 50]	528	[407, 924]	1032	[1537, 1482]	1536	[1889, 1106]

25	[51, 52]	529	[925, 926]	1033	[1503, 1494]	1537	[939, 1721]
26	[53, 54]	530	[927, 928]	1034	[190, 725]	1538	[1215, 1890]
27	[55, 56]	531	[929, 359]	1035	[1538, 1539]	1539	[1002, 1875]
28	[57, 58]	532	[930, 931]	1036	[1540, 793]	1540	[1891, 1172]
29	[59, 60]	533	[823, 932]	1037	[213, 1541]	1541	[1892, 925]
30	[61, 62]	534	[933, 934]	1038	[1542, 1543]	1542	[1018, 338]
31	[63, 64]	535	[427, 935]	1039	[1544, 216]	1543	[1227, 394]
32	[65, 66]	536	[936, 837]	1040	[519, 1545]	1544	[1780, 1730]
33	[67, 68]	537	[937, 824]	1041	[762, 1456]	1545	[909, 412]
34	[69, 70]	538	[938, 939]	1042	[1543, 205]	1546	[1126, 1008]
35	[71, 72]	539	[940, 941]	1043	[680, 1546]	1547	[274, 1731]
36	[73, 74]	540	[942, 827]	1044	[1547, 1548]	1548	[1036, 93]
37	[75, 76]	541	[364, 943]	1045	[52, 195]	1549	[1893, 1857]
38	[77, 78]	542	[944, 449]	1046	[483, 665]	1550	[1894, 1100]
39	[79, 80]	543	[888, 267]	1047	[1549, 1275]	1551	[1746, 1219]
40	[81, 82]	544	[945, 946]	1048	[676, 1396]	1552	[1883, 1714]
41	[83, 84]	545	[117, 947]	1049	[1550, 1551]	1553	[1223, 1850]
42	[85, 86]	546	[948, 949]	1050	[601, 1552]	1554	[1837, 1895]
43	[87, 88]	547	[354, 950]	1051	[754, 1553]	1555	[1896, 355]
44	[89, 90]	548	[951, 952]	1052	[1554, 179]	1556	[1897, 435]
45	[91, 92]	549	[826, 806]	1053	[1328, 1361]	1557	[1898, 1899]
46	[93, 94]	550	[818, 953]	1054	[1555, 759]	1558	[1261, 1759]
47	[95, 96]	551	[947, 954]	1055	[1556, 1557]	1559	[1900, 1860]
48	[97, 98]	552	[955, 956]	1056	[131, 520]	1560	[1880, 1810]
49	[99, 100]	553	[957, 103]	1057	[1558, 1559]	1561	[1792, 1896]
50	[101, 102]	554	[958, 959]	1058	[1560, 226]	1562	[1890, 1866]
51	[103, 104]	555	[960, 961]	1059	[1561, 549]	1563	[1753, 1884]
52	[90, 105]	556	[962, 812]	1060	[42, 1562]	1564	[1901, 277]
53	[106, 107]	557	[963, 964]	1061	[715, 1563]	1565	[1214, 1881]
54	[108, 109]	558	[377, 965]	1062	[1470, 1564]	1566	[1217, 1842]
55	[110, 111]	559	[297, 966]	1063	[746, 1565]	1567	[889, 454]
56	[112, 113]	560	[967, 955]	1064	[748, 1566]	1568	[247, 1224]
57	[114, 115]	561	[968, 969]	1065	[1567, 1568]	1569	[1840, 271]
58	[116, 117]	562	[970, 930]	1066	[1569, 1570]	1570	[1760, 1269]
59	[118, 119]	563	[971, 5]	1067	[1571, 1374]	1571	[1791, 1787]
60	[120, 121]	564	[972, 973]	1068	[737, 576]	1572	[1225, 1902]
61	[122, 123]	565	[974, 975]	1069	[744, 1310]	1573	[1903, 856]
62	[124, 125]	566	[976, 977]	1070	[1531, 1360]	1574	[1904, 1905]
63	[126, 127]	567	[350, 978]	1071	[1533, 1298]	1575	[1878, 1906]
64	[128, 129]	568	[979, 980]	1072	[470, 1378]	1576	[1192, 1815]
65	[130, 131]	569	[870, 981]	1073	[1276, 1572]	1577	[1046, 433]
66	[132, 133]	570	[982, 983]	1074	[1573, 589]	1578	[1140, 1082]
67	[134, 135]	571	[872, 984]	1075	[1574, 193]	1579	[1080, 1127]
68	[136, 137]	572	[804, 985]	1076	[1575, 1576]	1580	[1813, 1907]

69	[138, 139]	573	[986, 987]	1077	[674, 1327]	1581	[28, 1049]
70	[140, 141]	574	[874, 850]	1078	[1577, 1454]	1582	[1163, 1801]
71	[142, 143]	575	[450, 988]	1079	[1578, 1463]	1583	[935, 1783]
72	[144, 145]	576	[989, 990]	1080	[1345, 1579]	1584	[340, 1176]
73	[146, 147]	577	[991, 992]	1081	[159, 1580]	1585	[1773, 1061]
74	[148, 149]	578	[905, 993]	1082	[1487, 1581]	1586	[311, 1769]
75	[150, 151]	579	[994, 995]	1083	[1582, 160]	1587	[1090, 1114]
76	[152, 153]	580	[996, 972]	1084	[731, 1583]	1588	[1006, 869]
77	[154, 155]	581	[72, 997]	1085	[1584, 1585]	1589	[1158, 1908]
78	[156, 157]	582	[928, 998]	1086	[698, 784]	1590	[815, 1909]
79	[158, 159]	583	[24, 999]	1087	[161, 1586]	1591	[1797, 1752]
80	[160, 161]	584	[448, 808]	1088	[1525, 1587]	1592	[82, 1834]
81	[162, 163]	585	[1000, 1001]	1089	[1581, 1588]	1593	[309, 1851]
82	[164, 165]	586	[123, 1002]	1090	[1320, 1394]	1594	[1876, 1778]
83	[166, 167]	587	[1003, 1004]	1091	[1512, 1589]	1595	[1864, 1784]
84	[168, 169]	588	[1005, 1006]	1092	[1317, 1590]	1596	[413, 1790]
85	[170, 171]	589	[1007, 108]	1093	[1562, 1521]	1597	[1869, 1807]
86	[172, 173]	590	[1008, 1007]	1094	[613, 1591]	1598	[283, 300]
87	[174, 175]	591	[370, 1009]	1095	[242, 233]	1599	[335, 1183]
88	[176, 177]	592	[1010, 1011]	1096	[1510, 572]	1600	[1910, 1203]
89	[178, 41]	593	[1012, 330]	1097	[752, 557]	1601	[1782, 1911]
90	[179, 180]	594	[1013, 1014]	1098	[1483, 671]	1602	[1912, 1861]
91	[181, 182]	595	[1015, 1016]	1099	[1592, 1593]	1603	[1882, 1142]
92	[183, 184]	596	[1017, 948]	1100	[180, 1365]	1604	[1913, 1260]
93	[185, 186]	597	[868, 920]	1101	[1594, 1367]	1605	[395, 906]
94	[187, 188]	598	[323, 1018]	1102	[1389, 1369]	1606	[1902, 1025]
95	[189, 190]	599	[1019, 29]	1103	[1595, 1371]	1607	[1858, 262]
96	[191, 192]	600	[1020, 860]	1104	[217, 1596]	1608	[921, 4]
97	[193, 194]	601	[1021, 438]	1105	[1523, 516]	1609	[1045, 1914]
98	[195, 196]	602	[439, 1022]	1106	[1597, 1598]	1610	[66, 1915]
99	[197, 198]	603	[1023, 1024]	1107	[149, 1599]	1611	[1241, 1904]
100	[199, 200]	604	[1025, 1026]	1108	[719, 1600]	1612	[1895, 17]
101	[201, 202]	605	[16, 122]	1109	[540, 1601]	1613	[389, 1159]
102	[203, 204]	606	[18, 1027]	1110	[215, 1350]	1614	[1838, 1253]
103	[205, 206]	607	[1028, 1029]	1111	[1334, 1354]	1615	[1821, 1754]
104	[207, 208]	608	[444, 1030]	1112	[1602, 1495]	1616	[1887, 1916]
105	[209, 210]	609	[830, 1031]	1113	[1333, 1603]	1617	[1916, 1917]
106	[211, 212]	610	[1032, 803]	1114	[1375, 1604]	1618	[1775, 353]
107	[60, 213]	611	[1033, 951]	1115	[1423, 1605]	1619	[838, 1918]
108	[214, 215]	612	[1034, 1035]	1116	[1606, 1341]	1620	[1173, 1205]
109	[216, 217]	613	[964, 1036]	1117	[1607, 474]	1621	[1919, 1898]
110	[218, 219]	614	[1037, 1038]	1118	[1498, 543]	1622	[119, 1889]
111	[220, 221]	615	[1039, 87]	1119	[1608, 795]	1623	[879, 1056]
112	[222, 223]	616	[281, 1040]	1120	[767, 1542]	1624	[1917, 1920]

113	[224, 225]	617	[1041, 1042]	1121	[1593, 1609]	1625	[1921, 1862]
114	[226, 227]	618	[373, 1043]	1122	[619, 651]	1626	[997, 1922]
115	[228, 229]	619	[1044, 1045]	1123	[62, 655]	1627	[1128, 1923]
116	[139, 191]	620	[261, 1046]	1124	[1610, 1611]	1628	[899, 73]
117	[230, 231]	621	[805, 1047]	1125	[1612, 1613]	1629	[1055, 852]
118	[232, 33]	622	[411, 1048]	1126	[1326, 178]	1630	[1924, 933]
119	[233, 234]	623	[1049, 1050]	1127	[582, 594]	1631	[1899, 1912]
120	[235, 236]	624	[941, 19]	1128	[695, 579]	1632	[1906, 332]
121	[237, 238]	625	[1051, 1052]	1129	[1308, 1344]	1633	[886, 1925]
122	[32, 239]	626	[1053, 904]	1130	[1304, 1614]	1634	[1926, 1924]
123	[34, 240]	627	[1054, 1055]	1131	[1411, 1540]	1635	[455, 1927]
124	[241, 242]	628	[1056, 280]	1132	[1615, 1547]	1636	[1928, 1929]
125	[243, 9]	629	[1057, 908]	1133	[1616, 1273]	1637	[1766, 430]
126	[244, 245]	630	[1058, 1059]	1134	[1617, 1493]	1638	[318, 1831]
127	[246, 247]	631	[30, 1060]	1135	[1618, 1619]	1639	[1859, 81]
128	[248, 249]	632	[977, 341]	1136	[1517, 1620]	1640	[1873, 1724]
129	[250, 251]	633	[1061, 1062]	1137	[1621, 1556]	1641	[451, 1847]
130	[252, 253]	634	[1063, 1064]	1138	[1557, 1622]	1642	[287, 1872]
131	[254, 255]	635	[1065, 1066]	1139	[1623, 1416]	1643	[1816, 1930]
132	[256, 257]	636	[1067, 880]	1140	[1425, 1615]	1644	[358, 1931]
133	[258, 259]	637	[1068, 1069]	1141	[1506, 1624]	1645	[1932, 1740]
134	[260, 261]	638	[1070, 1071]	1142	[779, 1625]	1646	[1923, 77]
135	[262, 263]	639	[1072, 1073]	1143	[1383, 1626]	1647	[1103, 878]
136	[78, 264]	640	[1074, 1075]	1144	[206, 1627]	1648	[1817, 99]
137	[265, 266]	641	[1076, 1077]	1145	[1377, 1550]	1649	[1768, 1236]
138	[267, 268]	642	[932, 1078]	1146	[1628, 1390]	1650	[1856, 1767]
139	[269, 270]	643	[1079, 1080]	1147	[1629, 1567]	1651	[813, 1185]
140	[271, 272]	644	[425, 1051]	1148	[1630, 1631]	1652	[923, 1032]
141	[273, 274]	645	[387, 1081]	1149	[1449, 1355]	1653	[1761, 286]
142	[275, 276]	646	[1082, 1083]	1150	[177, 800]	1654	[303, 976]
143	[277, 278]	647	[1084, 1085]	1151	[1632, 1633]	1655	[1263, 1221]
144	[279, 256]	648	[1086, 1087]	1152	[240, 189]	1656	[393, 1037]
145	[280, 281]	649	[984, 1088]	1153	[1626, 1634]	1657	[1110, 1104]
146	[282, 283]	650	[104, 858]	1154	[1535, 776]	1658	[289, 408]
147	[284, 285]	651	[1043, 27]	1155	[1635, 1636]	1659	[1930, 1264]
148	[286, 287]	652	[1089, 1090]	1156	[1358, 1637]	1660	[1735, 326]
149	[288, 289]	653	[419, 1091]	1157	[1587, 1638]	1661	[919, 1833]
150	[290, 291]	654	[1092, 1093]	1158	[1491, 1639]	1662	[1026, 1070]
151	[292, 293]	655	[1094, 942]	1159	[1545, 1610]	1663	[327, 1231]
152	[294, 295]	656	[125, 1095]	1160	[1572, 1640]	1664	[1011, 1932]
153	[296, 297]	657	[92, 306]	1161	[1641, 612]	1665	[1095, 1926]
154	[298, 299]	658	[1096, 1097]	1162	[1485, 1497]	1666	[1229, 1824]
155	[300, 301]	659	[1098, 1099]	1163	[1642, 1643]	1667	[1143, 1848]
156	[302, 303]	660	[1100, 1101]	1164	[1644, 1645]	1668	[346, 938]

157	[304, 305]	661	[1102, 1103]	1165	[1614, 1618]	1669	[1135, 1900]
158	[306, 307]	662	[1104, 1105]	1166	[1440, 570]	1670	[1931, 23]
159	[308, 309]	663	[1106, 1107]	1167	[1646, 460]	1671	[1004, 1246]
160	[310, 311]	664	[1108, 1109]	1168	[623, 1412]	1672	[1749, 1901]
161	[312, 313]	665	[109, 1110]	1169	[1324, 607]	1673	[344, 282]
162	[314, 315]	666	[851, 1111]	1170	[1647, 498]	1674	[1905, 372]
163	[6, 316]	667	[1112, 1113]	1171	[1609, 1577]	1675	[1845, 1804]
164	[317, 318]	668	[1114, 1115]	1172	[713, 1513]	1676	[1062, 1897]
165	[319, 320]	669	[1116, 1117]	1173	[1349, 1616]	1677	[987, 940]
166	[321, 322]	670	[1118, 292]	1174	[1636, 1406]	1678	[950, 960]
167	[249, 323]	671	[1119, 1120]	1175	[1516, 1560]	1679	[1146, 996]
168	[324, 325]	672	[1121, 845]	1176	[1648, 1290]	1680	[1035, 1910]
169	[326, 327]	673	[1099, 900]	1177	[1565, 1509]	1681	[1911, 1820]
170	[328, 329]	674	[360, 894]	1178	[1637, 1649]	1682	[1174, 1812]
171	[330, 331]	675	[1122, 835]	1179	[1643, 1650]	1683	[1828, 1863]
172	[332, 333]	676	[836, 1123]	1180	[1651, 1652]	1684	[1212, 406]
173	[334, 335]	677	[966, 1124]	1181	[1414, 1653]	1685	[1059, 918]
174	[336, 337]	678	[1125, 378]	1182	[1654, 138]	1686	[1739, 290]
175	[338, 339]	679	[877, 1126]	1183	[1596, 472]	1687	[1836, 1933]
176	[313, 340]	680	[1127, 1128]	1184	[499, 1655]	1688	[1908, 1211]
177	[341, 342]	681	[993, 112]	1185	[1286, 1439]	1689	[1934, 1893]
178	[343, 344]	682	[1129, 1130]	1186	[1415, 1642]	1690	[959, 1913]
179	[345, 346]	683	[1131, 1023]	1187	[1656, 1537]	1691	[1886, 1065]
180	[347, 348]	684	[1132, 1072]	1188	[666, 1657]	1692	[980, 110]
181	[349, 350]	685	[1133, 1134]	1189	[1576, 1336]	1693	[1038, 1800]
182	[351, 352]	686	[362, 1135]	1190	[668, 1524]	1694	[1154, 1903]
183	[353, 354]	687	[1136, 862]	1191	[1657, 1342]	1695	[1855, 314]
184	[355, 356]	688	[859, 974]	1192	[1658, 1312]	1696	[1867, 1732]
185	[357, 358]	689	[954, 1137]	1193	[1649, 641]	1697	[1918, 252]
186	[359, 360]	690	[1138, 1139]	1194	[1331, 633]	1698	[913, 885]
187	[361, 362]	691	[926, 1140]	1195	[1659, 712]	1699	[1077, 841]
188	[363, 364]	692	[331, 376]	1196	[1435, 1319]	1700	[1207, 927]
189	[365, 366]	693	[1088, 1141]	1197	[1311, 1660]	1701	[102, 250]
190	[367, 368]	694	[1142, 1143]	1198	[1661, 1662]	1702	[1914, 958]
191	[369, 370]	695	[1130, 1144]	1199	[1663, 1335]	1703	[1907, 1737]
192	[371, 294]	696	[1145, 1146]	1200	[717, 462]	1704	[1935, 136]
193	[372, 373]	697	[1147, 1148]	1201	[1460, 740]	1705	[1627, 53]
194	[374, 375]	698	[1149, 1150]	1202	[659, 1664]	1706	[771, 627]
195	[376, 83]	699	[1151, 1152]	1203	[1665, 555]	1707	[1699, 1694]
196	[315, 377]	700	[299, 447]	1204	[1666, 574]	1708	[1936, 1937]
197	[378, 379]	701	[1153, 1154]	1205	[10, 1617]	1709	[1938, 1646]
198	[380, 381]	702	[1155, 1156]	1206	[1402, 1667]	1710	[1297, 693]
199	[382, 383]	703	[1157, 1158]	1207	[1668, 1669]	1711	[196, 1339]
200	[384, 385]	704	[1159, 245]	1208	[733, 1670]	1712	[1443, 592]

201	[386, 387]	705	[1148, 1160]	1209	[513, 1628]	1713	[1570, 140]
202	[388, 389]	706	[401, 1161]	1210	[1671, 1672]	1714	[38, 1939]
203	[390, 391]	707	[1162, 1163]	1211	[210, 1673]	1715	[1940, 1555]
204	[4, 392]	708	[1164, 1165]	1212	[683, 1332]	1716	[467, 1346]
205	[253, 393]	709	[1166, 1167]	1213	[1611, 1674]	1717	[1941, 587]
206	[394, 395]	710	[1168, 916]	1214	[1492, 1675]	1718	[546, 551]
207	[396, 334]	711	[1169, 1170]	1215	[1676, 1677]	1719	[1289, 1942]
208	[397, 398]	712	[1171, 260]	1216	[548, 1678]	1720	[1943, 1654]
209	[399, 347]	713	[1172, 1173]	1217	[1679, 1680]	1721	[1299, 1348]
210	[400, 401]	714	[985, 897]	1218	[1489, 1468]	1722	[741, 1379]
211	[402, 403]	715	[1064, 1174]	1219	[192, 1681]	1723	[491, 1461]
212	[404, 405]	716	[1175, 374]	1220	[1551, 1471]	1724	[1691, 1944]
213	[406, 407]	717	[1176, 814]	1221	[628, 144]	1725	[1945, 1595]
214	[408, 409]	718	[1177, 1178]	1222	[696, 507]	1726	[1946, 773]
215	[410, 411]	719	[1179, 1180]	1223	[1682, 532]	1727	[1399, 1936]
216	[412, 413]	720	[917, 1181]	1224	[537, 1554]	1728	[686, 1668]
217	[414, 415]	721	[1182, 1108]	1225	[463, 1651]	1729	[610, 51]
218	[416, 417]	722	[1183, 1184]	1226	[1683, 635]	1730	[223, 643]
219	[418, 419]	723	[1185, 1186]	1227	[1473, 710]	1731	[1645, 170]
220	[420, 421]	724	[1187, 288]	1228	[664, 772]	1732	[1464, 1947]
221	[422, 414]	725	[366, 1188]	1229	[1684, 185]	1733	[1664, 1948]
222	[375, 423]	726	[865, 1189]	1230	[1518, 1431]	1734	[1287, 599]
223	[424, 425]	727	[1190, 1191]	1231	[1681, 1685]	1735	[1598, 1949]
224	[426, 427]	728	[1050, 1192]	1232	[1314, 1686]	1736	[517, 766]
225	[428, 429]	729	[1193, 1194]	1233	[1450, 1316]	1737	[1950, 1951]
226	[430, 79]	730	[1195, 1119]	1234	[1687, 1530]	1738	[1677, 538]
227	[431, 432]	731	[934, 1196]	1235	[1662, 637]	1739	[1952, 1504]
228	[433, 365]	732	[1197, 831]	1236	[1688, 1689]	1740	[1953, 1954]
229	[415, 434]	733	[1198, 1199]	1237	[1690, 1472]	1741	[2, 162]
230	[435, 65]	734	[825, 1200]	1238	[1600, 1648]	1742	[688, 197]
231	[436, 437]	735	[811, 1201]	1239	[624, 1691]	1743	[556, 1955]
232	[438, 439]	736	[1202, 1147]	1240	[1499, 749]	1744	[1272, 1679]
233	[440, 441]	737	[1048, 898]	1241	[578, 1692]	1745	[1438, 1666]
234	[442, 397]	738	[1203, 1204]	1242	[781, 1644]	1746	[1599, 1658]
235	[443, 444]	739	[1205, 1133]	1243	[1459, 481]	1747	[1956, 645]
236	[445, 446]	740	[1109, 328]	1244	[615, 1693]	1748	[1951, 660]
237	[447, 448]	741	[1206, 1207]	1245	[617, 1429]	1749	[1949, 622]
238	[449, 450]	742	[1009, 1208]	1246	[797, 687]	1750	[1957, 1663]
239	[64, 451]	743	[1209, 1210]	1247	[1694, 478]	1751	[1689, 458]
240	[68, 452]	744	[1211, 1212]	1248	[1680, 505]	1752	[786, 1687]
241	[453, 440]	745	[1213, 1214]	1249	[591, 1578]	1753	[40, 1659]
242	[454, 455]	746	[264, 1215]	1250	[1366, 1695]	1754	[1391, 702]
243	[417, 67]	747	[1216, 1217]	1251	[1613, 745]	1755	[531, 1534]
244	[456, 457]	748	[1141, 1218]	1252	[1696, 1597]	1756	[1639, 1958]

245	[458, 459]	749	[1219, 1220]	1253	[621, 1479]	1757	[562, 1528]
246	[460, 461]	750	[1221, 71]	1254	[1465, 1697]	1758	[1640, 1474]
247	[462, 463]	751	[263, 1076]	1255	[521, 1698]	1759	[1959, 565]
248	[464, 465]	752	[978, 1222]	1256	[701, 1699]	1760	[1650, 699]
249	[466, 467]	753	[1024, 1223]	1257	[1527, 1612]	1761	[1944, 1281]
250	[468, 469]	754	[1224, 1225]	1258	[1700, 534]	1762	[1591, 1433]
251	[204, 470]	755	[1226, 1227]	1259	[792, 1353]	1763	[1372, 1571]
252	[163, 471]	756	[1228, 1229]	1260	[471, 241]	1764	[1960, 1690]
253	[472, 473]	757	[1220, 892]	1261	[652, 1387]	1765	[523, 764]
254	[474, 475]	758	[1230, 887]	1262	[1653, 1538]	1766	[1467, 1321]
255	[151, 203]	759	[999, 819]	1263	[1548, 1395]	1767	[1961, 1688]
256	[476, 477]	760	[1231, 404]	1264	[1466, 1701]	1768	[1453, 1671]
257	[478, 479]	761	[1232, 1233]	1265	[1522, 639]	1769	[573, 714]
258	[50, 480]	762	[1200, 246]	1266	[1667, 1635]	1770	[1693, 235]
259	[481, 482]	763	[84, 1234]	1267	[1702, 553]	1771	[1962, 1963]
260	[54, 483]	764	[452, 1235]	1268	[1501, 8]	1772	[1514, 1964]
261	[484, 485]	765	[1069, 312]	1269	[1703, 1641]	1773	[1965, 13]
262	[486, 487]	766	[1236, 363]	1270	[1704, 67]	1774	[783, 780]
263	[487, 488]	767	[1237, 1238]	1271	[1117, 1705]	1775	[1966, 647]
264	[489, 490]	768	[320, 1239]	1272	[1706, 319]	1776	[634, 1676]
265	[225, 491]	769	[1240, 1241]	1273	[1707, 1708]	1777	[1967, 1277]
266	[492, 493]	770	[1137, 1054]	1274	[1709, 1193]	1778	[1968, 1621]
267	[494, 495]	771	[1093, 1242]	1275	[911, 1710]	1779	[1969, 568]
268	[496, 497]	772	[1243, 258]	1276	[385, 871]	1780	[1697, 718]
269	[498, 499]	773	[339, 1122]	1277	[844, 1058]	1781	[1427, 57]
270	[500, 501]	774	[1244, 1245]	1278	[1711, 1164]	1782	[238, 584]
271	[502, 166]	775	[1105, 937]	1279	[1120, 1712]	1783	[760, 466]
272	[503, 504]	776	[1246, 979]	1280	[1713, 361]	1784	[1337, 1970]
273	[505, 506]	777	[1247, 1248]	1281	[1714, 1715]	1785	[1948, 464]
274	[507, 508]	778	[820, 1249]	1282	[1242, 1716]	1786	[1580, 1480]
275	[509, 510]	779	[842, 1250]	1283	[434, 1717]	1787	[1351, 561]
276	[145, 511]	780	[1251, 1033]	1284	[1257, 989]	1788	[459, 1965]
277	[512, 513]	781	[1252, 816]	1285	[295, 1718]	1789	[1579, 1966]
278	[514, 515]	782	[1253, 1125]	1286	[1719, 1720]	1790	[736, 1953]
279	[516, 517]	783	[953, 1254]	1287	[1721, 1020]	1791	[1420, 559]
280	[518, 519]	784	[1255, 1256]	1288	[907, 1089]	1792	[200, 1488]
281	[520, 521]	785	[1150, 866]	1289	[1001, 1098]	1793	[1971, 1972]
282	[522, 523]	786	[995, 1195]	1290	[1722, 396]	1794	[1973, 1573]
283	[524, 525]	787	[952, 1015]	1291	[1723, 1711]	1795	[726, 1974]
284	[526, 527]	788	[381, 1257]	1292	[1180, 890]	1796	[1937, 1975]
285	[528, 529]	789	[1258, 431]	1293	[1724, 1213]	1797	[129, 211]
286	[530, 531]	790	[924, 1259]	1294	[1087, 443]	1798	[1633, 172]
287	[532, 533]	791	[1260, 1261]	1295	[1720, 386]	1799	[1536, 1561]
288	[534, 535]	792	[1107, 1262]	1296	[96, 1034]	1800	[1541, 528]

289	[536, 537]	793	[846, 1263]	1297	[98, 873]	1801	[1603, 1938]
290	[538, 539]	794	[1264, 296]	1298	[1725, 1726]	1802	[169, 214]
291	[239, 540]	795	[1235, 1265]	1299	[1256, 1727]	1803	[1963, 1976]
292	[541, 542]	796	[293, 1266]	1300	[903, 1728]	1804	[1364, 1943]
293	[543, 544]	797	[245, 902]	1301	[1729, 75]	1805	[1947, 1400]
294	[545, 546]	798	[1267, 298]	1302	[1238, 1240]	1806	[1977, 755]
295	[547, 548]	799	[832, 1197]	1303	[875, 418]	1807	[1978, 727]
296	[549, 550]	800	[983, 1268]	1304	[266, 1723]	1808	[198, 1442]
297	[544, 457]	801	[1269, 1182]	1305	[1097, 833]	1809	[1655, 1945]
298	[551, 552]	802	[1270, 232]	1306	[1265, 349]	1810	[1964, 1950]
299	[553, 554]	803	[1271, 1272]	1307	[1730, 351]	1811	[506, 1629]
300	[555, 556]	804	[1273, 1274]	1308	[1731, 116]	1812	[1566, 1469]
301	[557, 558]	805	[1275, 1276]	1309	[1732, 1733]	1813	[709, 476]
302	[559, 560]	806	[1277, 1278]	1310	[1734, 1735]	1814	[1362, 1959]
303	[561, 562]	807	[1279, 1280]	1311	[1736, 1187]	1815	[1955, 1979]
304	[563, 564]	808	[790, 35]	1312	[1737, 445]	1816	[1660, 1455]
305	[565, 566]	809	[730, 689]	1313	[1738, 1739]	1817	[1980, 199]
306	[182, 567]	810	[1281, 1282]	1314	[1740, 1741]	1818	[1605, 1606]
307	[184, 547]	811	[1283, 1284]	1315	[111, 847]	1819	[1458, 1294]
308	[568, 569]	812	[1285, 1286]	1316	[305, 1738]	1820	[1672, 1607]
309	[570, 571]	813	[1282, 1287]	1317	[1268, 1742]	1821	[1601, 1330]
310	[572, 573]	814	[1288, 1289]	1318	[1060, 1092]	1822	[175, 1946]
311	[574, 575]	815	[1290, 49]	1319	[301, 1743]	1823	[554, 1515]
312	[576, 512]	816	[1291, 1292]	1320	[1744, 1267]	1824	[1564, 761]
313	[577, 578]	817	[234, 1279]	1321	[822, 1745]	1825	[1413, 1967]
314	[579, 580]	818	[508, 1293]	1322	[270, 1094]	1826	[1430, 1702]
315	[581, 582]	819	[1294, 1295]	1323	[840, 91]	1827	[1583, 1981]
316	[12, 583]	820	[1296, 1297]	1324	[1733, 1145]	1828	[1481, 1665]
317	[584, 585]	821	[1298, 1299]	1325	[1262, 1746]	1829	[1673, 45]
318	[477, 586]	822	[504, 1300]	1326	[115, 1747]	1830	[597, 763]
319	[587, 588]	823	[480, 484]	1327	[1259, 896]	1831	[1559, 1602]
320	[589, 494]	824	[1301, 1302]	1328	[291, 1748]	1832	[586, 1323]
321	[590, 591]	825	[1303, 492]	1329	[1186, 1162]	1833	[1669, 1558]
322	[592, 593]	826	[704, 1304]	1330	[1743, 1749]	1834	[1322, 798]
323	[594, 595]	827	[1305, 729]	1331	[421, 1063]	1835	[1309, 1956]
324	[596, 597]	828	[1306, 1307]	1332	[272, 345]	1836	[1982, 1305]
325	[598, 187]	829	[662, 1308]	1333	[848, 1166]	1837	[1983, 1984]
326	[599, 600]	830	[147, 1309]	1334	[1748, 69]	1838	[1686, 1490]
327	[558, 601]	831	[1310, 1311]	1335	[1750, 1198]	1839	[1553, 1985]
328	[529, 602]	832	[1312, 577]	1336	[325, 944]	1840	[1698, 503]
329	[603, 604]	833	[1313, 1314]	1337	[1751, 1121]	1841	[1675, 667]
330	[605, 606]	834	[525, 1315]	1338	[1752, 1753]	1842	[1678, 787]
331	[607, 608]	835	[1316, 1317]	1339	[931, 991]	1843	[1624, 1962]
332	[202, 148]	836	[1318, 653]	1340	[1178, 89]	1844	[1986, 1388]

333	[609, 610]	837	[1319, 1320]	1341	[1144, 854]	1845	[1586, 47]
334	[186, 611]	838	[1321, 1322]	1342	[1111, 410]	1846	[1987, 1647]
335	[612, 613]	839	[1307, 181]	1343	[1754, 388]	1847	[1278, 1526]
336	[614, 615]	840	[1323, 1324]	1344	[1160, 1755]	1848	[1625, 1447]
337	[616, 617]	841	[1325, 1326]	1345	[1756, 1053]	1849	[1958, 631]
338	[618, 619]	842	[1327, 509]	1346	[1184, 336]	1850	[1529, 1569]
339	[620, 621]	843	[501, 684]	1347	[1757, 1758]	1851	[1620, 738]
340	[622, 623]	844	[229, 1328]	1348	[1759, 1760]	1852	[1984, 1592]
341	[533, 624]	845	[703, 1329]	1349	[1727, 1707]	1853	[1428, 716]
342	[625, 626]	846	[801, 522]	1350	[409, 1084]	1854	[588, 1484]
343	[627, 628]	847	[1330, 39]	1351	[1245, 970]	1855	[1590, 1700]
344	[495, 629]	848	[221, 1331]	1352	[992, 1761]	1856	[1385, 1969]
345	[608, 630]	849	[1332, 1333]	1353	[1762, 1132]	1857	[560, 675]
346	[631, 632]	850	[595, 1334]	1354	[1016, 1230]	1858	[775, 1575]
347	[633, 634]	851	[1335, 732]	1355	[1763, 101]	1859	[1954, 1983]
348	[635, 636]	852	[1336, 1337]	1356	[1764, 1713]	1860	[1619, 183]
349	[637, 638]	853	[1300, 1338]	1357	[1765, 1766]	1861	[1568, 753]
350	[639, 640]	854	[1339, 1340]	1358	[1767, 1768]	1862	[1638, 1445]
351	[641, 642]	855	[1341, 1303]	1359	[1769, 1719]	1863	[1988, 1971]
352	[643, 644]	856	[1342, 518]	1360	[1770, 1771]	1864	[1539, 1511]
353	[645, 646]	857	[1343, 201]	1361	[1170, 1772]	1865	[1397, 1684]
354	[647, 648]	858	[1344, 1345]	1362	[1014, 1773]	1866	[564, 136]
355	[649, 650]	859	[1346, 1347]	1363	[969, 310]	1867	[1401, 1630]
356	[651, 652]	860	[1348, 1349]	1364	[817, 1129]	1868	[1974, 1978]
357	[653, 654]	861	[1350, 1351]	1365	[1774, 1775]	1869	[769, 1544]
358	[655, 656]	862	[1302, 1288]	1366	[348, 1776]	1870	[1701, 128]
359	[46, 657]	863	[569, 1352]	1367	[1726, 1777]	1871	[1356, 207]
360	[636, 658]	864	[1353, 1325]	1368	[1778, 1252]	1872	[1631, 1961]
361	[527, 659]	865	[1354, 757]	1369	[1779, 1780]	1873	[1476, 1502]
362	[660, 661]	866	[1355, 1356]	1370	[1781, 1782]	1874	[1585, 1532]
363	[515, 662]	867	[1357, 1358]	1371	[1783, 1013]	1875	[1975, 1968]
364	[663, 664]	868	[642, 1359]	1372	[1784, 1010]	1876	[1505, 1696]
365	[665, 666]	869	[1360, 747]	1373	[1785, 317]	1877	[1670, 673]
366	[667, 668]	870	[1361, 1362]	1374	[1786, 420]	1878	[133, 468]
367	[669, 670]	871	[640, 1363]	1375	[1787, 302]	1879	[774, 677]
368	[671, 672]	872	[580, 778]	1376	[1708, 1247]	1880	[1393, 1301]
369	[673, 674]	873	[194, 618]	1377	[1755, 371]	1881	[1674, 1574]
370	[675, 676]	874	[493, 705]	1378	[1788, 1789]	1882	[1295, 1957]
371	[677, 678]	875	[721, 1364]	1379	[1790, 120]	1883	[1604, 156]
372	[679, 680]	876	[1365, 1366]	1380	[1742, 1791]	1884	[167, 1952]
373	[681, 55]	877	[1367, 1368]	1381	[329, 380]	1885	[1496, 1508]
374	[682, 683]	878	[1369, 1370]	1382	[423, 1792]	1886	[1989, 1283]
375	[684, 174]	879	[1371, 1372]	1383	[1793, 962]	1887	[212, 1988]
376	[606, 61]	880	[1373, 164]	1384	[956, 369]	1888	[796, 1306]

377	[685, 686]	881	[1374, 1375]	1385	[1794, 1795]	1889	[1939, 1486]
378	[687, 688]	882	[1376, 1377]	1386	[1796, 1151]	1890	[1942, 1608]
379	[689, 690]	883	[1378, 663]	1387	[807, 1797]	1891	[788, 1980]
380	[602, 691]	884	[1379, 1380]	1388	[1254, 1779]	1892	[1500, 1426]
381	[692, 649]	885	[1381, 692]	1389	[121, 1781]	1893	[1990, 1982]
382	[693, 694]	886	[1382, 1383]	1390	[867, 1155]	1894	[1280, 1444]
383	[141, 695]	887	[1384, 1385]	1391	[990, 1157]	1895	[1588, 218]
384	[236, 696]	888	[1386, 625]	1392	[943, 1729]	1896	[1985, 1318]
385	[697, 685]	889	[794, 1357]	1393	[1798, 1153]	1897	[1563, 1682]
386	[490, 698]	890	[1387, 1373]	1394	[105, 1799]	1898	[153, 1285]
387	[699, 700]	891	[1388, 1389]	1395	[1800, 284]	1899	[1991, 1960]
388	[219, 701]	892	[600, 603]	1396	[1801, 1802]	1900	[661, 1987]
389	[702, 703]	893	[1390, 1391]	1397	[1123, 357]	1901	[1552, 1683]
390	[137, 704]	894	[1392, 1393]	1398	[1152, 1798]	1902	[756, 243]
391	[705, 706]	895	[658, 500]	1399	[1091, 1803]	1903	[1634, 1986]
392	[8, 605]	896	[1394, 734]	1400	[1712, 321]	1904	[571, 1584]
393	[707, 590]	897	[1274, 1376]	1401	[379, 118]	1905	[1692, 681]
394	[708, 709]	898	[1395, 146]	1402	[1804, 442]	1906	[1652, 609]
395	[710, 711]	899	[1396, 1397]	1403	[1805, 1751]	1907	[1340, 1632]
396	[712, 713]	900	[1398, 1399]	1404	[884, 1003]	1908	[1976, 751]
397	[714, 715]	901	[1400, 502]	1405	[1806, 424]	1909	[575, 1990]
398	[716, 717]	902	[457, 1401]	1406	[1807, 1808]	1910	[1685, 1623]
399	[718, 708]	903	[694, 1402]	1407	[949, 1809]	1911	[1546, 1656]
400	[482, 719]	904	[1403, 1404]	1408	[1810, 279]	1912	[1972, 1977]
401	[720, 721]	905	[1405, 222]	1409	[1811, 1202]	1913	[1520, 1992]
402	[650, 722]	906	[1406, 1407]	1410	[1747, 453]	1914	[1329, 1457]
403	[473, 723]	907	[469, 1408]	1411	[1812, 1813]	1915	[1370, 1703]
404	[58, 724]	908	[1409, 735]	1412	[853, 1793]	1916	[1992, 1973]
405	[725, 726]	909	[585, 768]	1413	[834, 1232]	1917	[1981, 1421]
406	[727, 728]	910	[1410, 1411]	1414	[981, 1814]	1918	[1408, 614]
407	[729, 730]	911	[777, 545]	1415	[1815, 1816]	1919	[1407, 1991]
408	[535, 731]	912	[171, 1381]	1416	[861, 1817]	1920	[724, 742]
409	[461, 679]	913	[1412, 1382]	1417	[1027, 1818]	1921	[1589, 1594]
410	[732, 733]	914	[475, 1413]	1418	[1819, 1243]	1922	[1695, 1519]
411	[734, 142]	915	[700, 1398]	1419	[1820, 273]	1923	[465, 1940]
412	[735, 736]	916	[1352, 1414]	1420	[1078, 967]	1924	[690, 1392]
413	[485, 737]	917	[173, 1415]	1421	[20, 106]	1925	[604, 1661]
414	[738, 739]	918	[1416, 1417]	1422	[1717, 1706]	1926	[1419, 489]
415	[552, 2]	919	[1418, 791]	1423	[1191, 1021]	1927	[550, 1549]
416	[740, 741]	920	[678, 1419]	1424	[893, 1821]	1928	[629, 1403]
417	[742, 541]	921	[1359, 1420]	1425	[1210, 1785]	1929	[1622, 59]
418	[188, 743]	922	[1421, 536]	1426	[278, 1209]	1930	[1979, 1418]
419	[583, 744]	923	[1422, 1271]	1427	[1799, 1756]	1931	[1970, 1993]
420	[745, 746]	924	[728, 1423]	1428	[88, 1822]	1932	[743, 1941]

421	[747, 748]	925	[1424, 1425]	1429	[1040, 1255]	1933	[227, 1582]
422	[749, 750]	926	[1426, 596]	1430	[1042, 957]	1934	[1993, 1475]
423	[751, 752]	927	[542, 770]	1431	[1823, 1057]	1935	[1994, 265]
424	[753, 754]	928	[1417, 1427]	1432	[1824, 21]	1936	[1139, 1877]
425	[755, 756]	929	[44, 1428]	1433	[891, 1825]	1937	[1803, 1921]
426	[757, 758]	930	[1429, 1430]	1434	[398, 1136]	1938	[1222, 1116]
427	[759, 760]	931	[1431, 1432]	1435	[1826, 1744]	1939	[1071, 1019]
428	[761, 762]	932	[1433, 1434]	1436	[914, 1067]	1940	[1181, 1734]
429	[763, 526]	933	[155, 1435]	1437	[1022, 968]	1941	[1885, 1005]
430	[764, 720]	934	[657, 1436]	1438	[883, 1805]	1942	[352, 1995]
431	[765, 134]	935	[1437, 1409]	1439	[1718, 1216]	1943	[1789, 1175]
432	[766, 767]	936	[1436, 158]	1440	[1827, 1828]	1944	[1874, 124]
433	[768, 769]	937	[611, 11]	1441	[1716, 963]	1945	[1052, 1709]
434	[770, 771]	938	[632, 1438]	1442	[946, 1786]	1946	[337, 1244]
435	[772, 132]	939	[630, 1437]	1443	[392, 1012]	1947	[1841, 1868]
436	[773, 774]	940	[1439, 1440]	1444	[1829, 1830]	1948	[1199, 390]
437	[706, 775]	941	[1441, 209]	1445	[1771, 1068]	1949	[1927, 1171]
438	[776, 777]	942	[165, 598]	1446	[1831, 1832]	1950	[1161, 1891]
439	[778, 779]	943	[670, 150]	1447	[1250, 275]	1951	[1839, 1096]
440	[780, 781]	944	[1442, 1443]	1448	[857, 1763]	1952	[1705, 1996]
441	[782, 154]	945	[1444, 669]	1449	[1066, 1764]	1953	[1201, 1997]
442	[656, 783]	946	[1445, 1446]	1450	[1081, 915]	1954	[1204, 1765]
443	[784, 785]	947	[1447, 1291]	1451	[1833, 1823]	1955	[961, 1819]
444	[14, 786]	948	[511, 616]	1452	[1834, 843]	1956	[1998, 1774]
445	[787, 788]	949	[1448, 1449]	1453	[1835, 1836]	1957	[1925, 1750]
446	[539, 789]	950	[646, 1450]	1454	[368, 829]	1958	[1030, 1894]
447	[750, 790]	951	[1446, 1451]	1455	[881, 1837]	1959	[1772, 269]
448	[791, 792]	952	[1404, 1424]	1456	[1233, 1838]	1960	[1809, 1149]
449	[793, 794]	953	[1452, 1453]	1457	[1802, 1839]	1961	[1909, 1934]
450	[795, 796]	954	[231, 32]	1458	[1085, 1827]	1962	[1871, 64]
451	[127, 797]	955	[1454, 228]	1459	[828, 849]	1963	[1920, 922]
452	[135, 620]	956	[1455, 486]	1460	[1728, 1206]	1964	[1818, 1258]
453	[798, 782]	957	[1456, 127]	1461	[429, 1041]	1965	[1234, 1179]
454	[723, 799]	958	[497, 1448]	1462	[901, 1840]	1966	[80, 1086]
455	[800, 801]	959	[1457, 707]	1463	[1795, 1841]	1967	[1929, 1849]
456	[802, 379]	960	[648, 1458]	1464	[1134, 982]	1968	[1933, 1762]
457	[803, 804]	961	[1315, 1459]	1465	[1156, 1177]	1969	[1710, 1028]
458	[26, 805]	962	[1338, 1460]	1466	[1825, 1102]	1970	[1208, 1928]
459	[806, 807]	963	[1461, 1462]	1467	[1075, 343]	1971	[1715, 16]
460	[808, 809]	964	[208, 43]	1468	[1842, 1796]	1972	[973, 1226]
461	[810, 811]	965	[1463, 1464]	1469	[1218, 1843]	1973	[1879, 1757]
462	[812, 813]	966	[799, 1465]	1470	[975, 1844]	1974	[1188, 1190]
463	[814, 815]	967	[1434, 1466]	1471	[316, 1845]	1975	[76, 1919]
464	[816, 817]	968	[593, 1313]	1472	[863, 1168]	1976	[437, 1835]

465	[70, 818]	969	[691, 1467]	1473	[1846, 1169]	1977	[1865, 1228]
466	[819, 820]	970	[1468, 1386]	1474	[1847, 910]	1978	[1189, 426]
467	[821, 822]	971	[1469, 1470]	1475	[1848, 95]	1979	[1915, 304]
468	[251, 823]	972	[1471, 514]	1476	[285, 1237]	1980	[998, 384]
469	[824, 825]	973	[1472, 1473]	1477	[1849, 821]	1981	[1196, 936]
470	[391, 826]	974	[1474, 1343]	1478	[1850, 810]	1982	[988, 1888]
471	[827, 828]	975	[1363, 563]	1479	[1851, 422]	1983	[1997, 416]
472	[829, 830]	976	[1475, 1476]	1480	[1852, 1251]	1984	[809, 1852]
473	[831, 832]	977	[1477, 496]	1481	[383, 1788]	1985	[268, 97]
474	[833, 834]	978	[638, 765]	1482	[1745, 1138]	1986	[1922, 1892]
475	[835, 836]	979	[1478, 37]	1483	[1776, 1853]	1987	[1101, 876]
476	[837, 838]	980	[1479, 220]	1484	[1854, 1826]	1988	[1167, 265]
477	[839, 840]	981	[1480, 1481]	1485	[441, 1855]	1989	[1995, 382]
478	[841, 842]	982	[1482, 1483]	1486	[1194, 399]	1990	[1996, 882]
479	[843, 844]	983	[1432, 232]	1487	[1165, 1811]	1991	[1249, 1131]
480	[845, 846]	984	[1347, 1478]	1488	[1239, 1856]	1992	[342, 1854]
481	[847, 848]	985	[1484, 152]	1489	[1857, 1858]	1993	[1853, 1998]
482	[849, 839]	986	[672, 1485]	1490	[1741, 1859]	1994	[1999, 492]
483	[850, 851]	987	[1486, 1441]	1491	[855, 971]	1995	[567, 1452]
484	[852, 853]	988	[722, 1487]	1492	[1124, 1000]	1996	[711, 1989]
485	[854, 855]	989	[1488, 1489]	1493	[22, 929]	1997	[1284, 1422]
486	[856, 857]	990	[1490, 1491]	1494	[1830, 428]	1998	[157, 682]
487	[858, 859]	991	[626, 1492]	1495	[1860, 1846]	1999	[2000, 869]
488	[860, 861]	992	[1493, 530]	1496	[1861, 1806]	2000	[2001, 1361]
489	[862, 863]	993	[1494, 224]	1497	[100, 1039]	2001	[2002, 1014]
490	[864, 865]	994	[1495, 1496]	1498	[1862, 945]	2002	[2003, 1965]
491	[866, 867]	995	[1497, 1498]	1499	[1863, 1864]	2003	[2004, 27]
492	[307, 868]	996	[56, 1499]	1500	[276, 1017]	2004	[2005, 57]
493	[869, 870]	997	[143, 1500]	1501	[333, 1865]	2005	[2006, 116]
494	[871, 872]	998	[739, 697]	1502	[1866, 1118]	2006	[2007, 230]
495	[873, 874]	999	[48, 1296]	1503	[1758, 1079]	2007	[2008, 436]
496	[446, 875]	1000	[1293, 1501]	1504	[965, 1867]	2008	[2009, 706]
497	[257, 248]	1001	[1502, 1503]	1505	[1868, 1869]	2009	[2010, 1105]
498	[876, 877]	1002	[1504, 1505]	1506	[1870, 1871]	2010	[2011, 611]
499	[878, 879]	1003	[789, 1410]	1507	[1872, 1873]	2011	[2012, 23]
500	[880, 881]	1004	[785, 1506]	1508	[1047, 1044]	2012	[2013, 49]
501	[882, 883]	1005	[566, 1507]	1509	[1814, 1874]	2013	[2014, 101]
502	[107, 402]	1006	[1508, 1405]	1510	[1777, 436]	2014	[1, 203]
503	[94, 884]	1007	[1509, 176]	1511	[1248, 1736]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	336	0	672	1	1008	1	1344	0	1680	0
1	0	337	0	673	0	1009	1	1345	0	1681	0

2	0	338	0	674	0	1010	0	1346	1	1682	0
3	0	339	1	675	1	1011	1	1347	1	1683	0
4	1	340	1	676	0	1012	1	1348	1	1684	0
5	0	341	1	677	1	1013	1	1349	0	1685	1
6	1	342	0	678	1	1014	0	1350	1	1686	1
7	0	343	1	679	0	1015	0	1351	0	1687	0
8	0	344	1	680	1	1016	0	1352	0	1688	1
9	1	345	1	681	0	1017	0	1353	1	1689	1
10	0	346	1	682	1	1018	0	1354	0	1690	1
11	0	347	0	683	0	1019	1	1355	1	1691	1
12	1	348	1	684	1	1020	1	1356	1	1692	0
13	1	349	0	685	0	1021	1	1357	0	1693	0
14	1	350	1	686	0	1022	0	1358	0	1694	1
15	0	351	1	687	1	1023	0	1359	1	1695	1
16	0	352	1	688	1	1024	0	1360	0	1696	1
17	0	353	0	689	1	1025	0	1361	0	1697	0
18	1	354	0	690	1	1026	0	1362	0	1698	0
19	1	355	1	691	0	1027	1	1363	0	1699	0
20	1	356	1	692	1	1028	1	1364	1	1700	1
21	0	357	1	693	1	1029	0	1365	0	1701	0
22	0	358	0	694	0	1030	1	1366	1	1702	0
23	0	359	1	695	1	1031	1	1367	0	1703	1
24	1	360	0	696	1	1032	0	1368	1	1704	0
25	1	361	1	697	0	1033	0	1369	1	1705	1
26	1	362	0	698	0	1034	0	1370	1	1706	1
27	1	363	0	699	0	1035	0	1371	0	1707	0
28	0	364	1	700	1	1036	1	1372	1	1708	0
29	1	365	0	701	0	1037	0	1373	0	1709	1
30	1	366	0	702	1	1038	0	1374	0	1710	1
31	0	367	0	703	1	1039	0	1375	0	1711	1
32	1	368	1	704	1	1040	0	1376	0	1712	0
33	0	369	0	705	1	1041	0	1377	1	1713	0
34	1	370	1	706	1	1042	0	1378	1	1714	1
35	0	371	1	707	1	1043	1	1379	1	1715	1
36	1	372	0	708	0	1044	1	1380	1	1716	1
37	1	373	0	709	0	1045	1	1381	0	1717	0
38	1	374	1	710	0	1046	0	1382	0	1718	1
39	1	375	1	711	1	1047	1	1383	1	1719	1
40	0	376	1	712	1	1048	0	1384	0	1720	0
41	1	377	0	713	1	1049	0	1385	0	1721	0
42	0	378	1	714	0	1050	1	1386	1	1722	1
43	0	379	1	715	0	1051	0	1387	1	1723	1
44	1	380	0	716	0	1052	1	1388	0	1724	1
45	0	381	1	717	0	1053	1	1389	0	1725	1

46	1	382	1	718	0	1054	1	1390	0	1726	0
47	0	383	0	719	0	1055	0	1391	0	1727	0
48	0	384	1	720	1	1056	0	1392	0	1728	0
49	1	385	0	721	1	1057	0	1393	0	1729	0
50	1	386	1	722	0	1058	0	1394	0	1730	0
51	1	387	0	723	1	1059	1	1395	0	1731	1
52	1	388	0	724	1	1060	0	1396	0	1732	0
53	1	389	1	725	0	1061	0	1397	0	1733	1
54	0	390	0	726	0	1062	0	1398	1	1734	0
55	1	391	1	727	0	1063	1	1399	0	1735	0
56	1	392	0	728	1	1064	0	1400	0	1736	1
57	0	393	1	729	0	1065	1	1401	1	1737	1
58	0	394	0	730	1	1066	0	1402	1	1738	0
59	1	395	0	731	0	1067	0	1403	0	1739	1
60	1	396	1	732	1	1068	0	1404	1	1740	0
61	1	397	0	733	1	1069	1	1405	0	1741	0
62	0	398	0	734	1	1070	0	1406	1	1742	1
63	0	399	0	735	0	1071	1	1407	0	1743	1
64	1	400	1	736	1	1072	1	1408	0	1744	1
65	1	401	1	737	0	1073	0	1409	0	1745	1
66	0	402	1	738	1	1074	1	1410	0	1746	0
67	0	403	0	739	0	1075	1	1411	0	1747	0
68	1	404	0	740	1	1076	1	1412	0	1748	0
69	1	405	0	741	1	1077	0	1413	1	1749	1
70	1	406	0	742	1	1078	0	1414	1	1750	0
71	0	407	0	743	1	1079	1	1415	0	1751	1
72	1	408	0	744	1	1080	0	1416	1	1752	1
73	1	409	1	745	0	1081	1	1417	1	1753	0
74	1	410	1	746	1	1082	1	1418	1	1754	0
75	1	411	1	747	0	1083	0	1419	1	1755	1
76	0	412	0	748	0	1084	0	1420	0	1756	0
77	1	413	0	749	1	1085	1	1421	1	1757	1
78	1	414	1	750	0	1086	0	1422	0	1758	0
79	1	415	1	751	0	1087	1	1423	1	1759	1
80	0	416	1	752	0	1088	1	1424	0	1760	0
81	0	417	1	753	0	1089	0	1425	1	1761	0
82	0	418	0	754	0	1090	1	1426	0	1762	1
83	1	419	1	755	0	1091	0	1427	1	1763	1
84	0	420	0	756	0	1092	1	1428	1	1764	1
85	0	421	0	757	0	1093	1	1429	0	1765	0
86	0	422	1	758	1	1094	0	1430	0	1766	1
87	0	423	0	759	0	1095	1	1431	0	1767	0
88	1	424	0	760	0	1096	1	1432	1	1768	0
89	1	425	0	761	0	1097	0	1433	0	1769	1

90	1	426	0	762	0	1098	1	1434	0	1770	0
91	0	427	0	763	0	1099	0	1435	0	1771	1
92	0	428	0	764	0	1100	0	1436	1	1772	1
93	1	429	0	765	1	1101	0	1437	0	1773	0
94	1	430	0	766	1	1102	0	1438	1	1774	0
95	0	431	1	767	1	1103	0	1439	1	1775	0
96	0	432	1	768	0	1104	1	1440	0	1776	1
97	0	433	0	769	1	1105	0	1441	1	1777	1
98	1	434	0	770	0	1106	1	1442	1	1778	1
99	1	435	0	771	1	1107	0	1443	0	1779	1
100	1	436	1	772	0	1108	0	1444	1	1780	0
101	1	437	1	773	1	1109	1	1445	1	1781	1
102	0	438	1	774	0	1110	1	1446	0	1782	1
103	1	439	0	775	0	1111	0	1447	1	1783	0
104	1	440	1	776	1	1112	0	1448	0	1784	1
105	0	441	1	777	1	1113	1	1449	0	1785	0
106	1	442	1	778	0	1114	0	1450	1	1786	0
107	1	443	1	779	0	1115	1	1451	0	1787	0
108	0	444	1	780	1	1116	0	1452	0	1788	1
109	0	445	1	781	1	1117	0	1453	0	1789	0
110	1	446	1	782	1	1118	0	1454	1	1790	1
111	0	447	0	783	0	1119	1	1455	0	1791	0
112	1	448	0	784	1	1120	1	1456	1	1792	1
113	0	449	1	785	1	1121	1	1457	1	1793	1
114	0	450	0	786	1	1122	1	1458	1	1794	0
115	0	451	1	787	1	1123	0	1459	0	1795	0
116	0	452	0	788	1	1124	0	1460	0	1796	1
117	0	453	0	789	1	1125	1	1461	0	1797	1
118	1	454	1	790	1	1126	0	1462	0	1798	0
119	1	455	1	791	0	1127	1	1463	0	1799	1
120	1	456	0	792	0	1128	1	1464	0	1800	0
121	0	457	0	793	1	1129	1	1465	0	1801	0
122	1	458	1	794	1	1130	1	1466	1	1802	1
123	1	459	1	795	0	1131	0	1467	1	1803	1
124	0	460	1	796	1	1132	1	1468	1	1804	1
125	1	461	1	797	1	1133	0	1469	0	1805	0
126	0	462	0	798	0	1134	0	1470	0	1806	0
127	1	463	0	799	1	1135	0	1471	1	1807	1
128	1	464	1	800	1	1136	1	1472	0	1808	0
129	1	465	1	801	1	1137	0	1473	0	1809	1
130	1	466	1	802	0	1138	1	1474	1	1810	0
131	0	467	1	803	0	1139	0	1475	0	1811	0
132	0	468	1	804	0	1140	1	1476	0	1812	0
133	1	469	0	805	1	1141	0	1477	1	1813	0

134	0	470	1	806	1	1142	0	1478	1	1814	0
135	0	471	0	807	1	1143	1	1479	0	1815	0
136	1	472	1	808	1	1144	0	1480	1	1816	0
137	0	473	0	809	1	1145	1	1481	0	1817	0
138	1	474	0	810	1	1146	0	1482	1	1818	0
139	0	475	1	811	0	1147	0	1483	1	1819	1
140	1	476	0	812	0	1148	1	1484	0	1820	1
141	0	477	0	813	1	1149	0	1485	1	1821	1
142	0	478	0	814	0	1150	1	1486	0	1822	0
143	0	479	0	815	1	1151	0	1487	1	1823	0
144	1	480	1	816	1	1152	1	1488	1	1824	1
145	0	481	1	817	1	1153	0	1489	0	1825	1
146	1	482	1	818	1	1154	1	1490	0	1826	0
147	1	483	0	819	1	1155	1	1491	0	1827	0
148	1	484	1	820	0	1156	0	1492	0	1828	0
149	0	485	0	821	1	1157	1	1493	0	1829	1
150	1	486	0	822	0	1158	0	1494	0	1830	0
151	1	487	0	823	1	1159	1	1495	0	1831	0
152	0	488	1	824	0	1160	0	1496	0	1832	1
153	1	489	1	825	1	1161	1	1497	1	1833	0
154	1	490	1	826	1	1162	1	1498	0	1834	0
155	0	491	1	827	0	1163	0	1499	1	1835	0
156	1	492	1	828	0	1164	0	1500	0	1836	0
157	0	493	0	829	1	1165	1	1501	1	1837	1
158	1	494	1	830	1	1166	0	1502	1	1838	1
159	1	495	1	831	0	1167	1	1503	0	1839	0
160	0	496	1	832	1	1168	0	1504	0	1840	0
161	1	497	0	833	0	1169	1	1505	0	1841	0
162	0	498	0	834	1	1170	0	1506	0	1842	1
163	1	499	1	835	1	1171	1	1507	1	1843	0
164	0	500	0	836	0	1172	1	1508	1	1844	1
165	1	501	0	837	0	1173	0	1509	0	1845	0
166	1	502	1	838	0	1174	0	1510	1	1846	0
167	1	503	1	839	0	1175	0	1511	0	1847	1
168	0	504	0	840	0	1176	0	1512	0	1848	0
169	1	505	0	841	0	1177	0	1513	1	1849	1
170	0	506	0	842	0	1178	1	1514	1	1850	1
171	0	507	1	843	0	1179	0	1515	1	1851	0
172	0	508	1	844	1	1180	1	1516	0	1852	1
173	1	509	0	845	1	1181	1	1517	1	1853	1
174	0	510	0	846	1	1182	1	1518	1	1854	0
175	0	511	1	847	1	1183	0	1519	0	1855	1
176	1	512	0	848	1	1184	1	1520	1	1856	0
177	1	513	1	849	1	1185	1	1521	0	1857	0

178	1	514	0	850	0	1186	0	1522	0	1858	0
179	1	515	0	851	0	1187	1	1523	0	1859	0
180	0	516	1	852	1	1188	0	1524	1	1860	0
181	0	517	1	853	0	1189	1	1525	1	1861	0
182	1	518	0	854	0	1190	0	1526	0	1862	0
183	0	519	0	855	0	1191	1	1527	1	1863	1
184	1	520	0	856	0	1192	1	1528	0	1864	0
185	1	521	1	857	0	1193	0	1529	1	1865	0
186	1	522	1	858	0	1194	0	1530	0	1866	1
187	1	523	0	859	1	1195	1	1531	0	1867	1
188	0	524	0	860	1	1196	0	1532	0	1868	0
189	0	525	1	861	1	1197	1	1533	1	1869	1
190	0	526	1	862	1	1198	1	1534	1	1870	0
191	0	527	1	863	0	1199	0	1535	1	1871	1
192	1	528	0	864	1	1200	0	1536	1	1872	1
193	0	529	0	865	0	1201	0	1537	0	1873	0
194	1	530	1	866	1	1202	1	1538	0	1874	0
195	1	531	1	867	0	1203	1	1539	0	1875	0
196	1	532	0	868	0	1204	0	1540	1	1876	0
197	1	533	1	869	0	1205	0	1541	0	1877	1
198	0	534	0	870	0	1206	1	1542	0	1878	1
199	1	535	0	871	1	1207	1	1543	0	1879	0
200	1	536	1	872	1	1208	1	1544	0	1880	0
201	1	537	0	873	1	1209	1	1545	1	1881	0
202	0	538	1	874	0	1210	1	1546	0	1882	0
203	0	539	1	875	1	1211	1	1547	1	1883	1
204	1	540	1	876	0	1212	0	1548	1	1884	1
205	1	541	1	877	0	1213	0	1549	1	1885	0
206	0	542	1	878	1	1214	0	1550	0	1886	1
207	1	543	1	879	0	1215	0	1551	0	1887	0
208	0	544	1	880	0	1216	0	1552	1	1888	1
209	0	545	0	881	0	1217	0	1553	0	1889	1
210	1	546	1	882	0	1218	0	1554	1	1890	1
211	1	547	0	883	1	1219	1	1555	1	1891	1
212	0	548	0	884	1	1220	0	1556	0	1892	0
213	0	549	1	885	0	1221	0	1557	1	1893	1
214	0	550	1	886	0	1222	1	1558	0	1894	0
215	1	551	1	887	0	1223	0	1559	0	1895	1
216	0	552	1	888	1	1224	0	1560	0	1896	1
217	1	553	1	889	1	1225	0	1561	1	1897	0
218	1	554	0	890	1	1226	0	1562	1	1898	1
219	0	555	0	891	0	1227	0	1563	0	1899	1
220	0	556	1	892	1	1228	0	1564	1	1900	0
221	1	557	0	893	0	1229	0	1565	0	1901	1

222	1	558	0	894	0	1230	1	1566	0	1902	0
223	0	559	1	895	1	1231	0	1567	1	1903	1
224	0	560	0	896	0	1232	0	1568	0	1904	1
225	0	561	1	897	1	1233	1	1569	0	1905	0
226	0	562	1	898	0	1234	0	1570	0	1906	0
227	1	563	0	899	0	1235	0	1571	0	1907	1
228	0	564	1	900	1	1236	1	1572	0	1908	1
229	1	565	1	901	0	1237	1	1573	1	1909	0
230	0	566	0	902	0	1238	1	1574	1	1910	1
231	1	567	1	903	0	1239	1	1575	1	1911	0
232	1	568	1	904	0	1240	1	1576	1	1912	0
233	1	569	0	905	0	1241	0	1577	0	1913	1
234	1	570	1	906	1	1242	1	1578	1	1914	0
235	1	571	1	907	0	1243	0	1579	0	1915	1
236	1	572	0	908	0	1244	0	1580	0	1916	0
237	0	573	1	909	1	1245	0	1581	0	1917	0
238	1	574	0	910	0	1246	1	1582	0	1918	0
239	1	575	0	911	1	1247	1	1583	0	1919	0
240	1	576	1	912	0	1248	0	1584	1	1920	1
241	0	577	1	913	0	1249	1	1585	0	1921	0
242	1	578	0	914	1	1250	1	1586	0	1922	1
243	1	579	0	915	1	1251	1	1587	1	1923	1
244	0	580	1	916	0	1252	1	1588	1	1924	1
245	1	581	1	917	1	1253	1	1589	0	1925	0
246	1	582	1	918	1	1254	0	1590	1	1926	1
247	0	583	1	919	1	1255	1	1591	1	1927	1
248	1	584	0	920	1	1256	0	1592	0	1928	0
249	1	585	1	921	1	1257	1	1593	1	1929	1
250	1	586	1	922	1	1258	1	1594	0	1930	1
251	1	587	1	923	0	1259	0	1595	0	1931	1
252	1	588	0	924	1	1260	0	1596	0	1932	1
253	1	589	0	925	0	1261	0	1597	1	1933	1
254	0	590	1	926	0	1262	0	1598	0	1934	1
255	1	591	1	927	1	1263	1	1599	0	1935	0
256	0	592	0	928	1	1264	1	1600	1	1936	0
257	0	593	1	929	1	1265	0	1601	1	1937	1
258	1	594	1	930	0	1266	1	1602	0	1938	1
259	1	595	0	931	0	1267	0	1603	0	1939	1
260	0	596	0	932	0	1268	1	1604	1	1940	1
261	1	597	0	933	0	1269	1	1605	0	1941	0
262	0	598	1	934	0	1270	0	1606	0	1942	1
263	0	599	1	935	0	1271	0	1607	0	1943	0
264	1	600	1	936	1	1272	1	1608	1	1944	0
265	0	601	1	937	0	1273	0	1609	1	1945	1

266	1	602	0	938	1	1274	1	1610	0	1946	0
267	1	603	0	939	0	1275	1	1611	0	1947	0
268	1	604	0	940	1	1276	0	1612	1	1948	0
269	0	605	0	941	1	1277	1	1613	1	1949	1
270	0	606	1	942	1	1278	1	1614	1	1950	1
271	1	607	1	943	0	1279	1	1615	1	1951	0
272	1	608	1	944	1	1280	0	1616	0	1952	1
273	0	609	1	945	1	1281	1	1617	0	1953	0
274	1	610	0	946	1	1282	1	1618	0	1954	0
275	0	611	0	947	1	1283	0	1619	0	1955	0
276	0	612	0	948	1	1284	1	1620	0	1956	0
277	0	613	0	949	0	1285	0	1621	0	1957	0
278	0	614	0	950	1	1286	1	1622	1	1958	1
279	1	615	0	951	0	1287	0	1623	0	1959	1
280	0	616	0	952	1	1288	0	1624	0	1960	1
281	0	617	0	953	0	1289	1	1625	0	1961	0
282	1	618	0	954	1	1290	1	1626	0	1962	1
283	0	619	1	955	1	1291	1	1627	1	1963	1
284	1	620	1	956	0	1292	1	1628	0	1964	0
285	0	621	1	957	1	1293	1	1629	0	1965	0
286	1	622	1	958	0	1294	1	1630	1	1966	0
287	0	623	0	959	1	1295	0	1631	1	1967	1
288	0	624	1	960	0	1296	0	1632	0	1968	1
289	1	625	0	961	0	1297	1	1633	0	1969	1
290	1	626	1	962	1	1298	1	1634	1	1970	1
291	1	627	1	963	0	1299	0	1635	1	1971	1
292	1	628	0	964	0	1300	0	1636	0	1972	0
293	1	629	0	965	0	1301	0	1637	1	1973	0
294	0	630	0	966	1	1302	1	1638	0	1974	0
295	0	631	1	967	0	1303	1	1639	0	1975	0
296	1	632	1	968	1	1304	1	1640	0	1976	1
297	1	633	0	969	0	1305	0	1641	1	1977	0
298	1	634	1	970	1	1306	0	1642	0	1978	1
299	1	635	1	971	0	1307	0	1643	0	1979	1
300	0	636	0	972	1	1308	1	1644	0	1980	0
301	0	637	0	973	0	1309	0	1645	1	1981	0
302	1	638	0	974	1	1310	0	1646	1	1982	0
303	1	639	1	975	0	1311	1	1647	0	1983	1
304	0	640	1	976	0	1312	1	1648	0	1984	1
305	1	641	1	977	1	1313	0	1649	0	1985	1
306	1	642	0	978	0	1314	0	1650	0	1986	1
307	1	643	1	979	1	1315	0	1651	1	1987	0
308	1	644	0	980	0	1316	1	1652	0	1988	1
309	1	645	0	981	1	1317	1	1653	0	1989	1

310	0	646	1	982	1	1318	0	1654	1	1990	1
311	0	647	0	983	1	1319	0	1655	1	1991	1
312	1	648	0	984	1	1320	1	1656	1	1992	0
313	1	649	1	985	0	1321	0	1657	1	1993	1
314	0	650	1	986	1	1322	0	1658	1	1994	0
315	1	651	1	987	0	1323	0	1659	1	1995	1
316	1	652	0	988	0	1324	1	1660	0	1996	1
317	0	653	1	989	1	1325	0	1661	1	1997	1
318	0	654	1	990	0	1326	0	1662	0	1998	0
319	1	655	0	991	1	1327	0	1663	0	1999	0
320	0	656	1	992	0	1328	1	1664	1	2000	0
321	1	657	0	993	0	1329	0	1665	1	2001	0
322	0	658	1	994	0	1330	1	1666	0	2002	0
323	1	659	1	995	1	1331	0	1667	1	2003	0
324	0	660	0	996	1	1332	1	1668	1	2004	0
325	1	661	0	997	0	1333	1	1669	0	2005	0
326	1	662	1	998	0	1334	0	1670	1	2006	0
327	0	663	1	999	0	1335	0	1671	1	2007	0
328	0	664	0	1000	1	1336	1	1672	1	2008	0
329	0	665	0	1001	1	1337	1	1673	1	2009	0
330	0	666	0	1002	0	1338	1	1674	0	2010	0
331	1	667	0	1003	1	1339	0	1675	0	2011	0
332	0	668	0	1004	1	1340	1	1676	0	2012	0
333	1	669	0	1005	0	1341	0	1677	0	2013	0
334	1	670	0	1006	1	1342	0	1678	1	2014	0
335	0	671	1	1007	0	1343	0	1679	0		

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