PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z+1,z^6+z^3+z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z+1,z^6+z^3+z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}}\in\mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b):=\mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a, b) = (z+1, z^6+z^3+z+1)$*, the two power series* $\mathrm{CF}(a, b)$ *and* $\mathrm{CF}(b, a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4 + p_3(z)y^3 + p_2(z)y^2 + p_1(z)y + p_0(z) = 0$$

where, for $\mathrm{CF}(a, b)$*,*

$$\begin{aligned}
p_0(z) &= z^{20} + z^{17} + z^{14} + z^{12} + z^{11} + z^7 + z^4 + z^3 + z^2 + 1,\\
p_1(z) &= z^{23} + z^{22} + z^{19} + z^{16} + z^{12} + z^{11} + z^8 + z^7 + z^5 + z^3,\\
p_2(z) &= z^{24} + z^{22} + z^{20} + z^{18} + z^{16} + z^{14} + z^{12} + z^6 + z^4 + 1,\\
p_3(z) &= z^{23} + z^{22} + z^{19} + z^{16} + z^{12} + z^{11} + z^8 + z^7 + z^5 + z^3,\\
p_4(z) &= z^{22} + z^{20} + z^{18} + z^{17} + z^{11} + z^7 + z^6 + z^4 + z^3 + z^2 + 1,
\end{aligned}$$

and for $\mathrm{CF}(b, a)$*,*

$$\begin{aligned}
p_0(z) &= z^{17} + z^{16} + z^{14} + z^{11} + z^7 + z^4 + z^3 + z^2 + 1,\\
p_1(z) &= z^{23} + z^{22} + z^{19} + z^{16} + z^{12} + z^{11} + z^8 + z^7 + z^5 + z^3,\\
p_2(z) &= z^{24} + z^{22} + z^{20} + z^{18} + z^{16} + z^{14} + z^{12} + z^6 + z^4 + 1,\\
p_3(z) &= z^{23} + z^{22} + z^{19} + z^{16} + z^{12} + z^{11} + z^8 + z^7 + z^5 + z^3,\\
p_4(z) &= z^{22} + z^{18} + z^{17} + z^{16} + z^{12} + z^{11} + z^7 + z^6 + z^4 + z^3 + z^2 + 1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\ge 0}$ and $(Q_n(z))_{n\ge 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z) \\ P_{n-1}(z) & Q_{n-1}(z) \end{pmatrix} := \begin{pmatrix} t_n(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t_{n-1}(z) & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} t_0(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a, b) = \frac{P_n(z)}{Q_n(z)} \in \mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a, b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x) = {} & x^{\deg(t_{2^n-1})} \begin{pmatrix} t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(t_{2^n-2})} \begin{pmatrix} t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\
& \cdots \times x^{\deg(t_0)} \begin{pmatrix} t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},\\
W_n(x) = {} & x^{\deg(\bar t_{2^n-1})} \begin{pmatrix} \bar t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(\bar t_{2^n-2})} \begin{pmatrix} \bar t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\
& \cdots \times x^{\deg(\bar t_0)} \begin{pmatrix} \bar t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b, a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \ge 0$

$$\begin{aligned}
M_{n+1}(x) &= W_n(x) \cdot M_n(x),\\
W_{n+1}(x) &= M_n(x) \cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \dots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x, y) = \mathrm{Res}_z \left(\phi(W^o, 0, 0)(x, z),\ z^{12} \cdot \phi(M^o, 0, 0)(x, y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x, y)$. To prove that $\phi_0(x, y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and that $Q(x, y) := P(x, y)/\phi_0(x, y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 630, which proves that $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x, y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e, 0, 0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x, y) = \mathrm{Res}_z \left(\phi_0(x, z),\ \phi_1(x, y + z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e, 0, 0)$ is an irreducible factor of $S(x, y)$ of multiplicity μ, and that $Q(x, y) := S(x, y)/\phi(M^e, 0, 0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 770, and therefore $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e, 0, 0)$, we know that the two series are one and the same. $\square$

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^u M^e[:6u] + x^{2u} M^e[:5u] + x^{4u} M^e[:3u] + x^{6u} M^e[:u] \\ & + x^u M^e[:7u] + x^{2u} M^e[:6u] + x^{3u} M^e[:5u] + x^{5u} M^e[:3u] \\ & + x^{7u} M^e[:u] + x^{3u} M^e[:7u] + x^{4u} M^e[:6u] + x^{5u} M^e[:5u] \\ & + x^{7u} M^e[:3u] + x^{9u} M^e[:u] + x^{6u} M^e[:7u] + x^{7u} M^e[:6u] \\ & + x^{8u} M^e[:5u] + x^{10u} M^e[:3u] + x^{12u} M^e[:u] + x^{7u} M^e[:7u] \\ & + x^{12u} M^e[:2u] + (x^{7u} + x^{8u} + x^{10u} + x^{13u}) R^e, \end{aligned}$$

$$\begin{aligned}
W^e[:14u] &= W^e[:7u] + x^u W^e[:6u] + x^{2u} W^e[:5u] + x^{4u} W^e[:3u] + x^{6u} W^e[:u] \\
&\quad + x^u W^e[:7u] + x^{2u} W^e[:6u] + x^{3u} W^e[:5u] + x^{5u} W^e[:3u] \\
&\quad + x^{7u} W^e[:u] + x^{3u} W^e[:7u] + x^{4u} W^e[:6u] + x^{5u} W^e[:5u] \\
&\quad + x^{7u} W^e[:3u] + x^{9u} W^e[:u] + x^{6u} W^e[:7u] + x^{7u} W^e[:6u] \\
&\quad + x^{8u} W^e[:5u] + x^{10u} W^e[:3u] + x^{12u} W^e[:u] + x^{7u} W^e[:7u] \\
&\quad + x^{12u} W^e[:2u] + (x^{7u} + x^{8u} + x^{10u} + x^{13u}) R^e.
\end{aligned}$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}
M^o[:14u] &= M^o[:7u] + x^u M^o[:6u] + x^{2u} M^o[:5u] + x^{4u} M^o[:3u] + x^{6u} M^o[:u] \\
&\quad + x^u M^o[:7u] + x^{2u} M^o[:6u] + x^{3u} M^o[:5u] + x^{5u} M^o[:3u] \\
&\quad + x^{7u} M^o[:u] + x^{3u} M^o[:7u] + x^{4u} M^o[:6u] + x^{5u} M^o[:5u] \\
&\quad + x^{7u} M^o[:3u] + x^{9u} M^o[:u] + x^{6u} M^o[:7u] + x^{7u} M^o[:6u] \\
&\quad + x^{8u} M^o[:5u] + x^{10u} M^o[:3u] + x^{12u} M^o[:u] + x^{7u} M^o[:7u] \\
&\quad + x^{12u} M^o[:2u] + (x^{7u} + x^{8u} + x^{10u} + x^{13u}) R^o, \\
W^o[:14u] &= W^o[:7u] + x^u W^o[:6u] + x^{2u} W^o[:5u] + x^{4u} W^o[:3u] + x^{6u} W^o[:u] \\
&\quad + x^u W^o[:7u] + x^{2u} W^o[:6u] + x^{3u} W^o[:5u] + x^{5u} W^o[:3u] \\
&\quad + x^{7u} W^o[:u] + x^{3u} W^o[:7u] + x^{4u} W^o[:6u] + x^{5u} W^o[:5u] \\
&\quad + x^{7u} W^o[:3u] + x^{9u} W^o[:u] + x^{6u} W^o[:7u] + x^{7u} W^o[:6u] \\
&\quad + x^{8u} W^o[:5u] + x^{10u} W^o[:3u] + x^{12u} W^o[:u] + x^{7u} W^o[:7u] \\
&\quad + x^{12u} W^o[:2u] + (x^{7u} + x^{8u} + x^{10u} + x^{13u}) R^o.
\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:14u] &= T[:7u] + x^u T[:6u] + x^{2u} T[:5u] + x^{4u} T[:3u] + x^{6u} T[:u] + x^u T[:7u] \\
&\quad + x^{2u} T[:6u] + x^{3u} T[:5u] + x^{5u} T[:3u] + x^{7u} T[:u] + x^{3u} T[:7u] \\
&\quad + x^{4u} T[:6u] + x^{5u} T[:5u] + x^{7u} T[:3u] + x^{9u} T[:u] + x^{6u} T[:7u] \\
&\quad + x^{7u} T[:6u] + x^{8u} T[:5u] + x^{10u} T[:3u] + x^{12u} T[:u] + x^{7u} T[:7u] \\
&\quad + x^{12u} T[:2u] + (x^{7u} + x^{8u} + x^{10u} + x^{13u}).
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
x^7 u &= x^{6u} T[u:2u] + x^{4u} T[3u:4u] + x^{2u} T[5u:6u] + x^u T[6u:7u] \\
&\quad + T[7u:8u], \\
x^8 u &= x^{8u} T[:u] + x^{7u} T[u:2u] + x^{6u} T[2u:3u] + x^{5u} T[3u:4u] \\
&\quad + x^{4u} T[4u:5u] + x^{3u} T[5u:6u] + T[8u:9u], \\
0 &= x^{9u} T[:u] + x^{8u} T[u:2u] + x^{7u} T[2u:3u] + x^{6u} T[3u:4u] \\
&\quad + x^{5u} T[4u:5u] + x^{4u} T[5u:6u] + x^{3u} T[6u:7u] + T[9u:10u],
\end{aligned}$$

$$\begin{aligned}
x^10u &= x^{10u}T[:u] + x^{8u}T[2u:3u] + x^{6u}T[4u:5u] + T[10u:11u],\\
0 &= x^{10u}T[u:2u] + x^{8u}T[3u:4u] + x^{6u}T[5u:6u] + T[11u:12u],\\
0 &= x^{10u}T[2u:3u] + x^{8u}T[4u:5u] + x^{6u}T[6u:7u] + T[12u:13u],\\
x^13u &= x^{12u}T[u:2u] + x^{7u}T[6u:7u] + T[13u:14u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.7}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\ & + T[[101w]_2] + T[[1000w]_2],\end{aligned} \tag{2.8}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\ & + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2],\end{aligned} \tag{2.9}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[1010w]_2], \tag{2.10}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1011w]_2], \tag{2.11}$$

$$0 = T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[1w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.14}$$

$$\begin{aligned}1 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\ & + T[[101w]_2] + T[[1000w]_2],\end{aligned} \tag{2.15}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\ & + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2],\end{aligned} \tag{2.16}$$

$$1 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[1010w]_2], \tag{2.17}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1011w]_2], \tag{2.18}$$

$$0 = T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$

$$1 = T[[1w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1543 states; its transition function and output function can be found in the annex. Let $A(s,w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i,w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$\begin{aligned}0 = {} & \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110))\\ & + \tau(A(s,111)),\end{aligned} \tag{2.21}$$

$$\begin{aligned}0 = {} & \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100))\\ & + \tau(A(s,101)) + \tau(A(s,1000)),\end{aligned} \tag{2.22}$$

$$\begin{aligned}0 = {} & \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100))\\ & + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1001)),\end{aligned} \tag{2.23}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,1010)), \tag{2.24}$$
$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,1011)), \tag{2.25}$$
$$0 = \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.26}$$
$$0 = \tau(A(s,1)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$\begin{aligned} 1 &= \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) \\ &\quad + \tau(A(s,111)), \end{aligned} \tag{2.28}$$
$$\begin{aligned} 1 &= \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,101)) + \tau(A(s,1000)), \end{aligned} \tag{2.29}$$
$$\begin{aligned} 0 &= \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1001)), \end{aligned} \tag{2.30}$$
$$1 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,1010)), \tag{2.31}$$
$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,1011)), \tag{2.32}$$
$$0 = \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.33}$$
$$1 = \tau(A(s,1)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{26}), E_{26}) = (A(i, 0^{14}), E_{14})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 13$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned} M_{2k} &= M^e[:7u] + x^u M^e[:6u] + x^{2u} M^e[:5u] + x^{4u} M^e[:3u] + x^{6u} M^e[:u] \\ &\quad + x^{7u} R^e, \\ W_{2k} &= W^e[:7u] + x^u W^e[:6u] + x^{2u} W^e[:5u] + x^{4u} W^e[:3u] + x^{6u} W^e[:u] \\ &\quad + x^{7u} R^e. \end{aligned}$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$\begin{aligned} M_{2k+1} &= M^o[:7u] + x^u M^o[:6u] + x^{2u} M^o[:5u] + x^{4u} M^o[:3u] + x^{6u} M^o[:u] \\ &\quad + x^{7u} R^o, \\ W_{2k+1} &= W^o[:7u] + x^u W^o[:6u] + x^{2u} W^o[:5u] + x^{4u} W^o[:3u] + x^{6u} W^o[:u] \\ &\quad + x^{7u} R^o. \end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$\begin{aligned} M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\ M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}. \end{aligned}$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:7v] + x^v W^e[:6v] + x^{2v}W^e[:5v] + x^{4v}W^e[:3v] + x^{6v}W^e[:v] \\ &+ x^{7u}R^e\big) \times \big(M^e[:7v] + x^v M^e[:6v] + x^{2v}M^e[:5v] + x^{4v}M^e[:3v] \\ &+ x^{6v}M^e[:v] + x^{7u}R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} W^e[:14v]M^e[:14v] + x^{2v}W^e[:12v]M^e[:12v] + x^{4v}W^e[:10v]M^e[:10v] \\ + x^{8v}W^e[:6v]M^e[:6v] + x^{12v}W^e[:2v]M^e[:2v]. \end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n\cdot v]$ and $M^e[:n\cdot v]$ in the above expression by the reduction modulo $x^{n\cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n\cdot v : (n+1)\cdot v]/X^n, \\ b_n &:= M^e[n\cdot v : (n+1)\cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a,b)$; for $\mathrm{CF}(b,a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j,k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{24} + x^{22} + x^{21} + x^{20} + x^{17} + x^{13} + x^{12} + x^{10} + x^7 + x^4,\\
q_1(x) &= x^{21} + x^{19} + x^{17} + x^{16} + x^{13} + x^{12} + x^8 + x^5 + x^2 + x,\\
q_2(x) &= x^{24} + x^{20} + x^{18} + x^{12} + x^{10} + x^8 + x^6 + x^4 + x^2 + 1,\\
q_3(x) &= x^{21} + x^{19} + x^{17} + x^{16} + x^{13} + x^{12} + x^8 + x^5 + x^2 + x,\\
q_4(x) &= x^{24} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{13} + x^7 + x^6 + x^4 + x^2.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\mathrm{CF}(a, b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{144} + x^{140} + x^{138} + x^{132} + x^{128} + x^{126} + x^{124} + x^{122} + x^{120} + x^{118}\\
& + x^{116} + x^{114} + x^{110} + x^{108} + x^{100} + x^{96} + x^{94} + x^{88} + x^{82} + x^{78}\\
& + x^{76} + x^{72} + x^{70} + x^{62} + x^{60} + x^{56} + x^{54} + x^{50} + x^{48} + x^{44} + x^{42}\\
& + x^{40} + x^{38} + x^{34} + x^{30} + x^{24} + x^{22} + x^{16} + x^{12},\\
p_3(x) = {} & x^{138} + x^{132} + x^{126} + x^{120} + x^{116} + x^{114} + x^{108} + x^{102} + x^{100} + x^{98}\\
& + x^{96} + x^{94} + x^{84} + x^{78} + x^{76} + x^{74} + x^{70} + x^{68} + x^{66} + x^{60} + x^{56}\\
& + x^{48} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{24} + x^{22} + x^{16} + x^8,\\
p_6(x) = {} & x^{140} + x^{138} + x^{134} + x^{128} + x^{124} + x^{122} + x^{120} + x^{118} + x^{114} + x^{108}\\
& + x^{104} + x^{102} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{76} + x^{74} + x^{70}\\
& + x^{60} + x^{56} + x^{54} + x^{50} + x^{46} + x^{42} + x^{40} + x^{30} + x^{28} + x^{26} + x^{24}\\
& + x^{16} + x^{14} + x^{12} + x^6 + x^2 + 1,\\
p_9(x) = {} & x^{138} + x^{134} + x^{132} + x^{130} + x^{128} + x^{118} + x^{110} + x^{106} + x^{104} + x^{100}\\
& + x^{96} + x^{88} + x^{86} + x^{80} + x^{78} + x^{76} + x^{72} + x^{70} + x^{66} + x^{62} + x^{60}\\
& + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{26} + x^{24} + x^{16}\\
& + x^{12} + x^2,\\
p_{12}(x) = {} & x^{144} + x^{136} + x^{128} + x^{104} + x^{96} + x^{72} + x^{64} + x^{48} + x^{32} + x^{24} + x^{16}\\
& + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{122} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{110} \\
&\quad + x^{109} + x^{107} + x^{103} + x^{101} + x^{99} + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} \\
&\quad + x^{90} + x^{88} + x^{84} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{71} + x^{69} \\
&\quad + x^{67} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} + x^{59} + x^{57} + x^{54} + x^{51} + x^{49} \\
&\quad + x^{47} + x^{44} + x^{43} + x^{41} + x^{38} + x^{37} + x^{35} + x^{33} + x^{32} + x^{29} + x^{24} \\
&\quad + x^{23} + x^{22} + x^{19} + x^{18}, \\
p_3(x) &= x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{90} + x^{87} + x^{78} \\
&\quad + x^{75} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{51} \\
&\quad + x^{50} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} \\
&\quad + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{12} + x^{9}, \\
p_6(x) &= x^{111} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} \\
&\quad + x^{95} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{77} + x^{74} + x^{72} \\
&\quad + x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{51} + x^{50} \\
&\quad + x^{47} + x^{44} + x^{43} + x^{42} + x^{37} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{25} \\
&\quad + x^{24} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} + x^{16} + x^{13} + x^{11} + x^{10} + x^{9} \\
&\quad + x^{7} + x^{6}, \\
p_9(x) &= x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111} + x^{110} + x^{107} + x^{102} + x^{100} \\
&\quad + x^{99} + x^{98} + x^{97} + x^{95} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} \\
&\quad + x^{83} + x^{78} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63} \\
&\quad + x^{60} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{41} + x^{40} \\
&\quad + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} \\
&\quad + x^{25} + x^{23} + x^{22} + x^{19} + x^{16} + x^{13} + x^{8} + x^{6} + x^{5} + x^{3}, \\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112} \\
&\quad + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{65} \\
&\quad + x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{17} + x^{14} \\
&\quad + x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{122} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{110} \\
&\quad + x^{109} + x^{107} + x^{103} + x^{101} + x^{99} + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} \\
&\quad + x^{90} + x^{88} + x^{84} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{71} + x^{69} \\
&\quad + x^{67} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} + x^{59} + x^{57} + x^{54} + x^{51} + x^{49} \\
&\quad + x^{47} + x^{44} + x^{43} + x^{41} + x^{38} + x^{37} + x^{35} + x^{33} + x^{32} + x^{29} + x^{24}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{23} + x^{22} + x^{19} + x^{18},\\
p_3(x) &= x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{90} + x^{87} + x^{78}\\
&+ x^{75} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{51}\\
&+ x^{50} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35}\\
&+ x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{12} + x^{9},\\
p_6(x) &= x^{111} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96}\\
&+ x^{95} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{77} + x^{74} + x^{72}\\
&+ x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{51} + x^{50}\\
&+ x^{47} + x^{44} + x^{43} + x^{42} + x^{37} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{25}\\
&+ x^{24} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} + x^{16} + x^{13} + x^{11} + x^{10} + x^{9}\\
&+ x^{7} + x^{6},\\
p_9(x) &= x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111} + x^{110} + x^{107} + x^{102} + x^{100}\\
&+ x^{99} + x^{98} + x^{97} + x^{95} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85}\\
&+ x^{83} + x^{78} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63}\\
&+ x^{60} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{41} + x^{40}\\
&+ x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26}\\
&+ x^{25} + x^{23} + x^{22} + x^{19} + x^{16} + x^{13} + x^{8} + x^{6} + x^{5} + x^{3},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112}\\
&+ x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{65}\\
&+ x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{17} + x^{14}\\
&+ x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0].

For $\phi(M^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{114} + x^{110} + x^{106} + x^{96} + x^{90} + x^{88} + x^{86} + x^{84} + x^{78} + x^{76} + x^{72}\\
&+ x^{66} + x^{56} + x^{54} + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{30}\\
&+ x^{28} + x^{26} + x^{24},\\
p_3(x) &= x^{108} + x^{100} + x^{98} + x^{96} + x^{94} + x^{90} + x^{86} + x^{84} + x^{78} + x^{72} + x^{70}\\
&+ x^{68} + x^{64} + x^{60} + x^{56} + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{38} + x^{36}\\
&+ x^{34} + x^{32} + x^{26} + x^{24} + x^{18} + x^{16} + x^{8} + x^{6},\\
p_6(x) &= x^{110} + x^{108} + x^{102} + x^{98} + x^{96} + x^{94} + x^{82} + x^{80} + x^{78} + x^{72} + x^{52}\\
&+ x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28}\\
&+ x^{24} + x^{16} + x^{10} + x^{8} + x^{4} + 1,\\
p_9(x) &= x^{108} + x^{104} + x^{98} + x^{94} + x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{64}\\
&+ x^{56} + x^{52} + x^{50} + x^{46} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{26}\\
&+ x^{24} + x^{20} + x^{18} + x^{12} + x^{10} + x^{6} + x^{4} + x^{2},
\end{aligned}
$$

$$
\begin{aligned}
p_{12}(x) = {} & x^{114} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{88} + x^{84} + x^{82} + x^{80} + x^{50} \\
& + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + x^{2} \\
& + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {} & x^{114} + x^{110} + x^{106} + x^{102} + x^{92} + x^{90} + x^{86} + x^{84} + x^{80} + x^{78} + x^{74} \\
& + x^{72} + x^{70} + x^{62} + x^{58} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{32} \\
& + x^{30} + x^{28} + x^{16} + x^{14} + x^{12}, \\
p_3(x) = {} & x^{108} + x^{100} + x^{98} + x^{94} + x^{92} + x^{88} + x^{82} + x^{80} + x^{74} + x^{70} + x^{68} \\
& + x^{66} + x^{60} + x^{58} + x^{54} + x^{52} + x^{46} + x^{40} + x^{36} + x^{28} + x^{24} + x^{22} \\
& + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^{8}, \\
p_6(x) = {} & x^{110} + x^{108} + x^{98} + x^{96} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{76} \\
& + x^{74} + x^{66} + x^{64} + x^{58} + x^{56} + x^{54} + x^{52} + x^{46} + x^{44} + x^{42} + x^{40} \\
& + x^{34} + x^{30} + x^{28} + x^{22} + x^{20} + x^{16} + x^{14} + x^{12} + x^{8} + 1, \\
p_9(x) = {} & x^{108} + x^{104} + x^{98} + x^{94} + x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{64} \\
& + x^{56} + x^{52} + x^{50} + x^{46} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{26} \\
& + x^{24} + x^{20} + x^{18} + x^{12} + x^{10} + x^{6} + x^{4} + x^{2}, \\
p_{12}(x) = {} & x^{114} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{88} + x^{84} + x^{82} + x^{80} + x^{50} \\
& + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + x^{2} \\
& + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) = {} & x^{129} + x^{126} + x^{124} + x^{122} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111} \\
& + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} \\
& + x^{96} + x^{95} + x^{94} + x^{90} + x^{89} + x^{85} + x^{78} + x^{76} + x^{71} + x^{70} + x^{69} \\
& + x^{65} + x^{63} + x^{57} + x^{56} + x^{53} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{36} \\
& + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27}, \\
p_3(x) = {} & x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{90} + x^{87} + x^{78} \\
& + x^{75} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{51} \\
& + x^{50} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} \\
& + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{12} + x^{9}, \\
p_6(x) = {} & x^{111} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} \\
& + x^{95} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{77} + x^{74} + x^{72} \\
& + x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{51} + x^{50} \\
& + x^{47} + x^{44} + x^{43} + x^{42} + x^{37} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{25} \\
& + x^{24} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} + x^{16} + x^{13} + x^{11} + x^{10} + x^{9}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{7} + x^{6},\\
p_9(x) &= x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111} + x^{110} + x^{107} + x^{102} + x^{100}\\
&+ x^{99} + x^{98} + x^{97} + x^{95} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85}\\
&+ x^{83} + x^{78} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63}\\
&+ x^{60} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{41} + x^{40}\\
&+ x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26}\\
&+ x^{25} + x^{23} + x^{22} + x^{19} + x^{16} + x^{13} + x^{8} + x^{6} + x^{5} + x^{3},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112}\\
&+ x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{65}\\
&+ x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{17} + x^{14}\\
&+ x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{122} + x^{119} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111}\\
&+ x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100}\\
&+ x^{96} + x^{95} + x^{94} + x^{90} + x^{89} + x^{85} + x^{78} + x^{76} + x^{71} + x^{70} + x^{69}\\
&+ x^{65} + x^{63} + x^{57} + x^{56} + x^{53} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{36}\\
&+ x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27},\\
p_3(x) &= x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{90} + x^{87} + x^{78}\\
&+ x^{75} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{51}\\
&+ x^{50} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35}\\
&+ x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{12} + x^{9},\\
p_6(x) &= x^{111} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96}\\
&+ x^{95} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{77} + x^{74} + x^{72}\\
&+ x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{51} + x^{50}\\
&+ x^{47} + x^{44} + x^{43} + x^{42} + x^{37} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{25}\\
&+ x^{24} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} + x^{16} + x^{13} + x^{11} + x^{10} + x^{9}\\
&+ x^{7} + x^{6},\\
p_9(x) &= x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{111} + x^{110} + x^{107} + x^{102} + x^{100}\\
&+ x^{99} + x^{98} + x^{97} + x^{95} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85}\\
&+ x^{83} + x^{78} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{63}\\
&+ x^{60} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{47} + x^{44} + x^{41} + x^{40}\\
&+ x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26}\\
&+ x^{25} + x^{23} + x^{22} + x^{19} + x^{16} + x^{13} + x^{8} + x^{6} + x^{5} + x^{3},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112}
\end{aligned}
$$

$$+ x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{65} + x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{17} + x^{14} + x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{140} + x^{138} + x^{128} + x^{126} + x^{122} + x^{120} + x^{118} + x^{110} + x^{106}\\
&\quad + x^{100} + x^{98} + x^{94} + x^{92} + x^{86} + x^{78} + x^{72} + x^{70} + x^{66} + x^{62} + x^{60}\\
&\quad + x^{58} + x^{56} + x^{52} + x^{46} + x^{44} + x^{42},\\
p_3(x) &= x^{138} + x^{132} + x^{122} + x^{120} + x^{114} + x^{108} + x^{106} + x^{104} + x^{102} + x^{96}\\
&\quad + x^{92} + x^{88} + x^{84} + x^{80} + x^{78} + x^{76} + x^{58} + x^{54} + x^{52} + x^{50} + x^{46}\\
&\quad + x^{44} + x^{42} + x^{40} + x^{26} + x^{22} + x^{20} + x^{6},\\
p_6(x) &= x^{140} + x^{138} + x^{134} + x^{132} + x^{128} + x^{126} + x^{124} + x^{120} + x^{118} + x^{116}\\
&\quad + x^{114} + x^{112} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{90} + x^{84} + x^{76}\\
&\quad + x^{68} + x^{66} + x^{64} + x^{60} + x^{58} + x^{52} + x^{48} + x^{44} + x^{38} + x^{36} + x^{34}\\
&\quad + x^{32} + x^{28} + x^{14} + x^{12} + x^{4} + x^{2} + 1,\\
p_9(x) &= x^{138} + x^{134} + x^{132} + x^{130} + x^{128} + x^{118} + x^{110} + x^{106} + x^{104} + x^{100}\\
&\quad + x^{96} + x^{88} + x^{86} + x^{80} + x^{78} + x^{76} + x^{72} + x^{70} + x^{66} + x^{62} + x^{60}\\
&\quad + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{26} + x^{24} + x^{16}\\
&\quad + x^{12} + x^{2},\\
p_{12}(x) &= x^{144} + x^{136} + x^{128} + x^{104} + x^{96} + x^{72} + x^{64} + x^{48} + x^{32} + x^{24} + x^{16}\\
&\quad + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{115}\\
&\quad + x^{110} + x^{108} + x^{107} + x^{104} + x^{102} + x^{99} + x^{98} + x^{97} + x^{95} + x^{92}\\
&\quad + x^{91} + x^{90} + x^{89} + x^{86} + x^{84} + x^{82} + x^{80} + x^{79} + x^{78} + x^{73} + x^{70}\\
&\quad + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{58} + x^{53} + x^{52}\\
&\quad + x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{42} + x^{39} + x^{38} + x^{35} + x^{32} + x^{30}\\
&\quad + x^{29} + x^{28} + x^{25} + x^{23} + x^{22} + x^{17} + x^{16} + x^{15} + x^{13} + x^{12},\\
p_3(x) &= x^{117} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{103} + x^{102}\\
&\quad + x^{100} + x^{99} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{87} + x^{84}\\
&\quad + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{67}\\
&\quad + x^{66} + x^{62} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} + x^{49}\\
&\quad + x^{48} + x^{46} + x^{42} + x^{40} + x^{39} + x^{35} + x^{34} + x^{33} + x^{28} + x^{27} + x^{24}\\
&\quad + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{11} + x^{9} + x^{8},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{113} + x^{112} + x^{111}\\
&\quad + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100}\\
&\quad + x^{99} + x^{97} + x^{94} + x^{93} + x^{88} + x^{87} + x^{86} + x^{81} + x^{79} + x^{74} + x^{73}\\
&\quad + x^{71} + x^{70} + x^{67} + x^{66} + x^{60} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{49}\\
&\quad + x^{48} + x^{44} + x^{42} + x^{41} + x^{40} + x^{38} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29}\\
&\quad + x^{27} + x^{25} + x^{23} + x^{20} + x^{19} + x^{18} + x^{15} + x^{12} + x^{10} + x^{9} + x^{7} + x^{6}\\
&\quad + x^{5} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{105} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96}\\
&\quad + x^{89} + x^{86} + x^{85} + x^{84} + x^{53} + x^{50} + x^{49} + x^{48} + x^{41} + x^{38} + x^{36}\\
&\quad + x^{34} + x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{9} + x^{6} + x^{5}\\
&\quad + x^{4},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{105} + x^{102}\\
&\quad + x^{100} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} + x^{80} + x^{65}\\
&\quad + x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{53} + x^{52} + x^{41} + x^{38} + x^{36} + x^{34}\\
&\quad + x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{5}\\
&\quad + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{120} + x^{112} + x^{111} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101}\\
&\quad + x^{100} + x^{99} + x^{98} + x^{96} + x^{90} + x^{89} + x^{88} + x^{85} + x^{84} + x^{82} + x^{77}\\
&\quad + x^{74} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} + x^{55} + x^{54} + x^{53}\\
&\quad + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{33}\\
&\quad + x^{32} + x^{31} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{18},\\
p_3(x) &= x^{93} + x^{91} + x^{89} + x^{88} + x^{87} + x^{84} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76}\\
&\quad + x^{75} + x^{74} + x^{73} + x^{69} + x^{67} + x^{65} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59}\\
&\quad + x^{58} + x^{57} + x^{53} + x^{51} + x^{49} + x^{48} + x^{47} + x^{44} + x^{42} + x^{40} + x^{39}\\
&\quad + x^{36} + x^{34} + x^{32} + x^{31} + x^{28} + x^{26} + x^{24} + x^{23} + x^{20} + x^{18} + x^{16}\\
&\quad + x^{15} + x^{13} + x^{12} + x^{11} + x^{10} + x^{9},\\
p_6(x) &= x^{102} + x^{92} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{74} + x^{70} + x^{68} + x^{66}\\
&\quad + x^{62} + x^{60} + x^{56} + x^{54} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36} + x^{32}\\
&\quad + x^{24} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6},\\
p_9(x) &= x^{111} + x^{109} + x^{106} + x^{103} + x^{100} + x^{99} + x^{95} + x^{93} + x^{90} + x^{87} + x^{84}\\
&\quad + x^{83} + x^{47} + x^{45} + x^{42} + x^{39} + x^{36} + x^{35} + x^{31} + x^{29} + x^{26} + x^{23}\\
&\quad + x^{20} + x^{19} + x^{15} + x^{13} + x^{10} + x^{7} + x^{4} + x^{3},\\
p_{12}(x) &= x^{120} + x^{116} + x^{96} + x^{88} + x^{84} + x^{80} + x^{56} + x^{52} + x^{32} + x^{16} + x^{8}
\end{aligned}$$

$$+ x^4 + 1,$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{140} + x^{136} + x^{135} + x^{134} + x^{132} + x^{131} + x^{127} + x^{126} + x^{125} \\
&\quad + x^{124} + x^{123} + x^{121} + x^{120} + x^{119} + x^{118} + x^{116} + x^{115} + x^{114} \\
&\quad + x^{112} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} \\
&\quad + x^{96} + x^{94} + x^{86} + x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} \\
&\quad + x^{73} + x^{70} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} \\
&\quad + x^{45} + x^{44} + x^{43} + x^{41} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27}, \\
p_3(x) &= x^{117} + x^{115} + x^{112} + x^{109} + x^{107} + x^{106} + x^{105} + x^{102} + x^{100} + x^{99} \\
&\quad + x^{98} + x^{95} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79} \\
&\quad + x^{78} + x^{75} + x^{74} + x^{71} + x^{70} + x^{69} + x^{66} + x^{64} + x^{63} + x^{62} + x^{59} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{50} + x^{49} + x^{37} + x^{35} + x^{32} \\
&\quad + x^{29} + x^{27} + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{13} \\
&\quad + x^{12} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{126} + x^{122} + x^{116} + x^{114} + x^{110} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} \\
&\quad + x^{90} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{58} + x^{48} + x^{46} + x^{44} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{36} + x^{32} + x^{28} + x^{22} + x^{20} + x^{18} + x^{14} + x^{12} + x^{10} \\
&\quad + x^{8} + x^{6}, \\
p_9(x) &= x^{135} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{123} + x^{121} \\
&\quad + x^{120} + x^{118} + x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} \\
&\quad + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{102} + x^{100} + x^{96} + x^{93} + x^{90} \\
&\quad + x^{87} + x^{84} + x^{83} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} \\
&\quad + x^{59} + x^{57} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} \\
&\quad + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} \\
&\quad + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{22} \\
&\quad + x^{20} + x^{16} + x^{13} + x^{10} + x^{7} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{144} + x^{136} + x^{132} + x^{128} + x^{112} + x^{108} + x^{92} + x^{88} + x^{84} + x^{72} \\
&\quad + x^{68} + x^{64} + x^{44} + x^{40} + x^{36} + x^{28} + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{115} + x^{114} \\
&\quad + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{104} + x^{102} + x^{98} + x^{96} \\
&\quad + x^{93} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} \\
&\quad + x^{72} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{58} + x^{56} + x^{55} + x^{54}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{53} + x^{51} + x^{48} + x^{47} + x^{45} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33},\\
p_3(x) &= x^{117} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103}\\
&+ x^{101} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{84} + x^{82} + x^{80}\\
&+ x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63} + x^{60} + x^{59} + x^{57}\\
&+ x^{56} + x^{55} + x^{54} + x^{53} + x^{51} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44}\\
&+ x^{43} + x^{42} + x^{41} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{21} + x^{19}\\
&+ x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12},\\
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{114} + x^{110} + x^{109} + x^{108}\\
&+ x^{107} + x^{105} + x^{100} + x^{99} + x^{95} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85}\\
&+ x^{84} + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{61}\\
&+ x^{58} + x^{57} + x^{54} + x^{53} + x^{48} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29} + x^{27} + x^{25} + x^{15} + x^{12}\\
&+ x^{11} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{105} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96}\\
&+ x^{89} + x^{86} + x^{85} + x^{84} + x^{53} + x^{50} + x^{49} + x^{48} + x^{41} + x^{38} + x^{36}\\
&+ x^{34} + x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{9} + x^{6} + x^{5}\\
&+ x^{4},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{105} + x^{102}\\
&+ x^{100} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} + x^{80} + x^{65}\\
&+ x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{53} + x^{52} + x^{41} + x^{38} + x^{36} + x^{34}\\
&+ x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{5}\\
&+ x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{115}\\
&+ x^{110} + x^{108} + x^{107} + x^{104} + x^{102} + x^{99} + x^{98} + x^{97} + x^{95} + x^{92}\\
&+ x^{91} + x^{90} + x^{89} + x^{86} + x^{84} + x^{82} + x^{80} + x^{79} + x^{78} + x^{73} + x^{70}\\
&+ x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{58} + x^{53} + x^{52}\\
&+ x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{42} + x^{39} + x^{38} + x^{35} + x^{32} + x^{30}\\
&+ x^{29} + x^{28} + x^{25} + x^{23} + x^{22} + x^{17} + x^{16} + x^{15} + x^{13} + x^{12},\\
p_3(x) &= x^{117} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{103} + x^{102}\\
&+ x^{100} + x^{99} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{87} + x^{84}\\
&+ x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{67}\\
&+ x^{66} + x^{62} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} + x^{49}\\
&+ x^{48} + x^{46} + x^{42} + x^{40} + x^{39} + x^{35} + x^{34} + x^{33} + x^{28} + x^{27} + x^{24}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{11} + x^{9} + x^{8},\\
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{113} + x^{112} + x^{111}\\
&+ x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100}\\
&+ x^{99} + x^{97} + x^{94} + x^{93} + x^{88} + x^{87} + x^{86} + x^{81} + x^{79} + x^{74} + x^{73}\\
&+ x^{71} + x^{70} + x^{67} + x^{66} + x^{60} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{49}\\
&+ x^{48} + x^{44} + x^{42} + x^{41} + x^{40} + x^{38} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29}\\
&+ x^{27} + x^{25} + x^{23} + x^{20} + x^{19} + x^{18} + x^{15} + x^{12} + x^{10} + x^{9} + x^{7} + x^{6}\\
&+ x^{5} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{105} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96}\\
&+ x^{89} + x^{86} + x^{85} + x^{84} + x^{53} + x^{50} + x^{49} + x^{48} + x^{41} + x^{38} + x^{36}\\
&+ x^{34} + x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{9} + x^{6} + x^{5}\\
&+ x^{4},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{105} + x^{102}\\
&+ x^{100} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} + x^{80} + x^{65}\\
&+ x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{53} + x^{52} + x^{41} + x^{38} + x^{36} + x^{34}\\
&+ x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{5}\\
&+ x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{144} + x^{140} + x^{136} + x^{135} + x^{134} + x^{132} + x^{131} + x^{127} + x^{126} + x^{125}\\
&+ x^{124} + x^{123} + x^{121} + x^{120} + x^{119} + x^{118} + x^{116} + x^{115} + x^{114}\\
&+ x^{112} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100}\\
&+ x^{96} + x^{94} + x^{86} + x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74}\\
&+ x^{73} + x^{70} + x^{64} + x^{63} + x^{61} + x^{59} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51}\\
&+ x^{45} + x^{44} + x^{43} + x^{41} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27},\\
p_3(x) &= x^{117} + x^{115} + x^{112} + x^{109} + x^{107} + x^{106} + x^{105} + x^{102} + x^{100} + x^{99}\\
&+ x^{98} + x^{95} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79}\\
&+ x^{78} + x^{75} + x^{74} + x^{71} + x^{70} + x^{69} + x^{66} + x^{64} + x^{63} + x^{62} + x^{59}\\
&+ x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{50} + x^{49} + x^{37} + x^{35} + x^{32}\\
&+ x^{29} + x^{27} + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{13}\\
&+ x^{12} + x^{11} + x^{10} + x^{9},\\
p_6(x) &= x^{126} + x^{122} + x^{116} + x^{114} + x^{110} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94}\\
&+ x^{90} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{58} + x^{48} + x^{46} + x^{44} + x^{42}\\
&+ x^{40} + x^{38} + x^{36} + x^{32} + x^{28} + x^{22} + x^{20} + x^{18} + x^{14} + x^{12} + x^{10}\\
&+ x^{8} + x^{6},
\end{aligned}
$$

$$\begin{aligned}
p_9(x) = {} & x^{135} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{123} + x^{121} \\
& + x^{120} + x^{118} + x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} \\
& + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{102} + x^{100} + x^{96} + x^{93} + x^{90} \\
& + x^{87} + x^{84} + x^{83} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} \\
& + x^{59} + x^{57} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} \\
& + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} \\
& + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{22} \\
& + x^{20} + x^{16} + x^{13} + x^{10} + x^{7} + x^{4} + x^{3}, \\
p_{12}(x) = {} & x^{144} + x^{136} + x^{132} + x^{128} + x^{112} + x^{108} + x^{92} + x^{88} + x^{84} + x^{72} \\
& + x^{68} + x^{64} + x^{44} + x^{40} + x^{36} + x^{28} + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{120} + x^{112} + x^{111} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} \\
& + x^{100} + x^{99} + x^{98} + x^{96} + x^{90} + x^{89} + x^{88} + x^{85} + x^{84} + x^{82} + x^{77} \\
& + x^{74} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} + x^{55} + x^{54} + x^{53} \\
& + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{33} \\
& + x^{32} + x^{31} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{18}, \\
p_3(x) = {} & x^{93} + x^{91} + x^{89} + x^{88} + x^{87} + x^{84} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} \\
& + x^{75} + x^{74} + x^{73} + x^{69} + x^{67} + x^{65} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} \\
& + x^{58} + x^{57} + x^{53} + x^{51} + x^{49} + x^{48} + x^{47} + x^{44} + x^{42} + x^{40} + x^{39} \\
& + x^{36} + x^{34} + x^{32} + x^{31} + x^{28} + x^{26} + x^{24} + x^{23} + x^{20} + x^{18} + x^{16} \\
& + x^{15} + x^{13} + x^{12} + x^{11} + x^{10} + x^{9}, \\
p_6(x) = {} & x^{102} + x^{92} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{74} + x^{70} + x^{68} + x^{66} \\
& + x^{62} + x^{60} + x^{56} + x^{54} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36} + x^{32} \\
& + x^{24} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6}, \\
p_9(x) = {} & x^{111} + x^{109} + x^{106} + x^{103} + x^{100} + x^{99} + x^{95} + x^{93} + x^{90} + x^{87} + x^{84} \\
& + x^{83} + x^{47} + x^{45} + x^{42} + x^{39} + x^{36} + x^{35} + x^{31} + x^{29} + x^{26} + x^{23} \\
& + x^{20} + x^{19} + x^{15} + x^{13} + x^{10} + x^{7} + x^{4} + x^{3}, \\
p_{12}(x) = {} & x^{120} + x^{116} + x^{96} + x^{88} + x^{84} + x^{80} + x^{56} + x^{52} + x^{32} + x^{16} + x^{8} \\
& + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{115} + x^{114} \\
& + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{104} + x^{102} + x^{98} + x^{96} \\
& + x^{93} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{72} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{58} + x^{56} + x^{55} + x^{54}\\
&+ x^{53} + x^{51} + x^{48} + x^{47} + x^{45} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33},\\
p_3(x) &= x^{117} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103}\\
&+ x^{101} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{86} + x^{84} + x^{82} + x^{80}\\
&+ x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63} + x^{60} + x^{59} + x^{57}\\
&+ x^{56} + x^{55} + x^{54} + x^{53} + x^{51} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44}\\
&+ x^{43} + x^{42} + x^{41} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{21} + x^{19}\\
&+ x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12},\\
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{114} + x^{110} + x^{109} + x^{108}\\
&+ x^{107} + x^{105} + x^{100} + x^{99} + x^{95} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85}\\
&+ x^{84} + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71} + x^{68} + x^{65} + x^{62} + x^{61}\\
&+ x^{58} + x^{57} + x^{54} + x^{53} + x^{48} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{31} + x^{29} + x^{27} + x^{25} + x^{15} + x^{12}\\
&+ x^{11} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{105} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96}\\
&+ x^{89} + x^{86} + x^{85} + x^{84} + x^{53} + x^{50} + x^{49} + x^{48} + x^{41} + x^{38} + x^{36}\\
&+ x^{34} + x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{9} + x^{6} + x^{5}\\
&+ x^{4},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{105} + x^{102}\\
&+ x^{100} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} + x^{80} + x^{65}\\
&+ x^{62} + x^{61} + x^{60} + x^{57} + x^{54} + x^{53} + x^{52} + x^{41} + x^{38} + x^{36} + x^{34}\\
&+ x^{33} + x^{32} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{5}\\
&+ x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	386	[634, 635]	772	[1108, 593]	1158	[358, 878]
1	[3, 4]	387	[448, 636]	773	[566, 529]	1159	[1383, 1010]
2	[5, 6]	388	[637, 638]	774	[573, 140]	1160	[360, 997]
3	[7, 8]	389	[639, 640]	775	[1109, 1110]	1161	[1384, 1385]
4	[9, 10]	390	[490, 139]	776	[574, 1111]	1162	[1298, 1386]
5	[11, 12]	391	[641, 642]	777	[1068, 1112]	1163	[1294, 941]
6	[13, 14]	392	[624, 643]	778	[128, 37]	1164	[398, 1356]
7	[15, 16]	393	[33, 644]	779	[458, 1113]	1165	[903, 351]
8	[17, 18]	394	[496, 645]	780	[1114, 1115]	1166	[1387, 298]
9	[19, 20]	395	[550, 499]	781	[584, 461]	1167	[274, 1388]

10	[21, 22]	396	[470, 646]	782	[1116, 482]	1168	[905, 1389]
11	[23, 24]	397	[647, 648]	783	[445, 664]	1169	[1390, 1391]
12	[25, 26]	398	[649, 151]	784	[631, 1117]	1170	[1392, 774]
13	[27, 28]	399	[650, 460]	785	[492, 643]	1171	[1257, 1393]
14	[29, 30]	400	[651, 231]	786	[1118, 1119]	1172	[885, 357]
15	[31, 32]	401	[652, 502]	787	[1102, 496]	1173	[1394, 242]
16	[33, 34]	402	[653, 646]	788	[1120, 463]	1174	[1289, 1395]
17	[35, 36]	403	[653, 471]	789	[133, 1112]	1175	[1396, 932]
18	[37, 38]	404	[140, 476]	790	[660, 590]	1176	[1397, 1315]
19	[39, 40]	405	[446, 477]	791	[454, 561]	1177	[387, 294]
20	[41, 42]	406	[616, 654]	792	[587, 546]	1178	[792, 1284]
21	[43, 44]	407	[537, 486]	793	[218, 1121]	1179	[1398, 1399]
22	[45, 46]	408	[480, 640]	794	[1061, 170]	1180	[887, 1392]
23	[47, 48]	409	[655, 656]	795	[1122, 662]	1181	[1359, 714]
24	[49, 50]	410	[657, 658]	796	[174, 1123]	1182	[1400, 1401]
25	[51, 52]	411	[659, 575]	797	[1124, 503]	1183	[1402, 290]
26	[53, 54]	412	[660, 210]	798	[1125, 1070]	1184	[1403, 811]
27	[55, 56]	413	[661, 662]	799	[1126, 1127]	1185	[841, 813]
28	[57, 58]	414	[567, 663]	800	[1128, 658]	1186	[731, 1404]
29	[59, 60]	415	[565, 664]	801	[1129, 578]	1187	[1314, 759]
30	[61, 62]	416	[665, 554]	802	[558, 1113]	1188	[1405, 1406]
31	[63, 64]	417	[598, 666]	803	[1130, 159]	1189	[324, 865]
32	[65, 66]	418	[200, 194]	804	[1077, 1084]	1190	[963, 1407]
33	[67, 68]	419	[540, 540]	805	[1131, 1132]	1191	[1408, 1409]
34	[69, 70]	420	[667, 128]	806	[610, 58]	1192	[1410, 1411]
35	[71, 72]	421	[657, 668]	807	[1131, 1133]	1193	[1001, 922]
36	[73, 74]	422	[528, 567]	808	[1134, 1135]	1194	[935, 795]
37	[75, 76]	423	[669, 142]	809	[1136, 583]	1195	[760, 781]
38	[77, 78]	424	[670, 144]	810	[1137, 634]	1196	[840, 369]
39	[79, 80]	425	[570, 476]	811	[1138, 504]	1197	[1267, 752]
40	[81, 82]	426	[188, 671]	812	[1027, 1139]	1198	[693, 1412]
41	[83, 84]	427	[672, 673]	813	[39, 1140]	1199	[1413, 1328]
42	[85, 86]	428	[50, 674]	814	[542, 581]	1200	[1414, 1415]
43	[87, 88]	429	[151, 675]	815	[230, 1141]	1201	[1331, 845]
44	[89, 90]	430	[591, 607]	816	[1142, 1143]	1202	[1282, 806]
45	[91, 92]	431	[529, 676]	817	[1144, 1145]	1203	[839, 296]
46	[93, 94]	432	[677, 678]	818	[1028, 202]	1204	[866, 1416]
47	[95, 96]	433	[679, 680]	819	[1122, 1146]	1205	[1417, 1418]
48	[97, 98]	434	[278, 681]	820	[1050, 1147]	1206	[944, 948]
49	[99, 100]	435	[682, 683]	821	[1148, 1073]	1207	[991, 1400]
50	[101, 102]	436	[684, 685]	822	[1046, 1074]	1208	[1277, 1419]
51	[103, 104]	437	[686, 399]	823	[1096, 1149]	1209	[1420, 710]
52	[105, 106]	438	[687, 19]	824	[485, 538]	1210	[855, 370]
53	[107, 108]	439	[688, 689]	825	[545, 1030]	1211	[1421, 1422]

54	[109, 110]	440	[690, 691]	826	[1150, 1024]	1212	[1332, 1423]
55	[111, 112]	441	[692, 693]	827	[1151, 501]	1213	[249, 1293]
56	[113, 114]	442	[689, 21]	828	[1152, 1153]	1214	[1424, 700]
57	[115, 116]	443	[694, 695]	829	[551, 1153]	1215	[1011, 953]
58	[117, 118]	444	[696, 697]	830	[559, 1154]	1216	[773, 889]
59	[119, 120]	445	[108, 698]	831	[1155, 130]	1217	[110, 1425]
60	[121, 122]	446	[699, 388]	832	[649, 171]	1218	[1006, 1426]
61	[123, 124]	447	[700, 701]	833	[519, 1156]	1219	[18, 1427]
62	[125, 126]	448	[702, 703]	834	[1157, 592]	1220	[264, 1428]
63	[127, 128]	449	[704, 705]	835	[1071, 1158]	1221	[1429, 939]
64	[129, 130]	450	[706, 707]	836	[1159, 672]	1222	[1345, 1430]
65	[131, 132]	451	[708, 709]	837	[190, 1160]	1223	[1431, 1432]
66	[133, 134]	452	[710, 711]	838	[1129, 181]	1224	[1433, 1394]
67	[135, 136]	453	[371, 363]	839	[548, 223]	1225	[1370, 957]
68	[137, 138]	454	[712, 713]	840	[1161, 1140]	1226	[758, 113]
69	[139, 140]	455	[714, 715]	841	[167, 542]	1227	[1307, 1434]
70	[141, 142]	456	[106, 395]	842	[167, 580]	1228	[366, 329]
71	[143, 144]	457	[716, 717]	843	[1054, 157]	1229	[912, 258]
72	[145, 145]	458	[718, 393]	844	[516, 1115]	1230	[1385, 989]
73	[146, 147]	459	[719, 720]	845	[1042, 597]	1231	[1412, 4]
74	[148, 149]	460	[721, 721]	846	[162, 559]	1232	[406, 808]
75	[60, 150]	461	[112, 411]	847	[211, 211]	1233	[1435, 725]
76	[151, 152]	462	[301, 722]	848	[433, 1052]	1234	[1376, 943]
77	[153, 154]	463	[723, 724]	849	[1061, 1162]	1235	[713, 1436]
78	[155, 156]	464	[725, 704]	850	[1163, 1090]	1236	[815, 1437]
79	[13, 157]	465	[726, 727]	851	[1164, 512]	1237	[1438, 1439]
80	[158, 159]	466	[728, 729]	852	[1144, 1067]	1238	[383, 1014]
81	[160, 161]	467	[730, 731]	853	[1116, 1165]	1239	[1378, 876]
82	[162, 163]	468	[732, 733]	854	[445, 1166]	1240	[801, 1440]
83	[42, 164]	469	[100, 734]	855	[158, 1167]	1241	[1441, 1442]
84	[164, 165]	470	[735, 323]	856	[1054, 14]	1242	[1443, 1444]
85	[166, 167]	471	[736, 737]	857	[665, 595]	1243	[1352, 874]
86	[164, 168]	472	[260, 738]	858	[1085, 1165]	1244	[1445, 1446]
87	[169, 170]	473	[739, 740]	859	[621, 1135]	1245	[999, 101]
88	[171, 152]	474	[262, 741]	860	[189, 523]	1246	[1447, 1448]
89	[172, 173]	475	[425, 742]	861	[37, 1168]	1247	[1449, 986]
90	[174, 175]	476	[743, 744]	862	[1169, 1119]	1248	[691, 1003]
91	[131, 176]	477	[745, 746]	863	[228, 496]	1249	[268, 1420]
92	[177, 132]	478	[747, 748]	864	[1170, 1171]	1250	[1309, 83]
93	[178, 179]	479	[749, 321]	865	[1172, 1156]	1251	[255, 1450]
94	[180, 181]	480	[750, 751]	866	[172, 1107]	1252	[1292, 706]
95	[182, 183]	481	[752, 753]	867	[1173, 1120]	1253	[1, 1129]
96	[57, 184]	482	[754, 325]	868	[1101, 1089]	1254	[1109, 1127]
97	[185, 186]	483	[755, 756]	869	[527, 1174]	1255	[1205, 218]

98	[187, 188]	484	[757, 758]	870	[1091, 1175]	1256	[60, 1451]
99	[189, 190]	485	[759, 760]	871	[1176, 1177]	1257	[62, 620]
100	[181, 191]	486	[761, 277]	872	[1178, 555]	1258	[1240, 1072]
101	[192, 193]	487	[762, 763]	873	[647, 1179]	1259	[138, 1123]
102	[42, 194]	488	[764, 16]	874	[494, 1180]	1260	[656, 507]
103	[195, 190]	489	[765, 259]	875	[32, 1181]	1261	[1087, 1452]
104	[196, 197]	490	[766, 300]	876	[1098, 1182]	1262	[1248, 1453]
105	[198, 199]	491	[767, 768]	877	[1033, 467]	1263	[436, 138]
106	[41, 200]	492	[769, 770]	878	[1183, 549]	1264	[50, 10]
107	[201, 202]	493	[771, 772]	879	[225, 199]	1265	[1454, 1228]
108	[203, 204]	494	[98, 773]	880	[32, 1038]	1266	[1190, 589]
109	[205, 206]	495	[774, 775]	881	[1184, 1185]	1267	[627, 1455]
110	[207, 208]	496	[776, 777]	882	[639, 481]	1268	[148, 1121]
111	[209, 210]	497	[421, 778]	883	[570, 472]	1269	[578, 524]
112	[211, 145]	498	[779, 780]	884	[490, 573]	1270	[1456, 453]
113	[212, 213]	499	[781, 782]	885	[192, 1186]	1271	[128, 546]
114	[214, 215]	500	[783, 784]	886	[502, 442]	1272	[1457, 1453]
115	[216, 212]	501	[785, 786]	887	[219, 573]	1273	[1242, 577]
116	[217, 218]	502	[787, 736]	888	[634, 1187]	1274	[547, 1139]
117	[219, 139]	503	[788, 785]	889	[1184, 456]	1275	[625, 526]
118	[220, 221]	504	[789, 790]	890	[1188, 452]	1276	[587, 37]
119	[222, 223]	505	[791, 792]	891	[1189, 180]	1277	[1020, 1458]
120	[196, 224]	506	[362, 793]	892	[1190, 1191]	1278	[187, 212]
121	[225, 226]	507	[794, 331]	893	[169, 1162]	1279	[1055, 197]
122	[168, 227]	508	[795, 796]	894	[640, 1192]	1280	[577, 1459]
123	[228, 229]	509	[797, 798]	895	[1130, 1167]	1281	[1217, 1460]
124	[230, 231]	510	[799, 71]	896	[200, 164]	1282	[203, 1461]
125	[232, 233]	511	[800, 801]	897	[1193, 1194]	1283	[1462, 613]
126	[234, 235]	512	[422, 413]	898	[588, 1191]	1284	[641, 1463]
127	[236, 237]	513	[802, 803]	899	[659, 1111]	1285	[1017, 1464]
128	[238, 239]	514	[804, 805]	900	[1034, 1069]	1286	[1239, 1195]
129	[240, 241]	515	[806, 807]	901	[1159, 1070]	1287	[1465, 434]
130	[242, 243]	516	[808, 809]	902	[1195, 8]	1288	[1087, 1171]
131	[244, 245]	517	[392, 810]	903	[1196, 1045]	1289	[1212, 34]
132	[246, 247]	518	[811, 812]	904	[1157, 1108]	1290	[186, 1451]
133	[248, 249]	519	[813, 814]	905	[661, 1146]	1291	[1466, 512]
134	[250, 251]	520	[809, 815]	906	[1197, 1198]	1292	[1128, 668]
135	[252, 253]	521	[816, 237]	907	[1199, 1108]	1293	[1194, 234]
136	[254, 255]	522	[817, 818]	908	[1200, 1201]	1294	[1074, 1463]
137	[256, 257]	523	[339, 91]	909	[437, 479]	1295	[1467, 1017]
138	[258, 121]	524	[819, 820]	910	[487, 1202]	1296	[651, 1141]
139	[259, 260]	525	[821, 822]	911	[1026, 547]	1297	[1056, 504]
140	[261, 262]	526	[823, 824]	912	[1152, 552]	1298	[1089, 1021]
141	[263, 264]	527	[825, 380]	913	[1024, 171]	1299	[198, 226]

142	[265, 266]	528	[826, 827]	914	[1203, 1044]	1300	[1233, 1460]
143	[267, 268]	529	[828, 829]	915	[520, 527]	1301	[1150, 649]
144	[269, 270]	530	[738, 766]	916	[547, 1204]	1302	[1221, 1050]
145	[271, 272]	531	[740, 117]	917	[1137, 1081]	1303	[562, 1234]
146	[273, 274]	532	[830, 831]	918	[1124, 442]	1304	[1468, 1054]
147	[275, 276]	533	[832, 833]	919	[1205, 148]	1305	[163, 560]
148	[277, 278]	534	[834, 835]	920	[1148, 1206]	1306	[484, 636]
149	[279, 280]	535	[836, 426]	921	[1207, 182]	1307	[43, 465]
150	[281, 282]	536	[104, 837]	922	[519, 1208]	1308	[581, 42]
151	[283, 284]	537	[838, 839]	923	[556, 638]	1309	[543, 1469]
152	[285, 286]	538	[86, 840]	924	[61, 605]	1310	[1164, 1065]
153	[287, 119]	539	[313, 841]	925	[1209, 1210]	1311	[670, 569]
154	[20, 288]	540	[102, 842]	926	[1199, 592]	1312	[467, 1069]
155	[289, 105]	541	[541, 541]	927	[136, 1174]	1313	[1470, 1101]
156	[290, 291]	542	[419, 749]	928	[1211, 445]	1314	[1471, 641]
157	[292, 293]	543	[843, 844]	929	[1161, 40]	1315	[1250, 536]
158	[294, 295]	544	[845, 846]	930	[1057, 505]	1316	[1106, 173]
159	[296, 297]	545	[423, 847]	931	[1120, 506]	1317	[1125, 672]
160	[298, 299]	546	[237, 848]	932	[1212, 644]	1318	[1472, 1206]
161	[300, 301]	547	[849, 850]	933	[510, 1213]	1319	[1473, 232]
162	[302, 303]	548	[851, 852]	934	[1214, 453]	1320	[434, 604]
163	[304, 305]	549	[711, 853]	935	[137, 174]	1321	[1017, 1474]
164	[306, 307]	550	[854, 855]	936	[1215, 1216]	1322	[1058, 1195]
165	[308, 309]	551	[856, 857]	937	[1207, 1098]	1323	[1041, 1172]
166	[307, 310]	552	[28, 858]	938	[1217, 1218]	1324	[1073, 464]
167	[122, 311]	553	[859, 860]	939	[1035, 1219]	1325	[141, 1475]
168	[312, 313]	554	[861, 862]	940	[632, 208]	1326	[579, 1469]
169	[314, 308]	555	[715, 804]	941	[234, 508]	1327	[1476, 1177]
170	[126, 315]	556	[94, 863]	942	[190, 1220]	1328	[1477, 1036]
171	[316, 317]	557	[310, 289]	943	[1209, 1100]	1329	[1189, 1129]
172	[318, 319]	558	[864, 367]	944	[1221, 155]	1330	[1206, 464]
173	[309, 320]	559	[865, 866]	945	[503, 1222]	1331	[1170, 1452]
174	[321, 322]	560	[867, 868]	946	[1053, 13]	1332	[1034, 146]
175	[323, 324]	561	[869, 870]	947	[207, 633]	1333	[482, 1247]
176	[325, 326]	562	[871, 718]	948	[455, 1185]	1334	[1478, 458]
177	[327, 328]	563	[872, 873]	949	[1223, 1202]	1335	[1478, 558]
178	[329, 330]	564	[874, 875]	950	[229, 497]	1336	[451, 1479]
179	[331, 332]	565	[876, 694]	951	[1224, 654]	1337	[1457, 1249]
180	[333, 246]	566	[877, 690]	952	[1126, 1110]	1338	[1169, 1480]
181	[4, 334]	567	[878, 879]	953	[186, 150]	1339	[1470, 1088]
182	[335, 336]	568	[16, 880]	954	[1051, 564]	1340	[525, 626]
183	[88, 337]	569	[881, 882]	955	[1225, 1101]	1341	[1481, 1026]
184	[338, 339]	570	[883, 884]	956	[1226, 629]	1342	[1471, 1074]
185	[340, 341]	571	[396, 885]	957	[1196, 1227]	1343	[1172, 1208]

186	[342, 343]	572	[844, 886]	958	[53, 1228]	1344	[1467, 1076]
187	[344, 345]	573	[722, 887]	959	[532, 619]	1345	[509, 1180]
188	[346, 281]	574	[888, 372]	960	[1229, 1230]	1346	[1482, 502]
189	[347, 348]	575	[889, 890]	961	[1231, 1232]	1347	[1081, 1187]
190	[349, 350]	576	[891, 892]	962	[1233, 1218]	1348	[637, 557]
191	[351, 352]	577	[777, 65]	963	[439, 1234]	1349	[1188, 1479]
192	[353, 354]	578	[893, 376]	964	[1235, 1087]	1350	[1048, 1483]
193	[355, 356]	579	[894, 895]	965	[1216, 492]	1351	[1484, 1232]
194	[357, 358]	580	[896, 85]	966	[136, 1236]	1352	[1136, 32]
195	[359, 360]	581	[311, 306]	967	[1237, 1238]	1353	[1481, 534]
196	[361, 362]	582	[897, 898]	968	[222, 45]	1354	[1097, 182]
197	[363, 364]	583	[899, 424]	969	[1239, 1059]	1355	[614, 1149]
198	[365, 366]	584	[900, 901]	970	[1104, 614]	1356	[1044, 1227]
199	[367, 368]	585	[70, 765]	971	[182, 1182]	1357	[1485, 1143]
200	[369, 312]	586	[902, 903]	972	[212, 671]	1358	[1466, 1065]
201	[370, 371]	587	[904, 905]	973	[218, 149]	1359	[601, 1486]
202	[372, 373]	588	[906, 907]	974	[1240, 1158]	1360	[1472, 1073]
203	[374, 375]	589	[848, 908]	975	[155, 1147]	1361	[1487, 55]
204	[376, 377]	590	[909, 910]	976	[1065, 196]	1362	[1193, 656]
205	[378, 379]	591	[911, 123]	977	[1241, 1133]	1363	[1066, 1145]
206	[380, 381]	592	[320, 912]	978	[521, 1079]	1364	[592, 1488]
207	[382, 383]	593	[780, 913]	979	[1242, 623]	1365	[1489, 599]
208	[384, 385]	594	[914, 915]	980	[546, 1168]	1366	[1062, 1198]
209	[386, 387]	595	[916, 917]	981	[1080, 634]	1367	[1244, 1229]
210	[388, 389]	596	[918, 919]	982	[582, 32]	1368	[442, 1222]
211	[272, 390]	597	[72, 374]	983	[1083, 1078]	1369	[1200, 1238]
212	[391, 392]	598	[920, 921]	984	[600, 1129]	1370	[555, 1051]
213	[393, 394]	599	[922, 923]	985	[216, 188]	1371	[1089, 1458]
214	[395, 396]	600	[924, 893]	986	[1047, 492]	1372	[623, 1459]
215	[397, 398]	601	[925, 926]	987	[1163, 568]	1373	[1194, 507]
216	[399, 400]	602	[927, 928]	988	[1237, 1201]	1374	[1231, 1490]
217	[401, 279]	603	[929, 896]	989	[1243, 433]	1375	[640, 1082]
218	[402, 403]	604	[680, 930]	990	[1099, 1210]	1376	[608, 1117]
219	[404, 261]	605	[931, 861]	991	[1244, 1245]	1377	[1241, 1132]
220	[405, 406]	606	[932, 933]	992	[503, 443]	1378	[1216, 624]
221	[407, 408]	607	[842, 342]	993	[1246, 1175]	1379	[1197, 1063]
222	[409, 410]	608	[373, 344]	994	[1165, 1247]	1380	[577, 1491]
223	[411, 412]	609	[812, 934]	995	[1155, 478]	1381	[652, 1124]
224	[413, 414]	610	[886, 935]	996	[1165, 483]	1382	[1492, 648]
225	[415, 416]	611	[936, 937]	997	[180, 578]	1383	[1178, 1243]
226	[345, 417]	612	[938, 27]	998	[650, 510]	1384	[481, 1192]
227	[418, 419]	613	[939, 940]	999	[1183, 514]	1385	[1114, 517]
228	[420, 421]	614	[879, 418]	1000	[1024, 151]	1386	[504, 1252]
229	[92, 335]	615	[941, 942]	1001	[1040, 605]	1387	[185, 60]

230	[280, 422]	616	[943, 944]	1002	[1248, 1249]	1388	[1214, 446]
231	[375, 423]	617	[945, 946]	1003	[1250, 1251]	1389	[1108, 1488]
232	[424, 425]	618	[947, 754]	1004	[1211, 565]	1390	[1493, 1047]
233	[426, 427]	619	[948, 949]	1005	[163, 1154]	1391	[1487, 629]
234	[428, 429]	620	[124, 950]	1006	[1224, 617]	1392	[669, 1475]
235	[430, 431]	621	[385, 361]	1007	[1081, 635]	1393	[1245, 1230]
236	[432, 433]	622	[951, 688]	1008	[460, 511]	1394	[441, 571]
237	[434, 435]	623	[952, 111]	1009	[500, 1186]	1395	[1076, 1464]
238	[436, 174]	624	[953, 954]	1010	[1151, 469]	1396	[1094, 1461]
239	[437, 438]	625	[955, 956]	1011	[1243, 1051]	1397	[1485, 1494]
240	[439, 440]	626	[957, 771]	1012	[1055, 224]	1398	[1027, 1204]
241	[441, 214]	627	[958, 823]	1013	[1057, 1252]	1399	[1118, 1480]
242	[442, 443]	628	[720, 732]	1014	[1037, 1095]	1400	[188, 213]
243	[444, 445]	629	[959, 883]	1015	[229, 645]	1401	[1245, 1495]
244	[446, 447]	630	[76, 960]	1016	[1253, 995]	1402	[565, 1166]
245	[448, 449]	631	[961, 99]	1017	[1254, 789]	1403	[534, 1027]
246	[450, 438]	632	[962, 125]	1018	[293, 1255]	1404	[611, 163]
247	[451, 452]	633	[685, 963]	1019	[847, 952]	1405	[1496, 135]
248	[453, 447]	634	[833, 269]	1020	[1256, 1257]	1406	[1497, 1096]
249	[454, 179]	635	[964, 965]	1021	[1258, 1259]	1407	[1498, 1245]
250	[455, 456]	636	[966, 967]	1022	[410, 1260]	1408	[527, 1236]
251	[457, 458]	637	[756, 968]	1023	[965, 1261]	1409	[1468, 13]
252	[459, 460]	638	[681, 929]	1024	[998, 285]	1410	[1499, 1044]
253	[160, 461]	639	[969, 970]	1025	[1262, 730]	1411	[1040, 62]
254	[462, 463]	640	[971, 355]	1026	[1263, 250]	1412	[1462, 1032]
255	[464, 465]	641	[118, 69]	1027	[1264, 767]	1413	[1500, 555]
256	[466, 467]	642	[972, 973]	1028	[1265, 769]	1414	[1501, 216]
257	[468, 469]	643	[974, 975]	1029	[1266, 1267]	1415	[518, 1172]
258	[470, 471]	644	[884, 799]	1030	[770, 1268]	1416	[1050, 156]
259	[472, 473]	645	[328, 976]	1031	[1269, 1270]	1417	[1500, 1243]
260	[472, 474]	646	[698, 977]	1032	[803, 240]	1418	[1025, 1031]
261	[473, 475]	647	[978, 979]	1033	[1271, 764]	1419	[523, 1160]
262	[476, 474]	648	[980, 981]	1034	[863, 275]	1420	[201, 1029]
263	[453, 477]	649	[982, 983]	1035	[1272, 1273]	1421	[1502, 216]
264	[129, 478]	650	[984, 800]	1036	[1274, 1275]	1422	[1503, 434]
265	[450, 479]	651	[985, 836]	1037	[291, 238]	1423	[217, 148]
266	[480, 481]	652	[986, 788]	1038	[64, 1271]	1424	[1215, 1047]
267	[482, 483]	653	[6, 384]	1039	[1276, 931]	1425	[52, 1251]
268	[484, 449]	654	[987, 988]	1040	[733, 81]	1426	[1504, 628]
269	[485, 486]	655	[989, 794]	1041	[295, 1277]	1427	[1456, 446]
270	[487, 488]	656	[907, 265]	1042	[1278, 1263]	1428	[656, 234]
271	[489, 490]	657	[990, 991]	1043	[741, 776]	1429	[1482, 1124]
272	[476, 473]	658	[992, 993]	1044	[1009, 687]	1430	[1076, 1474]
273	[35, 491]	659	[994, 995]	1045	[1279, 1280]	1431	[1501, 187]

274	[492, 493]	660	[996, 93]	1046	[116, 972]	1432	[1093, 1061]
275	[494, 495]	661	[997, 998]	1047	[330, 974]	1433	[1505, 1049]
276	[496, 497]	662	[814, 999]	1048	[1281, 1282]	1434	[523, 1220]
277	[498, 499]	663	[827, 1000]	1049	[784, 1283]	1435	[1142, 1494]
278	[500, 193]	664	[697, 1001]	1050	[403, 828]	1436	[146, 1506]
279	[468, 501]	665	[1002, 982]	1051	[1284, 980]	1437	[583, 1181]
280	[502, 503]	666	[1003, 430]	1052	[678, 1285]	1438	[1103, 1040]
281	[464, 44]	667	[1000, 75]	1053	[1286, 1287]	1439	[1465, 603]
282	[504, 505]	668	[850, 1004]	1054	[1288, 1289]	1440	[1138, 1057]
283	[462, 506]	669	[1005, 1006]	1055	[1290, 747]	1441	[1493, 1216]
284	[507, 508]	670	[1007, 791]	1056	[1291, 1292]	1442	[1473, 545]
285	[509, 495]	671	[400, 1008]	1057	[775, 349]	1443	[667, 587]
286	[510, 511]	672	[322, 1009]	1058	[1293, 1294]	1444	[1507, 1486]
287	[512, 196]	673	[1010, 1011]	1059	[1295, 1296]	1445	[466, 1034]
288	[513, 514]	674	[1012, 1013]	1060	[1297, 1298]	1446	[1492, 1179]
289	[498, 515]	675	[1014, 1015]	1061	[1299, 761]	1447	[1496, 520]
290	[516, 517]	676	[857, 918]	1062	[1300, 679]	1448	[1503, 603]
291	[467, 146]	677	[1016, 1017]	1063	[895, 757]	1449	[1508, 1120]
292	[518, 519]	678	[1018, 1019]	1064	[1301, 397]	1450	[623, 1491]
293	[520, 136]	679	[585, 36]	1065	[1302, 909]	1451	[120, 1509]
294	[521, 522]	680	[507, 235]	1066	[1303, 834]	1452	[1436, 1510]
295	[195, 523]	681	[581, 200]	1067	[334, 1304]	1453	[1363, 1511]
296	[181, 524]	682	[1020, 1021]	1068	[1305, 1306]	1454	[1512, 17]
297	[525, 526]	683	[45, 1022]	1069	[729, 1307]	1455	[1306, 382]
298	[135, 527]	684	[1023, 1024]	1070	[288, 402]	1456	[1513, 1514]
299	[528, 529]	685	[1025, 206]	1071	[868, 252]	1457	[1515, 428]
300	[530, 531]	686	[1026, 1027]	1072	[1308, 1309]	1458	[1316, 1516]
301	[532, 221]	687	[580, 41]	1073	[299, 726]	1459	[1444, 1276]
302	[533, 534]	688	[1028, 1029]	1074	[790, 739]	1460	[348, 978]
303	[535, 232]	689	[232, 1030]	1075	[1310, 1266]	1461	[1517, 1518]
304	[52, 536]	690	[205, 1031]	1076	[66, 1311]	1462	[1426, 825]
305	[537, 538]	691	[612, 1032]	1077	[1312, 333]	1463	[429, 1519]
306	[227, 539]	692	[1033, 1034]	1078	[390, 899]	1464	[995, 838]
307	[540, 541]	693	[1035, 1036]	1079	[245, 859]	1465	[379, 1398]
308	[165, 166]	694	[1037, 1019]	1080	[1313, 1314]	1466	[1520, 1290]
309	[194, 165]	695	[583, 1038]	1081	[1315, 1316]	1467	[1521, 1522]
310	[539, 542]	696	[1039, 1040]	1082	[751, 67]	1468	[1523, 292]
311	[541, 540]	697	[1041, 519]	1083	[915, 1317]	1469	[983, 1524]
312	[227, 167]	698	[1042, 1043]	1084	[336, 959]	1470	[30, 1256]
313	[166, 539]	699	[1044, 1045]	1085	[1318, 1319]	1471	[1407, 869]
314	[543, 544]	700	[1046, 641]	1086	[798, 945]	1472	[1525, 1343]
315	[223, 46]	701	[1047, 624]	1087	[1311, 743]	1473	[241, 927]
316	[545, 233]	702	[1048, 1049]	1088	[68, 1258]	1474	[1522, 1373]
317	[546, 38]	703	[563, 1050]	1089	[1320, 987]	1475	[923, 1526]

318	[534, 547]	704	[1051, 1052]	1090	[1296, 1321]	1476	[940, 1264]
319	[548, 45]	705	[1053, 1054]	1091	[1322, 1323]	1477	[1511, 1295]
320	[513, 549]	706	[512, 1055]	1092	[1324, 254]	1478	[1527, 1528]
321	[550, 515]	707	[1056, 1057]	1093	[860, 916]	1479	[1261, 678]
322	[551, 552]	708	[1058, 1059]	1094	[412, 404]	1480	[1425, 1348]
323	[553, 554]	709	[1060, 1061]	1095	[744, 1325]	1481	[1529, 849]
324	[555, 433]	710	[55, 630]	1096	[1326, 1308]	1482	[1260, 864]
325	[556, 557]	711	[596, 1043]	1097	[1255, 966]	1483	[1418, 990]
326	[457, 558]	712	[1062, 1063]	1098	[1325, 271]	1484	[26, 304]
327	[559, 560]	713	[1064, 571]	1099	[1327, 316]	1485	[1530, 1396]
328	[178, 561]	714	[1065, 1055]	1100	[1328, 925]	1486	[954, 1531]
329	[562, 440]	715	[459, 510]	1101	[926, 881]	1487	[1317, 1408]
330	[563, 155]	716	[1039, 61]	1102	[1329, 817]	1488	[1382, 797]
331	[433, 564]	717	[1060, 169]	1103	[1330, 1331]	1489	[1532, 1533]
332	[444, 565]	718	[629, 56]	1104	[319, 1274]	1490	[892, 107]
333	[566, 567]	719	[586, 128]	1105	[416, 1332]	1491	[831, 287]
334	[8, 568]	720	[1066, 1067]	1106	[976, 787]	1492	[1534, 1344]
335	[143, 569]	721	[1068, 134]	1107	[343, 1326]	1493	[1411, 699]
336	[139, 570]	722	[531, 490]	1108	[829, 1299]	1494	[1535, 1303]
337	[571, 572]	723	[1069, 147]	1109	[1333, 951]	1495	[1446, 900]
338	[573, 570]	724	[1070, 673]	1110	[239, 1334]	1496	[778, 1288]
339	[574, 575]	725	[9, 674]	1111	[824, 1335]	1497	[979, 1371]
340	[576, 577]	726	[1071, 1072]	1112	[305, 1336]	1498	[1536, 1535]
341	[578, 191]	727	[529, 663]	1113	[717, 924]	1499	[1273, 1279]
342	[579, 544]	728	[1073, 43]	1114	[1337, 103]	1500	[1537, 971]
343	[580, 581]	729	[1074, 642]	1115	[341, 1338]	1501	[701, 391]
344	[582, 583]	730	[1075, 1076]	1116	[1339, 755]	1502	[1428, 95]
345	[584, 161]	731	[1077, 1078]	1117	[317, 1340]	1503	[968, 682]
346	[585, 491]	732	[618, 1079]	1118	[1341, 1342]	1504	[1283, 1320]
347	[586, 587]	733	[1075, 1017]	1119	[1343, 77]	1505	[1538, 273]
348	[588, 589]	734	[1080, 1081]	1120	[350, 338]	1506	[753, 1539]
349	[177, 176]	735	[481, 1082]	1121	[354, 832]	1507	[1540, 1541]
350	[209, 590]	736	[1083, 1084]	1122	[746, 1297]	1508	[901, 723]
351	[591, 154]	737	[1085, 482]	1123	[257, 1312]	1509	[223, 1022]
352	[592, 593]	738	[473, 530]	1124	[1344, 1345]	1510	[571, 215]
353	[458, 594]	739	[489, 219]	1125	[1346, 109]	1511	[1134, 622]
354	[553, 595]	740	[474, 475]	1126	[1347, 64]	1512	[1476, 1542]
355	[596, 597]	741	[475, 531]	1127	[1348, 1349]	1513	[533, 1026]
356	[598, 599]	742	[475, 489]	1128	[1350, 904]	1514	[1226, 55]
357	[165, 227]	743	[531, 219]	1129	[703, 819]	1515	[1454, 54]
358	[539, 580]	744	[530, 489]	1130	[1351, 1352]	1516	[567, 676]
359	[600, 180]	745	[1086, 1087]	1131	[1353, 1354]	1517	[1069, 1506]
360	[601, 602]	746	[1088, 1089]	1132	[1355, 89]	1518	[1229, 1495]
361	[603, 604]	747	[8, 1090]	1133	[1356, 407]	1519	[1070, 1105]

362	[605, 606]	748	[1091, 47]	1134	[1357, 1355]	1520	[1059, 8]
363	[153, 607]	749	[194, 168]	1135	[898, 378]	1521	[655, 1194]
364	[138, 175]	750	[1092, 135]	1136	[1358, 1359]	1522	[1507, 602]
365	[558, 594]	751	[1093, 169]	1137	[1360, 1361]	1523	[1092, 520]
366	[608, 609]	752	[1094, 204]	1138	[1362, 1363]	1524	[1489, 666]
367	[610, 184]	753	[1018, 1095]	1139	[835, 1364]	1525	[1195, 1163]
368	[611, 559]	754	[1096, 615]	1140	[96, 1365]	1526	[1223, 488]
369	[168, 166]	755	[1097, 1098]	1141	[742, 1254]	1527	[1502, 187]
370	[612, 613]	756	[1088, 1020]	1142	[818, 346]	1528	[1497, 614]
371	[614, 615]	757	[1099, 1100]	1143	[80, 947]	1529	[1059, 1163]
372	[616, 617]	758	[1064, 214]	1144	[1366, 1367]	1530	[1176, 1542]
373	[618, 522]	759	[1101, 1020]	1145	[1368, 1369]	1531	[1246, 47]
374	[220, 619]	760	[1102, 229]	1146	[84, 314]	1532	[1225, 1088]
375	[145, 211]	761	[542, 41]	1147	[873, 1370]	1533	[535, 545]
376	[605, 620]	762	[1103, 61]	1148	[913, 87]	1534	[1484, 1490]
377	[214, 572]	763	[1104, 1096]	1149	[1371, 1372]	1535	[1477, 1219]
378	[621, 622]	764	[1098, 183]	1150	[1373, 283]	1536	[1023, 649]
379	[576, 623]	765	[140, 472]	1151	[1374, 340]	1537	[1206, 43]
380	[624, 493]	766	[474, 530]	1152	[1375, 1376]	1538	[1504, 1455]
381	[625, 626]	767	[62, 606]	1153	[253, 1377]	1539	[460, 1213]
382	[627, 628]	768	[672, 1105]	1154	[937, 1378]	1540	[1505, 1483]
383	[629, 630]	769	[603, 435]	1155	[1379, 1302]	1541	[1498, 1229]
384	[631, 609]	770	[171, 675]	1156	[284, 1380]	1542	[852, 1538]
385	[632, 633]	771	[1106, 1107]	1157	[1381, 1382]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	258	1	516	0	774	1	1032	0	1290	0
1	0	259	0	517	0	775	0	1033	1	1291	0
2	0	260	0	518	1	776	0	1034	0	1292	0
3	0	261	0	519	0	777	0	1035	1	1293	0
4	0	262	1	520	0	778	1	1036	0	1294	1
5	0	263	1	521	1	779	1	1037	1	1295	0
6	0	264	1	522	1	780	1	1038	1	1296	0
7	0	265	0	523	0	781	0	1039	0	1297	0
8	0	266	0	524	0	782	0	1040	1	1298	0
9	0	267	0	525	0	783	0	1041	0	1299	0
10	1	268	1	526	0	784	1	1042	1	1300	1
11	0	269	1	527	0	785	1	1043	1	1301	0
12	0	270	1	528	0	786	0	1044	0	1302	1
13	0	271	0	529	0	787	1	1045	0	1303	0
14	0	272	1	530	0	788	0	1046	0	1304	0
15	0	273	0	531	1	789	1	1047	0	1305	0
16	1	274	1	532	1	790	1	1048	0	1306	1

17	0	275	1	533	1	791	1	1049	1	1307	1
18	1	276	0	534	0	792	0	1050	0	1308	0
19	0	277	1	535	0	793	0	1051	0	1309	1
20	1	278	0	536	1	794	0	1052	0	1310	1
21	1	279	0	537	0	795	0	1053	1	1311	1
22	0	280	1	538	0	796	0	1054	1	1312	1
23	0	281	0	539	1	797	0	1055	0	1313	0
24	1	282	1	540	1	798	1	1056	0	1314	1
25	0	283	1	541	0	799	1	1057	0	1315	1
26	0	284	0	542	1	800	0	1058	0	1316	1
27	0	285	0	543	1	801	0	1059	0	1317	1
28	0	286	1	544	1	802	0	1060	0	1318	1
29	0	287	0	545	0	803	0	1061	0	1319	1
30	0	288	0	546	0	804	1	1062	1	1320	0
31	0	289	1	547	0	805	0	1063	0	1321	0
32	0	290	0	548	1	806	1	1064	0	1322	0
33	1	291	1	549	0	807	0	1065	1	1323	0
34	0	292	1	550	0	808	0	1066	0	1324	0
35	0	293	0	551	1	809	0	1067	0	1325	1
36	0	294	1	552	0	810	1	1068	0	1326	0
37	1	295	0	553	1	811	1	1069	1	1327	1
38	0	296	0	554	1	812	1	1070	0	1328	0
39	0	297	0	555	1	813	0	1071	1	1329	1
40	1	298	1	556	1	814	1	1072	0	1330	1
41	1	299	0	557	1	815	1	1073	0	1331	0
42	1	300	0	558	0	816	1	1074	1	1332	0
43	1	301	1	559	1	817	1	1075	1	1333	0
44	0	302	1	560	1	818	1	1076	1	1334	1
45	0	303	0	561	0	819	0	1077	1	1335	1
46	0	304	0	562	0	820	0	1078	1	1336	0
47	0	305	0	563	0	821	0	1079	0	1337	1
48	1	306	0	564	1	822	0	1080	0	1338	1
49	1	307	1	565	1	823	0	1081	1	1339	0
50	1	308	1	566	1	824	1	1082	1	1340	0
51	0	309	1	567	1	825	0	1083	0	1341	0
52	0	310	1	568	1	826	0	1084	0	1342	1
53	0	311	0	569	0	827	1	1085	1	1343	1
54	1	312	0	570	1	828	0	1086	1	1344	0
55	0	313	1	571	1	829	1	1087	1	1345	0
56	0	314	1	572	0	830	1	1088	0	1346	1
57	0	315	1	573	1	831	0	1089	0	1347	1
58	0	316	0	574	0	832	1	1090	0	1348	0
59	0	317	0	575	0	833	0	1091	0	1349	1
60	1	318	0	576	1	834	0	1092	0	1350	0

61	0	319	1	577	0	835	1	1093	1	1351	0
62	1	320	0	578	1	836	0	1094	1	1352	0
63	0	321	0	579	0	837	1	1095	0	1353	0
64	1	322	1	580	0	838	0	1096	0	1354	1
65	0	323	1	581	0	839	1	1097	1	1355	1
66	1	324	1	582	1	840	1	1098	1	1356	0
67	1	325	1	583	1	841	0	1099	1	1357	0
68	0	326	0	584	0	842	0	1100	0	1358	0
69	0	327	1	585	1	843	1	1101	1	1359	0
70	1	328	0	586	1	844	0	1102	1	1360	1
71	0	329	0	587	0	845	1	1103	1	1361	1
72	0	330	0	588	0	846	1	1104	1	1362	1
73	0	331	1	589	1	847	1	1105	0	1363	0
74	1	332	0	590	1	848	1	1106	1	1364	0
75	1	333	1	591	1	849	0	1107	0	1365	1
76	1	334	0	592	0	850	1	1108	1	1366	1
77	0	335	0	593	1	851	1	1109	0	1367	0
78	1	336	0	594	1	852	1	1110	1	1368	1
79	0	337	1	595	0	853	0	1111	1	1369	0
80	1	338	1	596	0	854	0	1112	0	1370	1
81	1	339	0	597	0	855	1	1113	0	1371	0
82	1	340	1	598	0	856	1	1114	1	1372	1
83	1	341	1	599	0	857	0	1115	1	1373	0
84	0	342	0	600	0	858	1	1116	0	1374	1
85	1	343	0	601	0	859	1	1117	0	1375	0
86	0	344	1	602	1	860	1	1118	0	1376	0
87	1	345	0	603	1	861	1	1119	1	1377	1
88	0	346	1	604	0	862	1	1120	0	1378	1
89	0	347	1	605	0	863	0	1121	1	1379	0
90	0	348	0	606	0	864	0	1122	0	1380	0
91	0	349	1	607	0	865	1	1123	0	1381	0
92	1	350	0	608	0	866	0	1124	0	1382	0
93	0	351	1	609	1	867	1	1125	1	1383	0
94	1	352	0	610	1	868	1	1126	1	1384	1
95	0	353	1	611	0	869	0	1127	0	1385	1
96	0	354	1	612	0	870	0	1128	0	1386	1
97	1	355	0	613	1	871	0	1129	0	1387	1
98	1	356	0	614	1	872	0	1130	0	1388	0
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100	0	358	1	616	0	874	1	1132	1	1390	1
101	1	359	0	617	0	875	0	1133	0	1391	1
102	1	360	0	618	0	876	1	1134	0	1392	0
103	0	361	1	619	1	877	1	1135	0	1393	1
104	1	362	0	620	1	878	1	1136	0	1394	1

105	0	363	0	621	1	879	1	1137	1	1395	1
106	1	364	1	622	1	880	0	1138	1	1396	1
107	0	365	0	623	1	881	0	1139	1	1397	0
108	0	366	0	624	0	882	1	1140	0	1398	1
109	1	367	1	625	1	883	1	1141	1	1399	0
110	0	368	0	626	1	884	1	1142	1	1400	1
111	0	369	0	627	0	885	1	1143	1	1401	1
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113	0	371	1	629	1	887	0	1145	1	1403	0
114	0	372	0	630	1	888	0	1146	0	1404	0
115	0	373	0	631	1	889	0	1147	1	1405	1
116	0	374	0	632	1	890	1	1148	0	1406	0
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124	1	382	0	640	0	898	0	1156	0	1414	0
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130	1	388	0	646	1	904	0	1162	0	1420	0
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132	0	390	1	648	0	906	0	1164	1	1422	0
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135	1	393	1	651	0	909	1	1167	1	1425	0
136	1	394	0	652	0	910	1	1168	1	1426	1
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145	0	403	0	661	1	919	1	1177	0	1435	1
146	0	404	0	662	1	920	0	1178	0	1436	0
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148	1	406	0	664	0	922	0	1180	0	1438	1

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153	0	411	1	669	0	927	1	1185	0	1443	0
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157	1	415	1	673	1	931	0	1189	1	1447	1
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161	0	419	1	677	0	935	0	1193	1	1451	1
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164	0	422	0	680	0	938	0	1196	1	1454	1
165	1	423	0	681	0	939	1	1197	0	1455	1
166	1	424	1	682	1	940	1	1198	1	1456	1
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170	1	428	1	686	1	944	1	1202	0	1460	0
171	0	429	1	687	0	945	0	1203	1	1461	1
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230	1	488	1	746	0	1004	1	1262	0	1520	0
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248	1	506	0	764	1	1022	1	1280	0	1538	1
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251	0	509	0	767	1	1025	0	1283	1	1541	1
252	1	510	1	768	1	1026	1	1284	0	1542	1
253	1	511	0	769	1	1027	1	1285	0		
254	1	512	0	770	0	1028	1	1286	1		
255	0	513	0	771	1	1029	1	1287	1		
256	0	514	1	772	1	1030	0	1288	1		
257	0	515	1	773	1	1031	1	1289	0		

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