PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z^6+z^5+z^3+1, z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z^6+z^5+z^3+1, z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^6+z^5+z^3+1, z+1)$, *the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$, *with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$p_0(z)=z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^4+z^3+z^2+z+1,$$
$$p_1(z)=z^{23}+z^{22}+z^{18}+z^{17}+z^{15}+z^{11}+z^{10}+z^8+z^7+z^6+z^5+z^3+z^2+z,$$
$$p_2(z)=z^{24}+z^{22}+z^8+z^4+z^2+1,$$
$$p_3(z)=z^{23}+z^{22}+z^{18}+z^{17}+z^{15}+z^{11}+z^{10}+z^8+z^7+z^6+z^5+z^3+z^2+z,$$
$$p_4(z)=z^{22}+z^{17}+z^{14}+z^{13}+z^{11}+z^{10}+z^9+z^8+z^4+z^3+z+1,$$

and for $\mathrm{CF}(b,a)$,

$$p_0(z)=z^{20}+z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^3+z^2+z+1,$$
$$p_1(z)=z^{23}+z^{22}+z^{18}+z^{17}+z^{15}+z^{11}+z^{10}+z^8+z^7+z^6+z^5+z^3+z^2+z,$$
$$p_2(z)=z^{24}+z^{22}+z^8+z^4+z^2+1,$$
$$p_3(z)=z^{23}+z^{22}+z^{18}+z^{17}+z^{15}+z^{11}+z^{10}+z^8+z^7+z^6+z^5+z^3+z^2+z,$$
$$p_4(z)=z^{22}+z^{20}+z^{17}+z^{14}+z^{13}+z^{11}+z^{10}+z^9+z^3+z+1.$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z) \\ P_{n-1}(z) & Q_{n-1}(z) \end{pmatrix} := \begin{pmatrix} t_n(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t_{n-1}(z) & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} t_0(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)} \in \mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$M_n(x)=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \cdots \times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},$$

$$W_n(x)=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \cdots \times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned} M_{n+1}(x) &= W_n(x) \cdot M_n(x), \\ W_{n+1}(x) &= M_n(x) \cdot W_n(x). \end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$\begin{aligned} M_n(x)_{0,1} &= x^{d_n} \tilde{P}_{2^n-1}(x), \\ M_n(x)_{0,0} &= x^{d_n} \tilde{Q}_{2^n-1}(x), \end{aligned}$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i,j \leq 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i,j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i,j \leq 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. $\square$

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {}& M^e[:7u] + x^{3u}M^e[:4u] + x^{5u}M^e[:2u] + x^{3u}M^e[:7u] + x^{6u}M^e[:4u] \\ & + x^{8u}M^e[:2u] + x^{5u}M^e[:7u] + x^{8u}M^e[:4u] + x^{10u}M^e[:2u] \\ & + x^{6u}M^e[:7u] + x^{9u}M^e[:4u] + x^{11u}M^e[:2u] + x^{7u}M^e[:7u] \\ & + x^{12u}M^e[:2u] + (x^{7u} + x^{10u} + x^{12u} + x^{13u})R^e, \end{aligned}$$

$$\begin{aligned}W^e[:14u] = {} & W^e[:7u] + x^{3u}W^e[:4u] + x^{5u}W^e[:2u] + x^{3u}W^e[:7u] + x^{6u}W^e[:4u] \\ & + x^{8u}W^e[:2u] + x^{5u}W^e[:7u] + x^{8u}W^e[:4u] + x^{10u}W^e[:2u] \\ & + x^{6u}W^e[:7u] + x^{9u}W^e[:4u] + x^{11u}W^e[:2u] + x^{7u}W^e[:7u] \\ & + x^{12u}W^e[:2u] + (x^{7u} + x^{10u} + x^{12u} + x^{13u})R^e.\end{aligned}$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}M^o[:14u] = {} & M^o[:7u] + x^{3u}M^o[:4u] + x^{5u}M^o[:2u] + x^{3u}M^o[:7u] + x^{6u}M^o[:4u] \\ & + x^{8u}M^o[:2u] + x^{5u}M^o[:7u] + x^{8u}M^o[:4u] + x^{10u}M^o[:2u] \\ & + x^{6u}M^o[:7u] + x^{9u}M^o[:4u] + x^{11u}M^o[:2u] + x^{7u}M^o[:7u] \\ & + x^{12u}M^o[:2u] + (x^{7u} + x^{10u} + x^{12u} + x^{13u})R^o, \\ W^o[:14u] = {} & W^o[:7u] + x^{3u}W^o[:4u] + x^{5u}W^o[:2u] + x^{3u}W^o[:7u] + x^{6u}W^o[:4u] \\ & + x^{8u}W^o[:2u] + x^{5u}W^o[:7u] + x^{8u}W^o[:4u] + x^{10u}W^o[:2u] \\ & + x^{6u}W^o[:7u] + x^{9u}W^o[:4u] + x^{11u}W^o[:2u] + x^{7u}W^o[:7u] \\ & + x^{12u}W^o[:2u] + (x^{7u} + x^{10u} + x^{12u} + x^{13u})R^o.\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}T[:14u] = {} & T[:7u] + x^{3u}T[:4u] + x^{5u}T[:2u] + x^{3u}T[:7u] + x^{6u}T[:4u] + x^{8u}T[:2u] \\ & + x^{5u}T[:7u] + x^{8u}T[:4u] + x^{10u}T[:2u] + x^{6u}T[:7u] + x^{9u}T[:4u] \\ & + x^{11u}T[:2u] + x^{7u}T[:7u] + x^{12u}T[:2u] \\ & + (x^{7u} + x^{10u} + x^{12u} + x^{13u}).\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}x^7u &= x^{7u}T[:u] + x^{5u}T[2u:3u] + x^{3u}T[4u:5u] + T[7u:8u], \\ 0 &= x^{7u}T[u:2u] + x^{5u}T[3u:4u] + x^{3u}T[5u:6u] + T[8u:9u], \\ 0 &= x^{9u}T[:u] + x^{7u}T[2u:3u] + x^{5u}T[4u:5u] + x^{3u}T[6u:7u] \\ & \quad + T[9u:10u], \\ x^10u &= x^{10u}T[:u] + x^{9u}T[u:2u] + x^{8u}T[2u:3u] + x^{7u}T[3u:4u] \\ & \quad + x^{6u}T[4u:5u] + x^{5u}T[5u:6u] + T[10u:11u], \\ 0 &= x^{11u}T[:u] + x^{10u}T[u:2u] + x^{9u}T[2u:3u] + x^{8u}T[3u:4u] \\ & \quad + x^{7u}T[4u:5u] + x^{6u}T[5u:6u] + x^{5u}T[6u:7u] + T[11u:12u], \\ x^12u &= x^{12u}T[:u] + x^{11u}T[u:2u] + x^{9u}T[3u:4u] + x^{7u}T[5u:6u] \\ & \quad + x^{6u}T[6u:7u] + T[12u:13u], \\ x^13u &= x^{12u}T[u:2u] + x^{7u}T[6u:7u] + T[13u:14u],\end{aligned}$$

which can be rewritten as

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.7}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.8}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.9}$$

$$\begin{aligned} 0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ & + T[[101w]_2] + T[[1010w]_2], \end{aligned} \tag{2.10}$$

$$\begin{aligned} 0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ & + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \end{aligned} \tag{2.11}$$

$$\begin{aligned} 0 = {} & T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ & + T[[1100w]_2], \end{aligned} \tag{2.12}$$

$$0 = T[[1w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$

$$\begin{aligned} 1 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ & + T[[101w]_2] + T[[1010w]_2], \end{aligned} \tag{2.17}$$

$$\begin{aligned} 0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ & + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \end{aligned} \tag{2.18}$$

$$\begin{aligned} 1 = {} & T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ & + T[[1100w]_2], \end{aligned} \tag{2.19}$$

$$1 = T[[1w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1614 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.21}$$

$$0 = \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 101)) + \tau(A(s, 1000)), \tag{2.22}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 110)) \\ & + \tau(A(s, 1001)), \end{aligned} \tag{2.23}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 101)) + \tau(A(s, 1010)), \end{aligned} \tag{2.24}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 101)) + \tau(A(s, 110)) + \tau(A(s, 1011)), \end{aligned} \tag{2.25}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 101)) \\ & + \tau(A(s, 110)) + \tau(A(s, 1100)), \end{aligned} \tag{2.26}$$

$$0=\tau(A(s,1))+\tau(A(s,110))+\tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$1=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,100))+\tau(A(s,111)), \tag{2.28}$$

$$0=\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,101))+\tau(A(s,1000)), \tag{2.29}$$

$$\begin{aligned}0=&\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,100))+\tau(A(s,110))\\&+\tau(A(s,1001)),\end{aligned} \tag{2.30}$$

$$\begin{aligned}1=&\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\&+\tau(A(s,101))+\tau(A(s,1010)),\end{aligned} \tag{2.31}$$

$$\begin{aligned}0=&\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\&+\tau(A(s,101))+\tau(A(s,110))+\tau(A(s,1011)),\end{aligned} \tag{2.32}$$

$$\begin{aligned}1=&\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,101))\\&+\tau(A(s,110))+\tau(A(s,1100)),\end{aligned} \tag{2.33}$$

$$1=\tau(A(s,1))+\tau(A(s,110))+\tau(A(s,1101)), \tag{2.34}$$

for $s=A(i,0^{2k})$. We find that $(A(i,0^{18}),E_{18})=(A(i,0^{16}),E_{16})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2\le k\le 9$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k\ge 2$, and $u=2^{2k-1}$,*

$$\begin{aligned}M_{2k}&=M^e[:7u]+x^{3u}M^e[:4u]+x^{5u}M^e[:2u]+x^{7u}R^e,\\W_{2k}&=W^e[:7u]+x^{3u}W^e[:4u]+x^{5u}W^e[:2u]+x^{7u}R^e.\end{aligned}$$

For all integer $k\ge 2$, and $u=2^{2k}$,

$$\begin{aligned}M_{2k+1}&=M^o[:7u]+x^{3u}M^o[:4u]+x^{5u}M^o[:2u]+x^{7u}R^o,\\W_{2k+1}&=W^o[:7u]+x^{3u}W^o[:4u]+x^{5u}W^o[:2u]+x^{7u}R^o.\end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n=2$, the identities can be verified directly. For $n\ge 2$, we claim that

$$\begin{aligned}M_{2k}\wedge W_{2k}&\Rightarrow M_{2k+1}\wedge W_{2k+1},\\M_{2k+1}\wedge W_{2k+1}&\Rightarrow M_{2k+2}\wedge W_{2k+2}.\end{aligned}$$

We give the proof of

$$M_{2k}\wedge W_{2k}\Rightarrow M_{2k+1},$$

the other ones being similar. Set $u=2^{2k}$ and $v=2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned}W_{2k}M_{2k}=&\left(W^e[:7v]+x^{3v}W^e[:4v]+x^{5v}W^e[:2v]+x^{7u}R^e\right)\times\left(M^e[:7v]\right.\\&\left.+x^{3v}M^e[:4v]+x^{5v}M^e[:2v]+x^{7u}R^e\right);\end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is

$O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$W^e[:14v]M^e[:14v] + x^{6v}W^e[:8v]M^e[:8v] + x^{10v}W^e[:4v]M^e[:4v].$$

For all $n \le 14$, replace the occurrences of $W^e[:n\cdot v]$ and $M^e[:n\cdot v]$ in the above expression by the reduction modulo $x^{n\cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n\cdot v : (n+1)\cdot v]/X^n, \\ b_n &:= M^e[n\cdot v : (n+1)\cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1,...,a_{14},b_1,...,b_{14},c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a,b)$; for $\mathrm{CF}(b,a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j,k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned} q_0(x) &= x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} \\ &\quad + x^7, \\ q_1(x) &= x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{13} + x^9 + x^7 + x^6 \\ &\quad + x^2 + x, \\ q_2(x) &= x^{24} + x^{22} + x^{20} + x^{16} + x^2 + 1, \\ q_3(x) &= x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{13} + x^9 + x^7 + x^6 \end{aligned}$$

$$
\begin{aligned}
&+ x^2 + x, \\
q_4(x) &= x^{24} + x^{23} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^7 + x^2.
\end{aligned}
$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\mathrm{CF}(a, b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{114} + x^{110} + x^{106} + x^{104} + x^{100} + x^{98} + x^{90} + x^{86} + x^{82} + x^{78} + x^{76} \\
&\quad + x^{74} + x^{72} + x^{70} + x^{68} + x^{62} + x^{60} + x^{54} + x^{46} + x^{42} + x^{20} + x^{18} \\
&\quad + x^{12}, \\
p_3(x) &= x^{112} + x^{110} + x^{108} + x^{102} + x^{100} + x^{98} + x^{92} + x^{90} + x^{84} + x^{82} + x^{80} \\
&\quad + x^{78} + x^{76} + x^{74} + x^{70} + x^{62} + x^{58} + x^{56} + x^{52} + x^{46} + x^{42} + x^{36} \\
&\quad + x^{34} + x^{32} + x^{28} + x^{20} + x^{16} + x^{12} + x^{10} + x^8, \\
p_6(x) &= x^{112} + x^{108} + x^{104} + x^{92} + x^{90} + x^{86} + x^{84} + x^{80} + x^{76} + x^{72} + x^{70} \\
&\quad + x^{66} + x^{56} + x^{54} + x^{48} + x^{44} + x^{42} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24} \\
&\quad + x^{22} + x^{18} + x^{16} + x^{14} + x^2 + 1, \\
p_9(x) &= x^{112} + x^{110} + x^{106} + x^{102} + x^{100} + x^{94} + x^{92} + x^{88} + x^{86} + x^{78} + x^{74} \\
&\quad + x^{68} + x^{66} + x^{60} + x^{54} + x^{52} + x^{50} + x^{48} + x^{40} + x^{38} + x^{36} + x^{34} \\
&\quad + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^8 + x^2, \\
p_{12}(x) &= x^{114} + x^{112} + x^{110} + x^{106} + x^{102} + x^{98} + x^{94} + x^{90} + x^{86} + x^{82} + x^{78} \\
&\quad + x^{74} + x^{70} + x^{66} + x^{62} + x^{58} + x^{54} + x^{48} + x^{18} + x^{16} + x^{14} + x^{10} \\
&\quad + x^6 + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{128} + x^{127} + x^{126} + x^{123} + x^{122} + x^{120} + x^{119} + x^{116} + x^{115} \\
&\quad + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{105} + x^{104} + x^{102} + x^{101} \\
&\quad + x^{100} + x^{98} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{85} + x^{84} + x^{81} \\
&\quad + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{71} + x^{70} + x^{69} + x^{68} + x^{64}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{62} + x^{61} + x^{60} + x^{58} + x^{57} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{42}\\
&+ x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{27},\\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105}\\
&+ x^{98} + x^{96} + x^{93} + x^{92} + x^{90} + x^{89} + x^{80} + x^{78} + x^{76} + x^{75} + x^{74}\\
&+ x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{58} + x^{56} + x^{53} + x^{52} + x^{50} + x^{49}\\
&+ x^{40} + x^{38} + x^{36} + x^{35} + x^{34} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{18}\\
&+ x^{16} + x^{13} + x^{12} + x^{10} + x^{9},\\
p_6(x) &= x^{123} + x^{122} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} + x^{105} + x^{104} + x^{103}\\
&+ x^{100} + x^{97} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} + x^{77} + x^{76} + x^{74}\\
&+ x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{57} + x^{55}\\
&+ x^{53} + x^{51} + x^{48} + x^{45} + x^{44} + x^{43} + x^{41} + x^{39} + x^{37} + x^{35} + x^{33}\\
&+ x^{30} + x^{21} + x^{20} + x^{19} + x^{17} + x^{14} + x^{9} + x^{8} + x^{6},\\
p_9(x) &= x^{126} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111}\\
&+ x^{108} + x^{106} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{86} + x^{84}\\
&+ x^{82} + x^{81} + x^{78} + x^{76} + x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{62}\\
&+ x^{58} + x^{57} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{44} + x^{43}\\
&+ x^{42} + x^{40} + x^{37} + x^{33} + x^{32} + x^{31} + x^{30} + x^{25} + x^{23} + x^{20} + x^{19}\\
&+ x^{18} + x^{17} + x^{15} + x^{13} + x^{10} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3},\\
p_{12}(x) &= x^{129} + x^{128} + x^{127} + x^{125} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112}\\
&+ x^{65} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{29} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13}\\
&+ x^{11} + x^{8} + x^{5} + x^{4} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1].

For $\phi(M^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{128} + x^{127} + x^{126} + x^{123} + x^{122} + x^{120} + x^{119} + x^{116} + x^{115}\\
&+ x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{105} + x^{104} + x^{102} + x^{101}\\
&+ x^{100} + x^{98} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{85} + x^{84} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{71} + x^{70} + x^{69} + x^{68} + x^{64}\\
&+ x^{62} + x^{61} + x^{60} + x^{58} + x^{57} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{42}\\
&+ x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{27},\\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105}\\
&+ x^{98} + x^{96} + x^{93} + x^{92} + x^{90} + x^{89} + x^{80} + x^{78} + x^{76} + x^{75} + x^{74}\\
&+ x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{58} + x^{56} + x^{53} + x^{52} + x^{50} + x^{49}\\
&+ x^{40} + x^{38} + x^{36} + x^{35} + x^{34} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{18}\\
&+ x^{16} + x^{13} + x^{12} + x^{10} + x^{9},
\end{aligned}
$$

$$\begin{aligned}
p_6(x) &= x^{123} + x^{122} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} + x^{105} + x^{104} + x^{103} \\
&\quad + x^{100} + x^{97} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} + x^{77} + x^{76} + x^{74} \\
&\quad + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{57} + x^{55} \\
&\quad + x^{53} + x^{51} + x^{48} + x^{45} + x^{44} + x^{43} + x^{41} + x^{39} + x^{37} + x^{35} + x^{33} \\
&\quad + x^{30} + x^{21} + x^{20} + x^{19} + x^{17} + x^{14} + x^{9} + x^{8} + x^{6}, \\
p_9(x) &= x^{126} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} \\
&\quad + x^{108} + x^{106} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{86} + x^{84} \\
&\quad + x^{82} + x^{81} + x^{78} + x^{76} + x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{62} \\
&\quad + x^{58} + x^{57} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{44} + x^{43} \\
&\quad + x^{42} + x^{40} + x^{37} + x^{33} + x^{32} + x^{31} + x^{30} + x^{25} + x^{23} + x^{20} + x^{19} \\
&\quad + x^{18} + x^{17} + x^{15} + x^{13} + x^{10} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{129} + x^{128} + x^{127} + x^{125} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} \\
&\quad + x^{65} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{33} \\
&\quad + x^{32} + x^{31} + x^{29} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13} \\
&\quad + x^{11} + x^{8} + x^{5} + x^{4} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{142} + x^{136} + x^{132} + x^{126} + x^{122} + x^{118} + x^{114} + x^{112} + x^{110} \\
&\quad + x^{108} + x^{106} + x^{104} + x^{98} + x^{92} + x^{86} + x^{80} + x^{76} + x^{74} + x^{72} + x^{70} \\
&\quad + x^{66} + x^{56} + x^{52} + x^{50} + x^{48} + x^{42}, \\
p_3(x) &= x^{142} + x^{138} + x^{136} + x^{134} + x^{132} + x^{126} + x^{124} + x^{122} + x^{114} + x^{110} \\
&\quad + x^{106} + x^{104} + x^{96} + x^{90} + x^{78} + x^{76} + x^{74} + x^{64} + x^{62} + x^{60} + x^{56} \\
&\quad + x^{54} + x^{44} + x^{42} + x^{34} + x^{24} + x^{22} + x^{20} + x^{14} + x^{6}, \\
p_6(x) &= x^{142} + x^{136} + x^{134} + x^{124} + x^{118} + x^{116} + x^{114} + x^{112} + x^{108} + x^{106} \\
&\quad + x^{102} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{82} + x^{78} + x^{74} + x^{70} + x^{68} \\
&\quad + x^{66} + x^{62} + x^{60} + x^{56} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34} + x^{30} + x^{26} \\
&\quad + x^{22} + x^{20} + x^{16} + x^{10} + x^{8} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{142} + x^{138} + x^{132} + x^{128} + x^{122} + x^{118} + x^{114} + x^{106} + x^{104} + x^{98} \\
&\quad + x^{96} + x^{94} + x^{88} + x^{78} + x^{76} + x^{66} + x^{54} + x^{50} + x^{44} + x^{40} + x^{38} \\
&\quad + x^{28} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^{12} + x^{10} + x^{2}, \\
p_{12}(x) &= x^{144} + x^{128} + x^{120} + x^{112} + x^{80} + x^{64} + x^{56} + x^{24} + x^{8} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{142} + x^{140} + x^{134} + x^{132} + x^{130} + x^{124} + x^{120} + x^{116} + x^{108} \\
&\quad + x^{106} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} + x^{88} + x^{84} + x^{66} + x^{60} + x^{58}
\end{aligned}$$

$$
\begin{aligned}
&+x^{56}+x^{54}+x^{52}+x^{48}+x^{46}+x^{36}+x^{34}+x^{32}+x^{28}+x^{24}+x^{22}\\
&+x^{20}+x^{16}+x^{12},\\
p_3(x)=&\,x^{142}+x^{132}+x^{130}+x^{128}+x^{122}+x^{120}+x^{116}+x^{114}+x^{112}+x^{108}\\
&+x^{102}+x^{98}+x^{96}+x^{94}+x^{92}+x^{88}+x^{82}+x^{80}+x^{78}+x^{76}+x^{74}\\
&+x^{72}+x^{70}+x^{64}+x^{62}+x^{60}+x^{56}+x^{54}+x^{50}+x^{48}+x^{46}+x^{44}\\
&+x^{40}+x^{38}+x^{36}+x^{30}+x^{28}+x^{20}+x^{18}+x^{8},\\
p_6(x)=&\,x^{142}+x^{140}+x^{138}+x^{136}+x^{132}+x^{130}+x^{128}+x^{126}+x^{124}+x^{118}\\
&+x^{114}+x^{110}+x^{108}+x^{100}+x^{98}+x^{96}+x^{94}+x^{92}+x^{86}+x^{84}\\
&+x^{80}+x^{78}+x^{74}+x^{66}+x^{62}+x^{60}+x^{56}+x^{54}+x^{52}+x^{46}+x^{44}\\
&+x^{40}+x^{36}+x^{28}+x^{26}+x^{20}+x^{18}+x^{14}+x^{12}+x^{10}+x^{8}+x^{6}+x^{2}\\
&+1,\\
p_9(x)=&\,x^{142}+x^{138}+x^{132}+x^{128}+x^{122}+x^{118}+x^{114}+x^{106}+x^{104}+x^{98}\\
&+x^{96}+x^{94}+x^{88}+x^{78}+x^{76}+x^{66}+x^{54}+x^{50}+x^{44}+x^{40}+x^{38}\\
&+x^{28}+x^{24}+x^{22}+x^{20}+x^{18}+x^{14}+x^{12}+x^{10}+x^{2},\\
p_{12}(x)=&\,x^{144}+x^{128}+x^{120}+x^{112}+x^{80}+x^{64}+x^{56}+x^{24}+x^{8}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x)=&\,x^{129}+x^{128}+x^{127}+x^{126}+x^{118}+x^{117}+x^{116}+x^{114}+x^{113}+x^{112}\\
&+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{102}+x^{100}+x^{97}+x^{95}+x^{94}\\
&+x^{93}+x^{87}+x^{85}+x^{83}+x^{81}+x^{79}+x^{75}+x^{71}+x^{69}+x^{68}+x^{67}\\
&+x^{66}+x^{65}+x^{63}+x^{62}+x^{57}+x^{56}+x^{55}+x^{53}+x^{51}+x^{50}+x^{48}\\
&+x^{47}+x^{45}+x^{41}+x^{36}+x^{35}+x^{34}+x^{32}+x^{31}+x^{26}+x^{25}+x^{24}\\
&+x^{20}+x^{18},\\
p_3(x)=&\,x^{120}+x^{118}+x^{116}+x^{115}+x^{114}+x^{110}+x^{109}+x^{107}+x^{106}+x^{105}\\
&+x^{98}+x^{96}+x^{93}+x^{92}+x^{90}+x^{89}+x^{80}+x^{78}+x^{76}+x^{75}+x^{74}\\
&+x^{70}+x^{69}+x^{67}+x^{66}+x^{65}+x^{58}+x^{56}+x^{53}+x^{52}+x^{50}+x^{49}\\
&+x^{40}+x^{38}+x^{36}+x^{35}+x^{34}+x^{30}+x^{29}+x^{27}+x^{26}+x^{25}+x^{18}\\
&+x^{16}+x^{13}+x^{12}+x^{10}+x^{9},\\
p_6(x)=&\,x^{123}+x^{122}+x^{120}+x^{117}+x^{116}+x^{115}+x^{112}+x^{105}+x^{104}+x^{103}\\
&+x^{100}+x^{97}+x^{96}+x^{95}+x^{92}+x^{91}+x^{90}+x^{88}+x^{77}+x^{76}+x^{74}\\
&+x^{73}+x^{72}+x^{71}+x^{69}+x^{66}+x^{63}+x^{62}+x^{61}+x^{59}+x^{57}+x^{55}\\
&+x^{53}+x^{51}+x^{48}+x^{45}+x^{44}+x^{43}+x^{41}+x^{39}+x^{37}+x^{35}+x^{33}\\
&+x^{30}+x^{21}+x^{20}+x^{19}+x^{17}+x^{14}+x^{9}+x^{8}+x^{6},\\
p_9(x)=&\,x^{126}+x^{122}+x^{121}+x^{120}+x^{119}+x^{117}+x^{115}+x^{114}+x^{112}+x^{111}\\
&+x^{108}+x^{106}+x^{103}+x^{100}+x^{99}+x^{97}+x^{96}+x^{95}+x^{86}+x^{84}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{82} + x^{81} + x^{78} + x^{76} + x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{62}\\
&+ x^{58} + x^{57} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{44} + x^{43}\\
&+ x^{42} + x^{40} + x^{37} + x^{33} + x^{32} + x^{31} + x^{30} + x^{25} + x^{23} + x^{20} + x^{19}\\
&+ x^{18} + x^{17} + x^{15} + x^{13} + x^{10} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3},\\
p_{12}(x) = {}& x^{129} + x^{128} + x^{127} + x^{125} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112}\\
&+ x^{65} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{29} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13}\\
&+ x^{11} + x^{8} + x^{5} + x^{4} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1, 1, 1].

For $\phi(W^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) = {}& x^{129} + x^{128} + x^{127} + x^{126} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{112}\\
&+ x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{102} + x^{100} + x^{97} + x^{95} + x^{94}\\
&+ x^{93} + x^{87} + x^{85} + x^{83} + x^{81} + x^{79} + x^{75} + x^{71} + x^{69} + x^{68} + x^{67}\\
&+ x^{66} + x^{65} + x^{63} + x^{62} + x^{57} + x^{56} + x^{55} + x^{53} + x^{51} + x^{50} + x^{48}\\
&+ x^{47} + x^{45} + x^{41} + x^{36} + x^{35} + x^{34} + x^{32} + x^{31} + x^{26} + x^{25} + x^{24}\\
&+ x^{20} + x^{18},\\
p_3(x) = {}& x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105}\\
&+ x^{98} + x^{96} + x^{93} + x^{92} + x^{90} + x^{89} + x^{80} + x^{78} + x^{76} + x^{75} + x^{74}\\
&+ x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{58} + x^{56} + x^{53} + x^{52} + x^{50} + x^{49}\\
&+ x^{40} + x^{38} + x^{36} + x^{35} + x^{34} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{18}\\
&+ x^{16} + x^{13} + x^{12} + x^{10} + x^{9},\\
p_6(x) = {}& x^{123} + x^{122} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} + x^{105} + x^{104} + x^{103}\\
&+ x^{100} + x^{97} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} + x^{77} + x^{76} + x^{74}\\
&+ x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63} + x^{62} + x^{61} + x^{59} + x^{57} + x^{55}\\
&+ x^{53} + x^{51} + x^{48} + x^{45} + x^{44} + x^{43} + x^{41} + x^{39} + x^{37} + x^{35} + x^{33}\\
&+ x^{30} + x^{21} + x^{20} + x^{19} + x^{17} + x^{14} + x^{9} + x^{8} + x^{6},\\
p_9(x) = {}& x^{126} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111}\\
&+ x^{108} + x^{106} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{86} + x^{84}\\
&+ x^{82} + x^{81} + x^{78} + x^{76} + x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{62}\\
&+ x^{58} + x^{57} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{44} + x^{43}\\
&+ x^{42} + x^{40} + x^{37} + x^{33} + x^{32} + x^{31} + x^{30} + x^{25} + x^{23} + x^{20} + x^{19}\\
&+ x^{18} + x^{17} + x^{15} + x^{13} + x^{10} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3},\\
p_{12}(x) = {}& x^{129} + x^{128} + x^{127} + x^{125} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112}\\
&+ x^{65} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{29} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13}
\end{aligned}
$$

$$+ x^{11} + x^{8} + x^{5} + x^{4} + x^{3} + 1,$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1, 1, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{114} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{94} + x^{92} + x^{90} \\
& + x^{88} + x^{82} + x^{78} + x^{76} + x^{64} + x^{62} + x^{60} + x^{58} + x^{50} + x^{48} + x^{46} \\
& + x^{36} + x^{34} + x^{32} + x^{24}, \\
p_3(x) = {} & x^{112} + x^{110} + x^{104} + x^{102} + x^{96} + x^{92} + x^{90} + x^{86} + x^{80} + x^{78} + x^{74} \\
& + x^{70} + x^{66} + x^{60} + x^{58} + x^{54} + x^{46} + x^{44} + x^{38} + x^{36} + x^{26} + x^{24} \\
& + x^{16} + x^{14} + x^{10} + x^{6}, \\
p_6(x) = {} & x^{112} + x^{110} + x^{108} + x^{104} + x^{102} + x^{96} + x^{94} + x^{92} + x^{86} + x^{82} + x^{78} \\
& + x^{76} + x^{74} + x^{72} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} \\
& + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{28} + x^{26} + x^{22} + x^{18} \\
& + x^{16} + x^{12} + x^{10} + x^{6} + x^{4} + x^{2} + 1, \\
p_9(x) = {} & x^{112} + x^{110} + x^{106} + x^{102} + x^{100} + x^{94} + x^{92} + x^{88} + x^{86} + x^{78} + x^{74} \\
& + x^{68} + x^{66} + x^{60} + x^{54} + x^{52} + x^{50} + x^{48} + x^{40} + x^{38} + x^{36} + x^{34} \\
& + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^{8} + x^{2}, \\
p_{12}(x) = {} & x^{114} + x^{112} + x^{110} + x^{106} + x^{102} + x^{98} + x^{94} + x^{90} + x^{86} + x^{82} + x^{78} \\
& + x^{74} + x^{70} + x^{66} + x^{62} + x^{58} + x^{54} + x^{48} + x^{18} + x^{16} + x^{14} + x^{10} \\
& + x^{6} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{129} + x^{128} + x^{126} + x^{124} + x^{123} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114} \\
& + x^{110} + x^{109} + x^{108} + x^{107} + x^{103} + x^{100} + x^{99} + x^{98} + x^{92} + x^{89} \\
& + x^{83} + x^{82} + x^{81} + x^{80} + x^{77} + x^{76} + x^{72} + x^{68} + x^{67} + x^{63} + x^{57} \\
& + x^{56} + x^{52} + x^{51} + x^{49} + x^{47} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} \\
& + x^{35} + x^{34} + x^{32} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{20} + x^{19} + x^{15} \\
& + x^{14} + x^{12}, \\
p_3(x) = {} & x^{125} + x^{124} + x^{122} + x^{120} + x^{119} + x^{116} + x^{114} + x^{111} + x^{110} + x^{109} \\
& + x^{108} + x^{107} + x^{104} + x^{98} + x^{96} + x^{94} + x^{93} + x^{91} + x^{89} + x^{87} + x^{84} \\
& + x^{80} + x^{79} + x^{77} + x^{68} + x^{65} + x^{64} + x^{63} + x^{60} + x^{59} + x^{54} + x^{52} \\
& + x^{49} + x^{48} + x^{47} + x^{44} + x^{43} + x^{42} + x^{38} + x^{34} + x^{33} + x^{32} + x^{30} \\
& + x^{29} + x^{27} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{17} + x^{16} + x^{12} + x^{11} \\
& + x^{10} + x^{8}, \\
p_6(x) = {} & x^{127} + x^{126} + x^{125} + x^{124} + x^{121} + x^{120} + x^{117} + x^{115} + x^{114} + x^{111} \\
& + x^{109} + x^{108} + x^{102} + x^{101} + x^{100} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91}
\end{aligned}$$

$$
\begin{aligned}
&+x^{85}+x^{83}+x^{81}+x^{80}+x^{79}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}\\
&+x^{71}+x^{70}+x^{69}+x^{68}+x^{66}+x^{64}+x^{62}+x^{60}+x^{58}+x^{54}+x^{53}\\
&+x^{51}+x^{50}+x^{47}+x^{45}+x^{43}+x^{42}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}\\
&+x^{32}+x^{28}+x^{25}+x^{23}+x^{21}+x^{18}+x^{17}+x^{16}+x^{12}+x^{11}+x^{7}\\
&+x^{6}+x^{5}+x^{3}+1,\\
p_9(x) ={}& x^{125}+x^{124}+x^{123}+x^{120}+x^{117}+x^{116}+x^{115}+x^{112}+x^{105}+x^{104}\\
&+x^{103}+x^{101}+x^{99}+x^{96}+x^{89}+x^{88}+x^{87}+x^{85}+x^{83}+x^{80}+x^{73}\\
&+x^{72}+x^{71}+x^{69}+x^{67}+x^{64}+x^{61}+x^{60}+x^{59}+x^{57}+x^{55}+x^{52}\\
&+x^{29}+x^{28}+x^{27}+x^{24}+x^{21}+x^{20}+x^{19}+x^{16}+x^{13}+x^{12}+x^{11}\\
&+x^{9}+x^{7}+x^{4},\\
p_{12}(x) ={}& x^{129}+x^{128}+x^{127}+x^{124}+x^{105}+x^{104}+x^{103}+x^{101}+x^{99}+x^{96}\\
&+x^{89}+x^{88}+x^{87}+x^{85}+x^{83}+x^{80}+x^{73}+x^{72}+x^{71}+x^{69}+x^{67}\\
&+x^{65}+x^{63}+x^{60}+x^{57}+x^{56}+x^{55}+x^{53}+x^{51}+x^{48}+x^{33}+x^{32}\\
&+x^{31}+x^{28}+x^{17}+x^{16}+x^{15}+x^{12}+x^{9}+x^{8}+x^{7}+x^{5}+x^{3}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) ={}& x^{144}+x^{141}+x^{140}+x^{137}+x^{134}+x^{133}+x^{132}+x^{125}+x^{123}+x^{118}\\
&+x^{117}+x^{113}+x^{112}+x^{109}+x^{108}+x^{107}+x^{105}+x^{100}+x^{99}+x^{98}\\
&+x^{95}+x^{93}+x^{88}+x^{86}+x^{85}+x^{82}+x^{81}+x^{79}+x^{78}+x^{77}+x^{76}\\
&+x^{72}+x^{71}+x^{69}+x^{65}+x^{61}+x^{59}+x^{57}+x^{55}+x^{54}+x^{53}+x^{52}\\
&+x^{51}+x^{50}+x^{48}+x^{46}+x^{45}+x^{43}+x^{42}+x^{40}+x^{37}+x^{35}+x^{34}\\
&+x^{32}+x^{30}+x^{29}+x^{27},\\
p_3(x) ={}& x^{135}+x^{134}+x^{132}+x^{130}+x^{129}+x^{128}+x^{122}+x^{119}+x^{117}+x^{113}\\
&+x^{112}+x^{109}+x^{108}+x^{107}+x^{105}+x^{102}+x^{101}+x^{100}+x^{98}+x^{95}\\
&+x^{94}+x^{93}+x^{91}+x^{88}+x^{82}+x^{79}+x^{77}+x^{73}+x^{72}+x^{69}+x^{68}\\
&+x^{67}+x^{65}+x^{62}+x^{61}+x^{60}+x^{58}+x^{55}+x^{54}+x^{53}+x^{51}+x^{48}\\
&+x^{42}+x^{39}+x^{37}+x^{33}+x^{32}+x^{29}+x^{28}+x^{27}+x^{25}+x^{22}+x^{21}\\
&+x^{20}+x^{18}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9},\\
p_6(x) ={}& x^{138}+x^{136}+x^{132}+x^{130}+x^{126}+x^{122}+x^{116}+x^{112}+x^{108}+x^{106}\\
&+x^{100}+x^{98}+x^{96}+x^{94}+x^{92}+x^{84}+x^{72}+x^{64}+x^{62}+x^{60}+x^{56}\\
&+x^{46}+x^{44}+x^{38}+x^{36}+x^{34}+x^{28}+x^{20}+x^{18}+x^{16}+x^{12}+x^{10}\\
&+x^{8}+x^{6},\\
p_9(x) ={}& x^{141}+x^{140}+x^{139}+x^{134}+x^{133}+x^{131}+x^{127}+x^{125}+x^{122}+x^{120}\\
&+x^{118}+x^{117}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}\\
&+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87}\\
&+ x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{74} + x^{73}\\
&+ x^{72} + x^{71} + x^{68} + x^{66} + x^{65} + x^{64} + x^{62} + x^{60} + x^{59} + x^{57} + x^{55}\\
&+ x^{52} + x^{51} + x^{45} + x^{44} + x^{43} + x^{38} + x^{37} + x^{35} + x^{31} + x^{28} + x^{27}\\
&+ x^{26} + x^{24} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{12} + x^{11} + x^{9} + x^{7} + x^{4}\\
&+ x^{3},\\
p_{12}(x) &= x^{144} + x^{140} + x^{136} + x^{124} + x^{120} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92}\\
&+ x^{88} + x^{84} + x^{68} + x^{64} + x^{52} + x^{44} + x^{40} + x^{32} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{117} + x^{116} + x^{114} + x^{113} + x^{109} + x^{106} + x^{102} + x^{101} + x^{100}\\
&+ x^{98} + x^{97} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{87} + x^{84} + x^{83}\\
&+ x^{82} + x^{78} + x^{75} + x^{70} + x^{64} + x^{60} + x^{58} + x^{57} + x^{52} + x^{50} + x^{48}\\
&+ x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{39} + x^{36} + x^{35} + x^{29} + x^{23} + x^{22}\\
&+ x^{21} + x^{18},\\
p_3(x) &= x^{111} + x^{110} + x^{108} + x^{106} + x^{105} + x^{104} + x^{99} + x^{96} + x^{95} + x^{94}\\
&+ x^{92} + x^{91} + x^{90} + x^{89} + x^{71} + x^{70} + x^{68} + x^{66} + x^{65} + x^{64} + x^{59}\\
&+ x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{31} + x^{30} + x^{28} + x^{26}\\
&+ x^{25} + x^{24} + x^{19} + x^{16} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^{9},\\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{106} + x^{100} + x^{98} + x^{96} + x^{90} + x^{84} + x^{80} + x^{76}\\
&+ x^{72} + x^{66} + x^{54} + x^{46} + x^{40} + x^{38} + x^{36} + x^{32} + x^{30} + x^{28} + x^{24}\\
&+ x^{20} + x^{16} + x^{8} + x^{6},\\
p_9(x) &= x^{117} + x^{116} + x^{115} + x^{110} + x^{109} + x^{107} + x^{105} + x^{104} + x^{101} + x^{94}\\
&+ x^{93} + x^{91} + x^{89} + x^{88} + x^{85} + x^{78} + x^{77} + x^{75} + x^{73} + x^{72} + x^{69}\\
&+ x^{62} + x^{61} + x^{59} + x^{57} + x^{56} + x^{52} + x^{51} + x^{21} + x^{20} + x^{19} + x^{14}\\
&+ x^{13} + x^{11} + x^{9} + x^{8} + x^{4} + x^{3},\\
p_{12}(x) &= x^{120} + x^{116} + x^{112} + x^{108} + x^{104} + x^{100} + x^{92} + x^{88} + x^{84} + x^{76}\\
&+ x^{72} + x^{68} + x^{60} + x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{128} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117}\\
&+ x^{116} + x^{115} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{104} + x^{101}\\
&+ x^{100} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{89} + x^{85} + x^{83} + x^{82}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65}\\
&+ x^{64} + x^{61} + x^{59} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{45}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{44} + x^{42} + x^{39} + x^{36} + x^{33},\\
p_3(x) = {}& x^{125} + x^{124} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{113}\\
&+ x^{111} + x^{99} + x^{94} + x^{93} + x^{90} + x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79}\\
&+ x^{78} + x^{76} + x^{75} + x^{73} + x^{70} + x^{69} + x^{67} + x^{65} + x^{63} + x^{60} + x^{58}\\
&+ x^{56} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{40} + x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{28} + x^{23} + x^{22} + x^{20} + x^{18}\\
&+ x^{17} + x^{12},\\
p_6(x) = {}& x^{127} + x^{126} + x^{122} + x^{119} + x^{117} + x^{116} + x^{115} + x^{114} + x^{111} + x^{109}\\
&+ x^{108} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{94} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{87} + x^{80} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69}\\
&+ x^{68} + x^{67} + x^{65} + x^{64} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53}\\
&+ x^{52} + x^{51} + x^{50} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{33} + x^{32}\\
&+ x^{30} + x^{29} + x^{27} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11}\\
&+ x^{5} + x^{4} + x^{3} + 1,\\
p_9(x) = {}& x^{125} + x^{124} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} + x^{105} + x^{104}\\
&+ x^{103} + x^{101} + x^{99} + x^{96} + x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{80} + x^{73}\\
&+ x^{72} + x^{71} + x^{69} + x^{67} + x^{64} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52}\\
&+ x^{29} + x^{28} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{16} + x^{13} + x^{12} + x^{11}\\
&+ x^{9} + x^{7} + x^{4},\\
p_{12}(x) = {}& x^{129} + x^{128} + x^{127} + x^{124} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96}\\
&+ x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{80} + x^{73} + x^{72} + x^{71} + x^{69} + x^{67}\\
&+ x^{65} + x^{63} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{51} + x^{48} + x^{33} + x^{32}\\
&+ x^{31} + x^{28} + x^{17} + x^{16} + x^{15} + x^{12} + x^{9} + x^{8} + x^{7} + x^{5} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {}& x^{129} + x^{128} + x^{126} + x^{124} + x^{123} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114}\\
&+ x^{110} + x^{109} + x^{108} + x^{107} + x^{103} + x^{100} + x^{99} + x^{98} + x^{92} + x^{89}\\
&+ x^{83} + x^{82} + x^{81} + x^{80} + x^{77} + x^{76} + x^{72} + x^{68} + x^{67} + x^{63} + x^{57}\\
&+ x^{56} + x^{52} + x^{51} + x^{49} + x^{47} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38}\\
&+ x^{35} + x^{34} + x^{32} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{20} + x^{19} + x^{15}\\
&+ x^{14} + x^{12},\\
p_3(x) = {}& x^{125} + x^{124} + x^{122} + x^{120} + x^{119} + x^{116} + x^{114} + x^{111} + x^{110} + x^{109}\\
&+ x^{108} + x^{107} + x^{104} + x^{98} + x^{96} + x^{94} + x^{93} + x^{91} + x^{89} + x^{87} + x^{84}\\
&+ x^{80} + x^{79} + x^{77} + x^{68} + x^{65} + x^{64} + x^{63} + x^{60} + x^{59} + x^{54} + x^{52}\\
&+ x^{49} + x^{48} + x^{47} + x^{44} + x^{43} + x^{42} + x^{38} + x^{34} + x^{33} + x^{32} + x^{30}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{29}+x^{27}+x^{24}+x^{23}+x^{22}+x^{20}+x^{19}+x^{17}+x^{16}+x^{12}+x^{11}\\
&+x^{10}+x^{8},\\
p_6(x)&=x^{127}+x^{126}+x^{125}+x^{124}+x^{121}+x^{120}+x^{117}+x^{115}+x^{114}+x^{111}\\
&+x^{109}+x^{108}+x^{102}+x^{101}+x^{100}+x^{96}+x^{94}+x^{93}+x^{92}+x^{91}\\
&+x^{85}+x^{83}+x^{81}+x^{80}+x^{79}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}\\
&+x^{71}+x^{70}+x^{69}+x^{68}+x^{66}+x^{64}+x^{62}+x^{60}+x^{58}+x^{54}+x^{53}\\
&+x^{51}+x^{50}+x^{47}+x^{45}+x^{43}+x^{42}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}\\
&+x^{32}+x^{28}+x^{25}+x^{23}+x^{21}+x^{18}+x^{17}+x^{16}+x^{12}+x^{11}+x^{7}\\
&+x^{6}+x^{5}+x^{3}+1,\\
p_9(x)&=x^{125}+x^{124}+x^{123}+x^{120}+x^{117}+x^{116}+x^{115}+x^{112}+x^{105}+x^{104}\\
&+x^{103}+x^{101}+x^{99}+x^{96}+x^{89}+x^{88}+x^{87}+x^{85}+x^{83}+x^{80}+x^{73}\\
&+x^{72}+x^{71}+x^{69}+x^{67}+x^{64}+x^{61}+x^{60}+x^{59}+x^{57}+x^{55}+x^{52}\\
&+x^{29}+x^{28}+x^{27}+x^{24}+x^{21}+x^{20}+x^{19}+x^{16}+x^{13}+x^{12}+x^{11}\\
&+x^{9}+x^{7}+x^{4},\\
p_{12}(x)&=x^{129}+x^{128}+x^{127}+x^{124}+x^{105}+x^{104}+x^{103}+x^{101}+x^{99}+x^{96}\\
&+x^{89}+x^{88}+x^{87}+x^{85}+x^{83}+x^{80}+x^{73}+x^{72}+x^{71}+x^{69}+x^{67}\\
&+x^{65}+x^{63}+x^{60}+x^{57}+x^{56}+x^{55}+x^{53}+x^{51}+x^{48}+x^{33}+x^{32}\\
&+x^{31}+x^{28}+x^{17}+x^{16}+x^{15}+x^{12}+x^{9}+x^{8}+x^{7}+x^{5}+x^{3}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x)&=x^{120}+x^{117}+x^{116}+x^{114}+x^{113}+x^{109}+x^{106}+x^{102}+x^{101}+x^{100}\\
&+x^{98}+x^{97}+x^{95}+x^{94}+x^{93}+x^{91}+x^{90}+x^{88}+x^{87}+x^{84}+x^{83}\\
&+x^{82}+x^{78}+x^{75}+x^{70}+x^{64}+x^{60}+x^{58}+x^{57}+x^{52}+x^{50}+x^{48}\\
&+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{39}+x^{36}+x^{35}+x^{29}+x^{23}+x^{22}\\
&+x^{21}+x^{18},\\
p_3(x)&=x^{111}+x^{110}+x^{108}+x^{106}+x^{105}+x^{104}+x^{99}+x^{96}+x^{95}+x^{94}\\
&+x^{92}+x^{91}+x^{90}+x^{89}+x^{71}+x^{70}+x^{68}+x^{66}+x^{65}+x^{64}+x^{59}\\
&+x^{56}+x^{55}+x^{54}+x^{52}+x^{51}+x^{50}+x^{49}+x^{31}+x^{30}+x^{28}+x^{26}\\
&+x^{25}+x^{24}+x^{19}+x^{16}+x^{15}+x^{14}+x^{12}+x^{11}+x^{10}+x^{9},\\
p_6(x)&=x^{114}+x^{112}+x^{108}+x^{106}+x^{100}+x^{98}+x^{96}+x^{90}+x^{84}+x^{80}+x^{76}\\
&+x^{72}+x^{66}+x^{54}+x^{46}+x^{40}+x^{38}+x^{36}+x^{32}+x^{30}+x^{28}+x^{24}\\
&+x^{20}+x^{16}+x^{8}+x^{6},\\
p_9(x)&=x^{117}+x^{116}+x^{115}+x^{110}+x^{109}+x^{107}+x^{105}+x^{104}+x^{101}+x^{94}\\
&+x^{93}+x^{91}+x^{89}+x^{88}+x^{85}+x^{78}+x^{77}+x^{75}+x^{73}+x^{72}+x^{69}\\
&+x^{62}+x^{61}+x^{59}+x^{57}+x^{56}+x^{52}+x^{51}+x^{21}+x^{20}+x^{19}+x^{14}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{13} + x^{11} + x^{9} + x^{8} + x^{4} + x^{3},\\
p_{12}(x) &= x^{120} + x^{116} + x^{112} + x^{108} + x^{104} + x^{100} + x^{92} + x^{88} + x^{84} + x^{76}\\
&+ x^{72} + x^{68} + x^{60} + x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1].

For $\phi(W^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{144} + x^{141} + x^{140} + x^{137} + x^{134} + x^{133} + x^{132} + x^{125} + x^{123} + x^{118}\\
&+ x^{117} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{100} + x^{99} + x^{98}\\
&+ x^{95} + x^{93} + x^{88} + x^{86} + x^{85} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76}\\
&+ x^{72} + x^{71} + x^{69} + x^{65} + x^{61} + x^{59} + x^{57} + x^{55} + x^{54} + x^{53} + x^{52}\\
&+ x^{51} + x^{50} + x^{48} + x^{46} + x^{45} + x^{43} + x^{42} + x^{40} + x^{37} + x^{35} + x^{34}\\
&+ x^{32} + x^{30} + x^{29} + x^{27},\\
p_3(x) &= x^{135} + x^{134} + x^{132} + x^{130} + x^{129} + x^{128} + x^{122} + x^{119} + x^{117} + x^{113}\\
&+ x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95}\\
&+ x^{94} + x^{93} + x^{91} + x^{88} + x^{82} + x^{79} + x^{77} + x^{73} + x^{72} + x^{69} + x^{68}\\
&+ x^{67} + x^{65} + x^{62} + x^{61} + x^{60} + x^{58} + x^{55} + x^{54} + x^{53} + x^{51} + x^{48}\\
&+ x^{42} + x^{39} + x^{37} + x^{33} + x^{32} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{21}\\
&+ x^{20} + x^{18} + x^{13} + x^{12} + x^{11} + x^{10} + x^{9},\\
p_6(x) &= x^{138} + x^{136} + x^{132} + x^{130} + x^{126} + x^{122} + x^{116} + x^{112} + x^{108} + x^{106}\\
&+ x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{84} + x^{72} + x^{64} + x^{62} + x^{60} + x^{56}\\
&+ x^{46} + x^{44} + x^{38} + x^{36} + x^{34} + x^{28} + x^{20} + x^{18} + x^{16} + x^{12} + x^{10}\\
&+ x^{8} + x^{6},\\
p_9(x) &= x^{141} + x^{140} + x^{139} + x^{134} + x^{133} + x^{131} + x^{127} + x^{125} + x^{122} + x^{120}\\
&+ x^{118} + x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108}\\
&+ x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98}\\
&+ x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87}\\
&+ x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{74} + x^{73}\\
&+ x^{72} + x^{71} + x^{68} + x^{66} + x^{65} + x^{64} + x^{62} + x^{60} + x^{59} + x^{57} + x^{55}\\
&+ x^{52} + x^{51} + x^{45} + x^{44} + x^{43} + x^{38} + x^{37} + x^{35} + x^{31} + x^{28} + x^{27}\\
&+ x^{26} + x^{24} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{12} + x^{11} + x^{9} + x^{7} + x^{4}\\
&+ x^{3},\\
p_{12}(x) &= x^{144} + x^{140} + x^{136} + x^{124} + x^{120} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92}\\
&+ x^{88} + x^{84} + x^{68} + x^{64} + x^{52} + x^{44} + x^{40} + x^{32} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1].

For $\phi(W^o, 1, 1)$:

$$
p_0(x) = x^{129} + x^{128} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117}
$$

$$+ x^{116} + x^{115} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{104} + x^{101}$$
$$+ x^{100} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{89} + x^{85} + x^{83} + x^{82}$$
$$+ x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65}$$
$$+ x^{64} + x^{61} + x^{59} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{45}$$
$$+ x^{44} + x^{42} + x^{39} + x^{36} + x^{33},$$

$$\begin{aligned}
p_3(x) = {} & x^{125} + x^{124} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{113} \\
& + x^{111} + x^{99} + x^{94} + x^{93} + x^{90} + x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79} \\
& + x^{78} + x^{76} + x^{75} + x^{73} + x^{70} + x^{69} + x^{67} + x^{65} + x^{63} + x^{60} + x^{58} \\
& + x^{56} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{43} + x^{42} + x^{41} \\
& + x^{40} + x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{28} + x^{23} + x^{22} + x^{20} + x^{18} \\
& + x^{17} + x^{12},
\end{aligned}$$

$$\begin{aligned}
p_6(x) = {} & x^{127} + x^{126} + x^{122} + x^{119} + x^{117} + x^{116} + x^{115} + x^{114} + x^{111} + x^{109} \\
& + x^{108} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{94} + x^{91} + x^{90} \\
& + x^{89} + x^{88} + x^{87} + x^{80} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} \\
& + x^{68} + x^{67} + x^{65} + x^{64} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} \\
& + x^{52} + x^{51} + x^{50} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{33} + x^{32} \\
& + x^{30} + x^{29} + x^{27} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11} \\
& + x^{5} + x^{4} + x^{3} + 1,
\end{aligned}$$

$$\begin{aligned}
p_9(x) = {} & x^{125} + x^{124} + x^{123} + x^{120} + x^{117} + x^{116} + x^{115} + x^{112} + x^{105} + x^{104} \\
& + x^{103} + x^{101} + x^{99} + x^{96} + x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{80} + x^{73} \\
& + x^{72} + x^{71} + x^{69} + x^{67} + x^{64} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52} \\
& + x^{29} + x^{28} + x^{27} + x^{24} + x^{21} + x^{20} + x^{19} + x^{16} + x^{13} + x^{12} + x^{11} \\
& + x^{9} + x^{7} + x^{4},
\end{aligned}$$

$$\begin{aligned}
p_{12}(x) = {} & x^{129} + x^{128} + x^{127} + x^{124} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96} \\
& + x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{80} + x^{73} + x^{72} + x^{71} + x^{69} + x^{67} \\
& + x^{65} + x^{63} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{51} + x^{48} + x^{33} + x^{32} \\
& + x^{31} + x^{28} + x^{17} + x^{16} + x^{15} + x^{12} + x^{9} + x^{8} + x^{7} + x^{5} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	404	[679, 542]	808	[1161, 1162]	1212	[1399, 804]
1	[3, 4]	405	[680, 681]	809	[568, 1163]	1213	[1423, 1424]
2	[5, 6]	406	[682, 683]	810	[1164, 160]	1214	[945, 1425]
3	[7, 8]	407	[684, 685]	811	[1143, 513]	1215	[1426, 105]

4	[9, 10]	408	[686, 687]	812	[1141, 152]	1216	[1427, 325]
5	[11, 12]	409	[688, 681]	813	[1165, 34]	1217	[1428, 1047]
6	[13, 14]	410	[689, 690]	814	[176, 1166]	1218	[410, 1362]
7	[15, 16]	411	[691, 692]	815	[156, 557]	1219	[1429, 1430]
8	[17, 18]	412	[693, 694]	816	[528, 524]	1220	[1431, 775]
9	[19, 20]	413	[695, 696]	817	[1167, 714]	1221	[1346, 1005]
10	[21, 22]	414	[220, 42]	818	[1168, 1169]	1222	[1432, 1433]
11	[23, 24]	415	[140, 697]	819	[1170, 32]	1223	[1434, 868]
12	[25, 26]	416	[698, 699]	820	[1171, 1172]	1224	[385, 946]
13	[27, 28]	417	[700, 186]	821	[168, 500]	1225	[1435, 1436]
14	[29, 30]	418	[701, 174]	822	[171, 1173]	1226	[1437, 1438]
15	[31, 32]	419	[702, 33]	823	[501, 618]	1227	[322, 1403]
16	[33, 34]	420	[498, 518]	824	[560, 1151]	1228	[1425, 881]
17	[35, 36]	421	[703, 704]	825	[588, 534]	1229	[1439, 1107]
18	[37, 38]	422	[705, 685]	826	[535, 1157]	1230	[1440, 1384]
19	[39, 40]	423	[706, 707]	827	[139, 602]	1231	[453, 274]
20	[41, 42]	424	[558, 708]	828	[1149, 538]	1232	[1441, 1442]
21	[43, 44]	425	[609, 540]	829	[638, 543]	1233	[1443, 1441]
22	[45, 46]	426	[709, 586]	830	[135, 1119]	1234	[1444, 980]
23	[47, 48]	427	[508, 710]	831	[1127, 1174]	1235	[900, 125]
24	[49, 50]	428	[648, 504]	832	[721, 220]	1236	[811, 1340]
25	[51, 52]	429	[711, 712]	833	[1124, 1175]	1237	[1404, 1445]
26	[53, 54]	430	[611, 222]	834	[546, 660]	1238	[841, 1079]
27	[55, 56]	431	[516, 713]	835	[580, 696]	1239	[876, 280]
28	[57, 58]	432	[216, 204]	836	[1176, 46]	1240	[84, 1439]
29	[59, 60]	433	[604, 714]	837	[1177, 1178]	1241	[1446, 1447]
30	[61, 62]	434	[715, 155]	838	[1177, 683]	1242	[923, 1448]
31	[63, 64]	435	[716, 466]	839	[1179, 613]	1243	[1449, 947]
32	[65, 66]	436	[603, 717]	840	[1135, 1180]	1244	[381, 1450]
33	[67, 68]	437	[218, 617]	841	[8, 694]	1245	[1451, 1452]
34	[69, 70]	438	[690, 718]	842	[1137, 1181]	1246	[1337, 1453]
35	[71, 72]	439	[11, 719]	843	[602, 697]	1247	[1105, 770]
36	[73, 74]	440	[720, 559]	844	[1182, 172]	1248	[845, 247]
37	[75, 76]	441	[547, 138]	845	[597, 501]	1249	[1454, 1407]
38	[77, 78]	442	[721, 41]	846	[1183, 1184]	1250	[1455, 951]
39	[79, 80]	443	[202, 659]	847	[55, 170]	1251	[1336, 827]
40	[81, 82]	444	[141, 40]	848	[209, 704]	1252	[1456, 1332]
41	[83, 84]	445	[477, 515]	849	[684, 1185]	1253	[985, 949]
42	[85, 86]	446	[722, 723]	850	[1186, 54]	1254	[1064, 1457]
43	[87, 88]	447	[496, 640]	851	[1187, 1188]	1255	[16, 1458]
44	[89, 90]	448	[724, 136]	852	[1189, 710]	1256	[1099, 812]
45	[91, 92]	449	[725, 726]	853	[190, 487]	1257	[1055, 899]
46	[93, 94]	450	[727, 728]	854	[671, 200]	1258	[1459, 1460]
47	[95, 96]	451	[729, 244]	855	[1190, 1172]	1259	[1355, 1461]

48	[97, 98]	452	[157, 702]	856	[180, 1191]	1260	[1462, 111]
49	[99, 100]	453	[730, 731]	857	[663, 485]	1261	[1463, 920]
50	[101, 102]	454	[181, 149]	858	[1192, 1193]	1262	[1464, 1465]
51	[103, 104]	455	[732, 733]	859	[645, 1194]	1263	[1466, 1467]
52	[105, 106]	456	[470, 734]	860	[163, 1195]	1264	[1327, 963]
53	[107, 108]	457	[591, 735]	861	[1190, 1196]	1265	[906, 924]
54	[109, 110]	458	[736, 737]	862	[1197, 519]	1266	[1468, 460]
55	[111, 112]	459	[483, 738]	863	[730, 653]	1267	[1421, 798]
56	[113, 114]	460	[739, 740]	864	[33, 1198]	1268	[1383, 1469]
57	[115, 116]	461	[741, 545]	865	[629, 1199]	1269	[1470, 75]
58	[117, 118]	462	[742, 743]	866	[1200, 167]	1270	[1471, 458]
59	[119, 120]	463	[744, 745]	867	[1142, 656]	1271	[1472, 855]
60	[121, 122]	464	[746, 747]	868	[1201, 633]	1272	[856, 1454]
61	[123, 124]	465	[748, 749]	869	[502, 1202]	1273	[1473, 1012]
62	[125, 126]	466	[750, 751]	870	[493, 493]	1274	[1474, 1375]
63	[127, 128]	467	[752, 753]	871	[1151, 547]	1275	[1475, 276]
64	[129, 130]	468	[754, 755]	872	[1203, 1204]	1276	[1476, 954]
65	[131, 132]	469	[756, 119]	873	[196, 1205]	1277	[1028, 1016]
66	[133, 134]	470	[757, 752]	874	[1206, 241]	1278	[420, 16]
67	[135, 136]	471	[277, 430]	875	[488, 1207]	1279	[1457, 1477]
68	[137, 138]	472	[251, 758]	876	[1145, 233]	1280	[912, 1075]
69	[139, 140]	473	[759, 760]	877	[635, 232]	1281	[1478, 1479]
70	[141, 142]	474	[761, 762]	878	[1208, 639]	1282	[1480, 1481]
71	[143, 144]	475	[763, 764]	879	[610, 516]	1283	[72, 1482]
72	[145, 146]	476	[765, 766]	880	[153, 234]	1284	[373, 1483]
73	[147, 148]	477	[767, 65]	881	[1155, 520]	1285	[1484, 1485]
74	[149, 150]	478	[768, 769]	882	[1209, 1210]	1286	[1486, 1350]
75	[151, 152]	479	[770, 771]	883	[1123, 1211]	1287	[1487, 1488]
76	[153, 154]	480	[772, 773]	884	[1212, 1213]	1288	[1482, 1333]
77	[146, 155]	481	[774, 409]	885	[46, 139]	1289	[753, 1489]
78	[156, 157]	482	[775, 411]	886	[1150, 141]	1290	[1490, 898]
79	[158, 159]	483	[776, 777]	887	[1187, 1214]	1291	[1491, 797]
80	[147, 160]	484	[778, 779]	888	[670, 189]	1292	[1479, 772]
81	[161, 162]	485	[780, 781]	889	[1215, 489]	1293	[1492, 1493]
82	[163, 164]	486	[782, 783]	890	[1216, 55]	1294	[1430, 756]
83	[165, 166]	487	[784, 785]	891	[1217, 1218]	1295	[976, 1494]
84	[167, 167]	488	[786, 787]	892	[582, 717]	1296	[1495, 1074]
85	[168, 161]	489	[788, 307]	893	[593, 475]	1297	[779, 978]
86	[169, 170]	490	[263, 789]	894	[1219, 1149]	1298	[927, 953]
87	[171, 172]	491	[790, 791]	895	[1220, 479]	1299	[273, 1009]
88	[173, 174]	492	[492, 492]	896	[615, 573]	1300	[1467, 1496]
89	[175, 176]	493	[792, 793]	897	[678, 1145]	1301	[392, 1497]
90	[177, 178]	494	[308, 382]	898	[1221, 1143]	1302	[1498, 942]
91	[179, 180]	495	[265, 794]	899	[1159, 465]	1303	[1499, 1500]

92	[181, 182]	496	[789, 795]	900	[565, 1222]	1304	[1501, 434]
93	[183, 184]	497	[796, 340]	901	[1223, 130]	1305	[1489, 1364]
94	[185, 186]	498	[797, 4]	902	[1224, 1225]	1306	[1345, 1502]
95	[187, 188]	499	[787, 309]	903	[1226, 1227]	1307	[301, 837]
96	[143, 189]	500	[798, 799]	904	[1228, 1125]	1308	[1503, 365]
97	[190, 191]	501	[351, 800]	905	[1179, 1229]	1309	[1483, 1504]
98	[192, 193]	502	[801, 121]	906	[1230, 11]	1310	[993, 1470]
99	[194, 195]	503	[802, 803]	907	[202, 43]	1311	[1505, 344]
100	[196, 197]	504	[804, 300]	908	[207, 692]	1312	[1410, 796]
101	[198, 58]	505	[805, 806]	909	[1231, 1232]	1313	[1506, 878]
102	[199, 200]	506	[807, 311]	910	[1233, 1231]	1314	[904, 1507]
103	[201, 202]	507	[808, 809]	911	[1210, 1183]	1315	[1402, 27]
104	[203, 204]	508	[116, 372]	912	[1234, 149]	1316	[1508, 1499]
105	[205, 206]	509	[810, 811]	913	[701, 619]	1317	[1509, 831]
106	[207, 208]	510	[812, 813]	914	[598, 183]	1318	[446, 1510]
107	[209, 210]	511	[814, 815]	915	[618, 173]	1319	[1419, 292]
108	[211, 212]	512	[816, 817]	916	[619, 578]	1320	[1511, 744]
109	[213, 52]	513	[414, 818]	917	[1170, 1235]	1321	[30, 874]
110	[214, 215]	514	[819, 802]	918	[1236, 1237]	1322	[1512, 1014]
111	[216, 217]	515	[820, 821]	919	[657, 473]	1323	[1513, 910]
112	[218, 219]	516	[822, 823]	920	[1238, 1180]	1324	[1102, 1446]
113	[220, 221]	517	[824, 369]	921	[1125, 1232]	1325	[70, 1348]
114	[222, 223]	518	[20, 825]	922	[723, 1181]	1326	[1359, 1514]
115	[224, 225]	519	[826, 824]	923	[539, 1239]	1327	[242, 1222]
116	[226, 227]	520	[827, 394]	924	[1229, 614]	1328	[557, 632]
117	[228, 229]	521	[828, 829]	925	[698, 1240]	1329	[1515, 1212]
118	[59, 230]	522	[68, 830]	926	[165, 1241]	1330	[236, 1210]
119	[231, 232]	523	[831, 832]	927	[184, 576]	1331	[1516, 1517]
120	[233, 234]	524	[833, 834]	928	[1196, 1242]	1332	[1118, 735]
121	[176, 235]	525	[835, 21]	929	[1243, 1244]	1333	[240, 1518]
122	[236, 237]	526	[395, 836]	930	[706, 1245]	1334	[192, 224]
123	[238, 239]	527	[837, 838]	931	[709, 606]	1335	[1519, 1520]
124	[240, 241]	528	[328, 839]	932	[1246, 214]	1336	[1120, 580]
125	[242, 243]	529	[840, 841]	933	[1247, 1128]	1337	[1246, 1307]
126	[133, 244]	530	[842, 843]	934	[691, 208]	1338	[8, 1255]
127	[245, 246]	531	[844, 845]	935	[723, 1138]	1339	[1236, 38]
128	[247, 83]	532	[846, 847]	936	[1248, 542]	1340	[161, 1521]
129	[248, 249]	533	[848, 826]	937	[523, 529]	1341	[1297, 1293]
130	[92, 29]	534	[849, 850]	938	[200, 1197]	1342	[43, 1315]
131	[250, 251]	535	[851, 852]	939	[173, 619]	1343	[1308, 679]
132	[252, 253]	536	[853, 854]	940	[183, 597]	1344	[1522, 588]
133	[254, 255]	537	[293, 447]	941	[1249, 163]	1345	[581, 1256]
134	[256, 257]	538	[855, 856]	942	[1201, 159]	1346	[1134, 527]
135	[258, 259]	539	[66, 331]	943	[37, 1237]	1347	[1523, 1319]

136	[260, 261]	540	[857, 858]	944	[1250, 151]	1348	[140, 665]
137	[262, 263]	541	[859, 860]	945	[1251, 1155]	1349	[1524, 667]
138	[264, 265]	542	[861, 862]	946	[1252, 166]	1350	[652, 731]
139	[266, 267]	543	[863, 810]	947	[688, 1253]	1351	[1278, 1202]
140	[94, 268]	544	[864, 865]	948	[1254, 1229]	1352	[645, 505]
141	[259, 269]	545	[866, 867]	949	[693, 1255]	1353	[1297, 1309]
142	[270, 271]	546	[868, 869]	950	[531, 1239]	1354	[1152, 612]
143	[272, 273]	547	[870, 871]	951	[516, 1256]	1355	[1123, 718]
144	[274, 275]	548	[872, 873]	952	[45, 1257]	1356	[1220, 590]
145	[276, 277]	549	[874, 875]	953	[700, 1258]	1357	[572, 616]
146	[278, 279]	550	[783, 876]	954	[175, 1184]	1358	[1522, 533]
147	[280, 281]	551	[366, 877]	955	[151, 1259]	1359	[1229, 1269]
148	[282, 283]	552	[878, 879]	956	[1260, 1261]	1360	[500, 162]
149	[284, 285]	553	[377, 880]	957	[1186, 1262]	1361	[628, 506]
150	[286, 287]	554	[881, 882]	958	[1263, 1264]	1362	[1525, 1186]
151	[288, 289]	555	[883, 884]	959	[1265, 55]	1363	[1320, 669]
152	[290, 291]	556	[885, 886]	960	[1197, 1155]	1364	[539, 532]
153	[292, 293]	557	[793, 67]	961	[1266, 1267]	1365	[1212, 1169]
154	[294, 295]	558	[887, 888]	962	[211, 50]	1366	[517, 490]
155	[296, 297]	559	[28, 889]	963	[1252, 1241]	1367	[639, 544]
156	[298, 299]	560	[791, 342]	964	[1183, 176]	1368	[1206, 1518]
157	[300, 301]	561	[795, 788]	965	[1196, 1268]	1369	[1313, 678]
158	[302, 303]	562	[890, 846]	966	[1133, 1123]	1370	[1277, 1195]
159	[304, 305]	563	[891, 892]	967	[613, 1269]	1371	[1217, 1312]
160	[281, 306]	564	[893, 894]	968	[1176, 1257]	1372	[481, 1526]
161	[307, 308]	565	[895, 896]	969	[577, 173]	1373	[1527, 477]
162	[309, 310]	566	[897, 863]	970	[1270, 214]	1374	[678, 552]
163	[311, 312]	567	[898, 780]	971	[699, 1179]	1375	[1528, 1294]
164	[313, 314]	568	[899, 900]	972	[522, 626]	1376	[62, 1529]
165	[315, 316]	569	[901, 902]	973	[1250, 1141]	1377	[500, 1521]
166	[317, 318]	570	[903, 904]	974	[1271, 1272]	1378	[600, 541]
167	[319, 320]	571	[905, 906]	975	[1273, 1212]	1379	[213, 1314]
168	[321, 322]	572	[762, 907]	976	[538, 531]	1380	[1319, 707]
169	[323, 324]	573	[908, 909]	977	[1274, 568]	1381	[1530, 1531]
170	[325, 326]	574	[910, 908]	978	[727, 564]	1382	[1532, 533]
171	[327, 328]	575	[911, 912]	979	[1275, 651]	1383	[690, 1211]
172	[329, 330]	576	[913, 914]	980	[606, 587]	1384	[1322, 1533]
173	[331, 332]	577	[88, 915]	981	[1168, 1213]	1385	[476, 1534]
174	[333, 334]	578	[916, 93]	982	[579, 695]	1386	[546, 1301]
175	[335, 336]	579	[917, 77]	983	[1276, 586]	1387	[581, 713]
176	[24, 337]	580	[379, 918]	984	[510, 1193]	1388	[1316, 1240]
177	[338, 339]	581	[451, 384]	985	[584, 526]	1389	[672, 1251]
178	[340, 341]	582	[919, 296]	986	[1184, 1166]	1390	[1203, 674]
179	[342, 343]	583	[920, 921]	987	[716, 651]	1391	[57, 1280]

180	[344, 345]	584	[922, 923]	988	[1249, 1277]	1392	[677, 1144]
181	[346, 347]	585	[924, 925]	989	[1191, 662]	1393	[1171, 1196]
182	[348, 349]	586	[926, 927]	990	[1278, 503]	1394	[1535, 1248]
183	[350, 351]	587	[303, 928]	991	[640, 1151]	1395	[1235, 1536]
184	[352, 353]	588	[383, 848]	992	[668, 1279]	1396	[1323, 471]
185	[354, 323]	589	[929, 930]	993	[1189, 509]	1397	[1208, 543]
186	[355, 356]	590	[931, 932]	994	[198, 1280]	1398	[1537, 723]
187	[357, 358]	591	[933, 905]	995	[1281, 193]	1399	[682, 1178]
188	[359, 360]	592	[934, 935]	996	[1282, 1283]	1400	[1296, 1538]
189	[361, 362]	593	[936, 937]	997	[1221, 512]	1401	[725, 128]
190	[363, 364]	594	[938, 454]	998	[626, 1191]	1402	[552, 153]
191	[365, 366]	595	[915, 87]	999	[1284, 1264]	1403	[1230, 527]
192	[367, 368]	596	[334, 778]	1000	[1285, 1224]	1404	[1184, 235]
193	[369, 370]	597	[939, 940]	1001	[1286, 1287]	1405	[715, 1289]
194	[371, 117]	598	[418, 352]	1002	[1192, 511]	1406	[1539, 522]
195	[372, 373]	599	[941, 859]	1003	[229, 712]	1407	[1540, 1541]
196	[374, 375]	600	[942, 943]	1004	[1141, 1259]	1408	[726, 1542]
197	[376, 377]	601	[944, 945]	1005	[586, 607]	1409	[1167, 605]
198	[378, 379]	602	[946, 319]	1006	[527, 719]	1410	[1276, 606]
199	[380, 381]	603	[947, 948]	1007	[467, 715]	1411	[600, 1543]
200	[382, 383]	604	[949, 950]	1008	[711, 555]	1412	[1113, 1544]
201	[384, 385]	605	[951, 952]	1009	[1288, 506]	1413	[726, 1545]
202	[386, 89]	606	[953, 350]	1010	[146, 1289]	1414	[225, 504]
203	[387, 388]	607	[954, 955]	1011	[1290, 1214]	1415	[521, 179]
204	[389, 390]	608	[956, 957]	1012	[1291, 1292]	1416	[643, 1546]
205	[391, 392]	609	[958, 282]	1013	[594, 201]	1417	[182, 150]
206	[393, 73]	610	[959, 960]	1014	[680, 1253]	1418	[1144, 552]
207	[394, 395]	611	[961, 748]	1015	[658, 43]	1419	[641, 1300]
208	[396, 397]	612	[370, 962]	1016	[729, 134]	1420	[1298, 709]
209	[398, 399]	613	[963, 913]	1017	[1165, 1198]	1421	[47, 549]
210	[124, 400]	614	[964, 965]	1018	[1264, 1293]	1422	[1172, 1242]
211	[401, 71]	615	[457, 966]	1019	[1274, 239]	1423	[1547, 1548]
212	[402, 403]	616	[324, 922]	1020	[1286, 1294]	1424	[53, 1262]
213	[404, 405]	617	[967, 968]	1021	[550, 227]	1425	[1288, 629]
214	[406, 278]	618	[353, 333]	1022	[632, 33]	1426	[1257, 601]
215	[407, 408]	619	[823, 969]	1023	[627, 1295]	1427	[1532, 588]
216	[409, 410]	620	[879, 970]	1024	[629, 507]	1428	[1226, 1549]
217	[411, 412]	621	[971, 432]	1025	[498, 178]	1429	[1207, 499]
218	[413, 414]	622	[972, 973]	1026	[1207, 168]	1430	[634, 231]
219	[415, 416]	623	[974, 386]	1027	[1234, 182]	1431	[1550, 1520]
220	[417, 418]	624	[862, 449]	1028	[464, 1160]	1432	[621, 1526]
221	[419, 420]	625	[975, 967]	1029	[1296, 1283]	1433	[741, 1149]
222	[421, 406]	626	[829, 976]	1030	[1284, 1297]	1434	[193, 225]
223	[422, 423]	627	[830, 750]	1031	[1146, 647]	1435	[566, 1544]

224	[337, 424]	628	[977, 808]	1032	[1298, 1276]	1436	[1531, 1225]
225	[425, 426]	629	[978, 926]	1033	[514, 39]	1437	[1551, 1517]
226	[427, 428]	630	[76, 979]	1034	[720, 708]	1438	[1304, 693]
227	[80, 429]	631	[980, 981]	1035	[623, 1299]	1439	[185, 1258]
228	[430, 431]	632	[982, 983]	1036	[1263, 1297]	1440	[1270, 1307]
229	[432, 433]	633	[984, 801]	1037	[224, 648]	1441	[504, 1194]
230	[434, 435]	634	[771, 438]	1038	[667, 1276]	1442	[1235, 1552]
231	[436, 437]	635	[104, 985]	1039	[722, 1137]	1443	[661, 1235]
232	[438, 439]	636	[877, 864]	1040	[655, 1300]	1444	[1515, 1168]
233	[440, 441]	637	[986, 987]	1041	[542, 220]	1445	[1539, 179]
234	[442, 19]	638	[988, 989]	1042	[206, 1301]	1446	[1291, 1533]
235	[443, 444]	639	[100, 990]	1043	[187, 195]	1447	[1219, 545]
236	[445, 446]	640	[991, 870]	1044	[1302, 1303]	1448	[612, 223]
237	[447, 448]	641	[992, 993]	1045	[199, 672]	1449	[1240, 1254]
238	[449, 402]	642	[994, 995]	1046	[484, 689]	1450	[1325, 1553]
239	[450, 451]	643	[996, 997]	1047	[1304, 8]	1451	[1317, 1548]
240	[452, 453]	644	[998, 999]	1048	[1305, 1306]	1452	[214, 1554]
241	[454, 455]	645	[424, 792]	1049	[703, 210]	1453	[1243, 1261]
242	[456, 457]	646	[1000, 1001]	1050	[705, 1185]	1454	[646, 726]
243	[458, 459]	647	[1002, 1003]	1051	[1307, 215]	1455	[1273, 1168]
244	[460, 461]	648	[368, 805]	1052	[1254, 613]	1456	[1311, 1218]
245	[462, 62]	649	[1004, 81]	1053	[1251, 519]	1457	[180, 627]
246	[129, 463]	650	[1005, 1006]	1054	[1182, 1173]	1458	[32, 1536]
247	[464, 465]	651	[794, 23]	1055	[1200, 1200]	1459	[736, 740]
248	[466, 467]	652	[249, 1007]	1056	[1308, 1248]	1460	[1527, 1534]
249	[468, 469]	653	[745, 1008]	1057	[1191, 1295]	1461	[612, 608]
250	[470, 471]	654	[1009, 1010]	1058	[1264, 1309]	1462	[484, 1133]
251	[472, 473]	655	[1011, 857]	1059	[1310, 197]	1463	[1233, 1125]
252	[474, 475]	656	[1012, 1013]	1060	[1277, 164]	1464	[1547, 1318]
253	[476, 477]	657	[1014, 1015]	1061	[632, 1165]	1465	[1267, 1555]
254	[478, 479]	658	[973, 359]	1062	[169, 56]	1466	[61, 1112]
255	[472, 480]	659	[1016, 866]	1063	[522, 180]	1467	[675, 1303]
256	[481, 482]	660	[1017, 1018]	1064	[1265, 169]	1468	[1324, 53]
257	[483, 484]	661	[1019, 1020]	1065	[1311, 1312]	1469	[533, 589]
258	[194, 188]	662	[832, 413]	1066	[1290, 1188]	1470	[1145, 153]
259	[35, 485]	663	[1021, 1022]	1067	[486, 191]	1471	[1556, 1319]
260	[486, 487]	664	[1023, 1024]	1068	[1313, 1144]	1472	[1557, 644]
261	[488, 489]	665	[267, 1025]	1069	[1240, 1179]	1473	[1266, 686]
262	[490, 491]	666	[1026, 1027]	1070	[51, 1314]	1474	[1558, 62]
263	[492, 493]	667	[969, 1028]	1071	[1267, 687]	1475	[1534, 515]
264	[491, 494]	668	[1029, 266]	1072	[1302, 676]	1476	[1140, 151]
265	[495, 496]	669	[375, 1030]	1073	[1281, 224]	1477	[1285, 1531]
266	[497, 498]	670	[1031, 419]	1074	[625, 498]	1478	[545, 562]
267	[499, 500]	671	[1032, 1033]	1075	[1115, 637]	1479	[1537, 1137]

268	[184, 501]	672	[279, 1034]	1076	[149, 631]	1480	[1559, 1299]
269	[502, 503]	673	[1035, 861]	1077	[9, 1175]	1481	[228, 711]
270	[504, 505]	674	[1007, 1036]	1078	[1172, 1268]	1482	[1530, 1224]
271	[506, 507]	675	[1037, 374]	1079	[659, 1315]	1483	[1528, 1287]
272	[508, 509]	676	[997, 376]	1080	[1316, 699]	1484	[1558, 1112]
273	[467, 146]	677	[1038, 1039]	1081	[490, 495]	1485	[1540, 1162]
274	[510, 511]	678	[441, 1040]	1082	[667, 709]	1486	[193, 648]
275	[512, 513]	679	[800, 417]	1083	[499, 161]	1487	[1147, 1143]
276	[514, 141]	680	[405, 1041]	1084	[1317, 1318]	1488	[1560, 1561]
277	[515, 516]	681	[106, 1042]	1085	[1319, 1245]	1489	[695, 664]
278	[49, 212]	682	[1043, 427]	1086	[1320, 1279]	1490	[1562, 1272]
279	[137, 517]	683	[1044, 1045]	1087	[1310, 1205]	1491	[1248, 721]
280	[177, 518]	684	[1046, 288]	1088	[1216, 169]	1492	[1223, 463]
281	[233, 154]	685	[1047, 1048]	1089	[1321, 1277]	1493	[239, 569]
282	[519, 520]	686	[838, 1049]	1090	[519, 1156]	1494	[543, 1158]
283	[521, 522]	687	[1050, 1051]	1091	[1164, 148]	1495	[1557, 1546]
284	[523, 524]	688	[255, 1052]	1092	[673, 1204]	1496	[1215, 1207]
285	[525, 526]	689	[358, 1053]	1093	[238, 568]	1497	[32, 1552]
286	[41, 221]	690	[1054, 1055]	1094	[563, 728]	1498	[651, 145]
287	[527, 12]	691	[935, 113]	1095	[1117, 571]	1499	[1131, 635]
288	[528, 529]	692	[431, 1056]	1096	[1322, 1292]	1500	[1560, 733]
289	[530, 219]	693	[364, 1002]	1097	[575, 738]	1501	[1321, 163]
290	[531, 532]	694	[1057, 1058]	1098	[1323, 734]	1502	[666, 1298]
291	[533, 534]	695	[1059, 984]	1099	[702, 1165]	1503	[1210, 175]
292	[535, 536]	696	[813, 1060]	1100	[635, 1154]	1504	[128, 1542]
293	[537, 491]	697	[1061, 1062]	1101	[1209, 237]	1505	[1516, 1563]
294	[538, 539]	698	[1063, 1064]	1102	[1324, 1186]	1506	[1534, 610]
295	[540, 541]	699	[332, 807]	1103	[1231, 1126]	1507	[1129, 1306]
296	[542, 41]	700	[1065, 919]	1104	[738, 1133]	1508	[200, 1251]
297	[543, 544]	701	[914, 916]	1105	[1247, 1174]	1509	[679, 721]
298	[545, 538]	702	[886, 69]	1106	[1238, 1136]	1510	[1564, 1553]
299	[205, 546]	703	[1066, 1059]	1107	[167, 1200]	1511	[1562, 1565]
300	[547, 517]	704	[1067, 1068]	1108	[699, 1254]	1512	[595, 658]
301	[548, 549]	705	[1069, 933]	1109	[1228, 1231]	1513	[1550, 1566]
302	[550, 551]	706	[1049, 107]	1110	[1325, 1326]	1514	[1524, 1298]
303	[552, 233]	707	[1070, 1071]	1111	[237, 1183]	1515	[1567, 1423]
304	[553, 554]	708	[1072, 1073]	1112	[867, 1327]	1516	[1568, 982]
305	[229, 555]	709	[1074, 998]	1113	[1328, 1021]	1517	[426, 1567]
306	[556, 557]	710	[1075, 1076]	1114	[1329, 883]	1518	[1569, 304]
307	[558, 559]	711	[892, 1077]	1115	[90, 814]	1519	[1570, 335]
308	[560, 561]	712	[950, 5]	1116	[1010, 1330]	1520	[345, 1468]
309	[562, 531]	713	[821, 1078]	1117	[1331, 321]	1521	[1041, 1571]
310	[39, 142]	714	[1079, 1080]	1118	[1332, 442]	1522	[970, 1459]
311	[563, 564]	715	[751, 1081]	1119	[1333, 1334]	1523	[932, 1451]

312	[565, 243]	716	[1082, 763]	1120	[360, 1023]	1524	[1572, 1476]
313	[566, 567]	717	[1077, 842]	1121	[1335, 1336]	1525	[1573, 1464]
314	[568, 569]	718	[1083, 1004]	1122	[758, 1337]	1526	[1039, 1574]
315	[570, 571]	719	[1084, 1085]	1123	[126, 939]	1527	[1575, 959]
316	[572, 573]	720	[1086, 1087]	1124	[1338, 415]	1528	[1576, 754]
317	[474, 574]	721	[1088, 85]	1125	[873, 1339]	1529	[1577, 1578]
318	[575, 484]	722	[1089, 1090]	1126	[1340, 1341]	1530	[1401, 1368]
319	[576, 577]	723	[1087, 1091]	1127	[747, 443]	1531	[312, 450]
320	[578, 183]	724	[1092, 272]	1128	[4, 1338]	1532	[1579, 929]
321	[579, 580]	725	[1093, 1044]	1129	[773, 1342]	1533	[1438, 1580]
322	[515, 581]	726	[1094, 1094]	1130	[1342, 1343]	1534	[268, 822]
323	[582, 583]	727	[1095, 387]	1131	[1344, 1345]	1535	[1581, 1088]
324	[584, 585]	728	[1096, 1097]	1132	[983, 1346]	1536	[64, 1582]
325	[586, 587]	729	[1098, 436]	1133	[777, 1083]	1537	[1030, 1352]
326	[588, 589]	730	[852, 1099]	1134	[1347, 1084]	1538	[1053, 958]
327	[478, 590]	731	[1091, 1100]	1135	[310, 290]	1539	[1583, 1398]
328	[591, 592]	732	[955, 986]	1136	[1348, 1349]	1540	[1584, 1031]
329	[593, 574]	733	[120, 1101]	1137	[1350, 1351]	1541	[1481, 1585]
330	[594, 595]	734	[948, 1102]	1138	[1352, 1353]	1542	[1001, 1586]
331	[501, 577]	735	[285, 1103]	1139	[999, 1057]	1543	[1377, 1587]
332	[596, 597]	736	[1104, 1105]	1140	[1354, 1355]	1544	[766, 1588]
333	[598, 596]	737	[299, 1106]	1141	[316, 389]	1545	[246, 1589]
334	[578, 596]	738	[1107, 1054]	1142	[1356, 338]	1546	[1414, 1000]
335	[599, 600]	739	[1108, 816]	1143	[1357, 833]	1547	[1590, 284]
336	[601, 602]	740	[1109, 1110]	1144	[871, 440]	1548	[1591, 840]
337	[138, 537]	741	[1111, 890]	1145	[769, 294]	1549	[1052, 1440]
338	[603, 583]	742	[462, 1112]	1146	[888, 1061]	1550	[1592, 1591]
339	[604, 605]	743	[1113, 567]	1147	[1358, 1359]	1551	[1593, 1109]
340	[606, 607]	744	[1114, 711]	1148	[1025, 1360]	1552	[1020, 1594]
341	[222, 608]	745	[1115, 1116]	1149	[940, 844]	1553	[843, 1595]
342	[561, 547]	746	[1117, 620]	1150	[1361, 270]	1554	[1453, 1596]
343	[548, 48]	747	[1118, 592]	1151	[1081, 264]	1555	[1597, 1598]
344	[609, 600]	748	[739, 737]	1152	[1362, 956]	1556	[1599, 1096]
345	[610, 581]	749	[622, 201]	1153	[1363, 393]	1557	[1600, 1393]
346	[611, 612]	750	[138, 490]	1154	[1364, 1365]	1558	[1477, 1577]
347	[613, 614]	751	[724, 1119]	1155	[1034, 1366]	1559	[1601, 1569]
348	[615, 616]	752	[1120, 695]	1156	[388, 1367]	1560	[781, 1017]
349	[530, 617]	753	[562, 539]	1157	[1368, 1369]	1561	[1090, 1602]
350	[576, 618]	754	[226, 551]	1158	[989, 1370]	1562	[1603, 1487]
351	[619, 598]	755	[145, 715]	1159	[1371, 1357]	1563	[966, 1354]
352	[174, 598]	756	[649, 630]	1160	[1372, 1373]	1564	[412, 286]
353	[174, 578]	757	[1121, 1122]	1161	[1374, 79]	1565	[428, 1361]
354	[570, 620]	758	[689, 1123]	1162	[1027, 361]	1566	[839, 346]
355	[621, 482]	759	[1124, 10]	1163	[1375, 1376]	1567	[1525, 53]

356	[622, 595]	760	[1125, 1126]	1164	[1100, 1377]	1568	[1604, 1549]
357	[623, 624]	761	[595, 202]	1165	[962, 262]	1569	[1114, 229]
358	[625, 177]	762	[1127, 1128]	1166	[336, 1378]	1570	[1519, 1566]
359	[626, 627]	763	[654, 546]	1167	[444, 396]	1571	[540, 1543]
360	[628, 629]	764	[1129, 1130]	1168	[343, 421]	1572	[237, 175]
361	[553, 630]	765	[179, 626]	1169	[1379, 1380]	1573	[1605, 1267]
362	[182, 631]	766	[1131, 231]	1170	[1381, 1072]	1574	[1305, 1130]
363	[157, 632]	767	[577, 701]	1171	[1382, 1383]	1575	[672, 1197]
364	[158, 633]	768	[1121, 1132]	1172	[937, 759]	1576	[466, 145]
365	[634, 635]	769	[599, 540]	1173	[1384, 1385]	1577	[1161, 1541]
366	[636, 637]	770	[1133, 690]	1174	[1106, 1386]	1578	[1531, 1606]
367	[46, 601]	771	[1134, 11]	1175	[1387, 1388]	1579	[1523, 706]
368	[638, 639]	772	[1135, 1136]	1176	[1389, 944]	1580	[1564, 1326]
369	[494, 640]	773	[1137, 1138]	1177	[1390, 17]	1581	[489, 168]
370	[641, 642]	774	[738, 689]	1178	[1391, 1392]	1582	[128, 1545]
371	[643, 644]	775	[1139, 693]	1179	[1393, 964]	1583	[1257, 139]
372	[225, 645]	776	[489, 499]	1180	[347, 1394]	1584	[557, 702]
373	[646, 128]	777	[1140, 1141]	1181	[1360, 1395]	1585	[732, 1561]
374	[468, 647]	778	[596, 184]	1182	[1396, 348]	1586	[1224, 1606]
375	[648, 645]	779	[1142, 132]	1183	[1397, 931]	1587	[506, 1199]
376	[649, 554]	780	[636, 1116]	1184	[448, 790]	1588	[1148, 60]
377	[512, 650]	781	[1143, 650]	1185	[1398, 934]	1589	[62, 1607]
378	[651, 467]	782	[1144, 1145]	1186	[1040, 1399]	1590	[201, 658]
379	[652, 653]	783	[1146, 469]	1187	[1400, 820]	1591	[1139, 8]
380	[477, 610]	784	[1147, 512]	1188	[755, 1401]	1592	[1608, 1122]
381	[654, 206]	785	[1148, 230]	1189	[399, 1402]	1593	[1551, 1563]
382	[561, 137]	786	[1149, 562]	1190	[1403, 1387]	1594	[239, 1163]
383	[655, 642]	787	[1150, 39]	1191	[108, 991]	1595	[1535, 679]
384	[597, 576]	788	[496, 560]	1192	[275, 1404]	1596	[1307, 1554]
385	[131, 656]	789	[491, 496]	1193	[1405, 1406]	1597	[1260, 1244]
386	[497, 177]	790	[640, 561]	1194	[907, 835]	1598	[686, 1555]
387	[657, 480]	791	[495, 494]	1195	[1407, 1408]	1599	[1556, 706]
388	[658, 659]	792	[537, 495]	1196	[896, 1409]	1600	[1271, 1565]
389	[525, 585]	793	[1151, 137]	1197	[1033, 1410]	1601	[1282, 1538]
390	[206, 660]	794	[494, 560]	1198	[1015, 1411]	1602	[1275, 466]
391	[661, 32]	795	[493, 492]	1199	[1412, 1413]	1603	[1559, 624]
392	[627, 662]	796	[1152, 222]	1200	[320, 767]	1604	[1609, 425]
393	[663, 36]	797	[203, 217]	1201	[96, 1414]	1605	[1610, 1597]
394	[580, 664]	798	[1153, 536]	1202	[880, 1415]	1606	[1485, 1611]
395	[602, 665]	799	[517, 537]	1203	[1416, 784]	1607	[743, 1612]
396	[659, 44]	800	[618, 701]	1204	[960, 1381]	1608	[1613, 1463]
397	[666, 667]	801	[231, 1154]	1205	[858, 1417]	1609	[1604, 1227]
398	[668, 669]	802	[1155, 1156]	1206	[1418, 302]	1610	[1605, 686]
399	[670, 144]	803	[556, 157]	1207	[1366, 1419]	1611	[1112, 1529]

400	[671, 672]	804	[1153, 1157]	1208	[1036, 1405]	1612	[1112, 1607]
401	[673, 674]	805	[601, 140]	1209	[1420, 1397]	1613	[1608, 1132]
402	[675, 676]	806	[639, 1158]	1210	[799, 1421]		
403	[677, 678]	807	[1159, 1160]	1211	[1022, 1422]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	270	0	540	1	810	1	1080	1	1350	0
1	0	271	0	541	1	811	1	1081	0	1351	1
2	1	272	0	542	1	812	1	1082	1	1352	1
3	0	273	0	543	1	813	1	1083	0	1353	0
4	1	274	1	544	0	814	0	1084	0	1354	0
5	1	275	0	545	1	815	1	1085	1	1355	1
6	0	276	1	546	0	816	0	1086	0	1356	0
7	0	277	0	547	1	817	0	1087	1	1357	1
8	0	278	0	548	0	818	0	1088	0	1358	1
9	1	279	0	549	0	819	1	1089	1	1359	0
10	1	280	0	550	1	820	0	1090	1	1360	0
11	1	281	0	551	1	821	1	1091	1	1361	0
12	1	282	1	552	1	822	1	1092	1	1362	0
13	0	283	0	553	0	823	1	1093	1	1363	0
14	0	284	1	554	0	824	1	1094	0	1364	1
15	0	285	0	555	1	825	0	1095	1	1365	1
16	0	286	0	556	0	826	1	1096	0	1366	1
17	0	287	0	557	0	827	0	1097	1	1367	0
18	0	288	0	558	1	828	0	1098	0	1368	0
19	1	289	0	559	0	829	0	1099	0	1369	0
20	0	290	0	560	1	830	0	1100	1	1370	1
21	1	291	1	561	1	831	1	1101	0	1371	0
22	1	292	1	562	0	832	0	1102	1	1372	0
23	1	293	1	563	0	833	0	1103	1	1373	0
24	0	294	1	564	1	834	0	1104	0	1374	1
25	1	295	1	565	0	835	0	1105	0	1375	0
26	1	296	1	566	1	836	0	1106	0	1376	1
27	0	297	1	567	0	837	0	1107	0	1377	0
28	0	298	1	568	0	838	0	1108	1	1378	0
29	0	299	0	569	1	839	0	1109	1	1379	0
30	1	300	1	570	0	840	1	1110	0	1380	1
31	0	301	0	571	0	841	0	1111	0	1381	1
32	1	302	1	572	1	842	0	1112	0	1382	0
33	0	303	1	573	0	843	1	1113	0	1383	0
34	0	304	0	574	0	844	0	1114	0	1384	0
35	0	305	0	575	1	845	1	1115	0	1385	1
36	0	306	0	576	0	846	1	1116	0	1386	0

37	0	307	1	577	1	847	0	1117	1	1387	0
38	0	308	1	578	1	848	1	1118	1	1388	1
39	1	309	0	579	1	849	1	1119	1	1389	0
40	1	310	1	580	0	850	0	1120	0	1390	0
41	0	311	0	581	0	851	1	1121	1	1391	0
42	1	312	0	582	1	852	1	1122	1	1392	1
43	1	313	1	583	0	853	1	1123	1	1393	0
44	0	314	0	584	1	854	0	1124	0	1394	0
45	1	315	0	585	0	855	1	1125	0	1395	0
46	0	316	1	586	0	856	1	1126	1	1396	0
47	1	317	0	587	1	857	1	1127	1	1397	1
48	1	318	1	588	0	858	0	1128	1	1398	0
49	0	319	0	589	0	859	1	1129	0	1399	1
50	1	320	1	590	1	860	0	1130	1	1400	1
51	1	321	1	591	0	861	1	1131	1	1401	1
52	0	322	0	592	1	862	1	1132	0	1402	1
53	1	323	1	593	1	863	1	1133	0	1403	1
54	0	324	1	594	1	864	0	1134	0	1404	1
55	0	325	0	595	0	865	1	1135	1	1405	1
56	1	326	0	596	1	866	1	1136	0	1406	1
57	0	327	1	597	1	867	0	1137	0	1407	0
58	1	328	0	598	0	868	0	1138	1	1408	0
59	0	329	1	599	0	869	0	1139	0	1409	0
60	0	330	1	600	0	870	1	1140	0	1410	0
61	1	331	1	601	1	871	0	1141	1	1411	0
62	1	332	1	602	1	872	0	1142	0	1412	0
63	0	333	0	603	0	873	0	1143	1	1413	0
64	0	334	1	604	1	874	0	1144	0	1414	1
65	1	335	0	605	1	875	0	1145	0	1415	0
66	1	336	1	606	1	876	0	1146	1	1416	0
67	0	337	0	607	0	877	1	1147	1	1417	0
68	0	338	0	608	1	878	1	1148	1	1418	0
69	0	339	1	609	1	879	1	1149	0	1419	1
70	0	340	1	610	1	880	1	1150	0	1420	0
71	0	341	0	611	1	881	0	1151	0	1421	1
72	1	342	1	612	1	882	0	1152	0	1422	1
73	0	343	0	613	1	883	1	1153	0	1423	1
74	1	344	1	614	1	884	1	1154	1	1424	1
75	0	345	1	615	0	885	0	1155	0	1425	1
76	1	346	1	616	1	886	0	1156	1	1426	1
77	0	347	1	617	1	887	1	1157	0	1427	0
78	1	348	0	618	0	888	1	1158	1	1428	0
79	1	349	0	619	1	889	1	1159	0	1429	1
80	0	350	0	620	1	890	0	1160	0	1430	0

81	1	351	1	621	1	891	0	1161	1	1431	0
82	0	352	0	622	0	892	1	1162	0	1432	1
83	0	353	0	623	1	893	1	1163	0	1433	0
84	0	354	0	624	1	894	1	1164	1	1434	1
85	1	355	1	625	1	895	0	1165	1	1435	1
86	1	356	0	626	0	896	0	1166	1	1436	0
87	1	357	1	627	0	897	1	1167	0	1437	0
88	1	358	1	628	0	898	0	1168	0	1438	1
89	0	359	0	629	1	899	0	1169	0	1439	0
90	0	360	0	630	1	900	0	1170	1	1440	1
91	1	361	0	631	1	901	1	1171	0	1441	0
92	1	362	0	632	1	902	1	1172	1	1442	0
93	0	363	1	633	1	903	0	1173	0	1443	0
94	0	364	1	634	0	904	1	1174	0	1444	0
95	1	365	0	635	1	905	0	1175	0	1445	1
96	0	366	1	636	1	906	1	1176	0	1446	1
97	1	367	0	637	1	907	0	1177	0	1447	1
98	0	368	0	638	0	908	0	1178	0	1448	1
99	0	369	1	639	0	909	1	1179	0	1449	0
100	0	370	1	640	0	910	0	1180	1	1450	0
101	1	371	0	641	1	911	1	1181	0	1451	0
102	1	372	1	642	1	912	0	1182	0	1452	1
103	1	373	0	643	0	913	0	1183	1	1453	0
104	1	374	0	644	0	914	0	1184	1	1454	0
105	0	375	0	645	1	915	0	1185	0	1455	1
106	0	376	1	646	0	916	1	1186	0	1456	1
107	1	377	0	647	0	917	1	1187	1	1457	1
108	1	378	1	648	0	918	1	1188	1	1458	1
109	0	379	0	649	1	919	1	1189	1	1459	0
110	1	380	1	650	0	920	0	1190	1	1460	0
111	0	381	1	651	1	921	0	1191	1	1461	1
112	1	382	1	652	0	922	1	1192	0	1462	1
113	1	383	0	653	0	923	1	1193	1	1463	0
114	0	384	1	654	1	924	0	1194	0	1464	1
115	0	385	1	655	0	925	0	1195	0	1465	1
116	0	386	0	656	1	926	0	1196	0	1466	1
117	1	387	1	657	1	927	0	1197	1	1467	0
118	0	388	1	658	1	928	0	1198	1	1468	1
119	0	389	0	659	0	929	0	1199	0	1469	1
120	0	390	1	660	1	930	0	1200	1	1470	0
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122	1	392	0	662	0	932	0	1202	1	1472	1
123	1	393	1	663	1	933	0	1203	0	1473	1
124	1	394	0	664	0	934	1	1204	1	1474	0

125	1	395	1	665	0	935	1	1205	0	1475	0
126	1	396	0	666	1	936	1	1206	0	1476	0
127	0	397	1	667	1	937	1	1207	1	1477	0
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151	0	421	0	691	1	961	1	1231	1	1501	1
152	0	422	0	692	1	962	1	1232	0	1502	1
153	1	423	0	693	1	963	1	1233	0	1503	1
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158	1	428	0	698	0	968	0	1238	0	1508	1
159	0	429	1	699	1	969	1	1239	0	1509	0
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161	1	431	1	701	0	971	1	1241	1	1511	0
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167	0	437	1	707	1	977	0	1247	0	1517	1
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172	1	442	0	712	0	982	1	1252	1	1522	1
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177	0	447	0	717	1	987	1	1257	1	1527	0
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180	1	450	1	720	0	990	1	1260	1	1530	1
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183	0	453	1	723	1	993	1	1263	1	1533	1
184	0	454	1	724	1	994	1	1264	1	1534	0
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192	0	462	0	732	0	1002	0	1272	1	1542	1
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211	1	481	0	751	1	1021	1	1291	1	1561	1
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228	1	498	1	768	1	1038	1	1308	1	1578	0
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257	0	527	0	797	1	1067	0	1337	0	1607	0
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267	0	537	1	807	0	1077	1	1347	0		
268	0	538	1	808	1	1078	1	1348	0		
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