PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z^5 + z + 1, z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a, b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z^5 + z + 1, z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

Date: runone at : 30/04/2020 17:28:40 .

2010 *Mathematics Subject Classification.* 11B85, 11J70, 11B50, 11Y65, 05A15.

Key words and phrases. automatic sequence, continued fraction, Thue-Morse sequence.

* This paper was automatically generated from a computer program developed by the authors, with 36.8 minutes CPU time used.

Theorem 1.1. *When* $(a,b)=(z^5+z+1,z^2+z+1)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{13}+z^{12}+z^{10}+z^9+z^8+z^7+z^6+z^5+z^4+z^3+1,\\
p_1(z)&=z^{18}+z^{17}+z^{13}+z^{11}+z^9+z^8+z^7+z^2,\\
p_2(z)&=z^{20}+z^{16}+z^{14}+z^8+1,\\
p_3(z)&=z^{18}+z^{17}+z^{13}+z^{11}+z^9+z^8+z^7+z^2,\\
p_4(z)&=z^{16}+z^{14}+z^{13}+z^{10}+z^9+z^8+z^7+z^6+z^5+z^3+1,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{13}+z^{10}+z^9+z^7+z^6+z^5+z^4+z^3+1,\\
p_1(z)&=z^{18}+z^{17}+z^{13}+z^{11}+z^9+z^8+z^7+z^2,\\
p_2(z)&=z^{20}+z^{16}+z^{14}+z^8+1,\\
p_3(z)&=z^{18}+z^{17}+z^{13}+z^{11}+z^9+z^8+z^7+z^2,\\
p_4(z)&=z^{16}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^7+z^6+z^5+z^3+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z \left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot \phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z \left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^u M^e[:6u] + x^{2u} M^e[:5u] + x^{5u} M^e[:2u] + x^u M^e[:7u] \\ & + x^{2u} M^e[:6u] + x^{3u} M^e[:5u] + x^{6u} M^e[:2u] + x^{3u} M^e[:7u] \\ & + x^{4u} M^e[:6u] + x^{5u} M^e[:5u] + x^{8u} M^e[:2u] + x^{4u} M^e[:7u] \\ & + x^{5u} M^e[:6u] + x^{6u} M^e[:5u] + x^{9u} M^e[:2u] + x^{5u} M^e[:7u] \\ & + x^{6u} M^e[:6u] + x^{7u} M^e[:5u] + x^{10u} M^e[:2u] + x^{6u} M^e[:7u] \\ & + x^{7u} M^e[:6u] + x^{8u} M^e[:5u] + x^{11u} M^e[:2u] + x^{8u} M^e[:6u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{9u}M^e[:5u] + x^{11u}M^e[:3u] + x^{12u}M^e[:2u] \\
&+ (x^{7u} + x^{8u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e, \\
W^e[:14u] &= W^e[:7u] + x^uW^e[:6u] + x^{2u}W^e[:5u] + x^{5u}W^e[:2u] + x^uW^e[:7u] \\
&+ x^{2u}W^e[:6u] + x^{3u}W^e[:5u] + x^{6u}W^e[:2u] + x^{3u}W^e[:7u] \\
&+ x^{4u}W^e[:6u] + x^{5u}W^e[:5u] + x^{8u}W^e[:2u] + x^{4u}W^e[:7u] \\
&+ x^{5u}W^e[:6u] + x^{6u}W^e[:5u] + x^{9u}W^e[:2u] + x^{5u}W^e[:7u] \\
&+ x^{6u}W^e[:6u] + x^{7u}W^e[:5u] + x^{10u}W^e[:2u] + x^{6u}W^e[:7u] \\
&+ x^{7u}W^e[:6u] + x^{8u}W^e[:5u] + x^{11u}W^e[:2u] + x^{8u}W^e[:6u] \\
&+ x^{9u}W^e[:5u] + x^{11u}W^e[:3u] + x^{12u}W^e[:2u] \\
&+ (x^{7u} + x^{8u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] &= M^o[:7u] + x^uM^o[:6u] + x^{2u}M^o[:5u] + x^{5u}M^o[:2u] + x^uM^o[:7u] \\
&+ x^{2u}M^o[:6u] + x^{3u}M^o[:5u] + x^{6u}M^o[:2u] + x^{3u}M^o[:7u] \\
&+ x^{4u}M^o[:6u] + x^{5u}M^o[:5u] + x^{8u}M^o[:2u] + x^{4u}M^o[:7u] \\
&+ x^{5u}M^o[:6u] + x^{6u}M^o[:5u] + x^{9u}M^o[:2u] + x^{5u}M^o[:7u] \\
&+ x^{6u}M^o[:6u] + x^{7u}M^o[:5u] + x^{10u}M^o[:2u] + x^{6u}M^o[:7u] \\
&+ x^{7u}M^o[:6u] + x^{8u}M^o[:5u] + x^{11u}M^o[:2u] + x^{8u}M^o[:6u] \\
&+ x^{9u}M^o[:5u] + x^{11u}M^o[:3u] + x^{12u}M^o[:2u] \\
&+ (x^{7u} + x^{8u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o, \\
W^o[:14u] &= W^o[:7u] + x^uW^o[:6u] + x^{2u}W^o[:5u] + x^{5u}W^o[:2u] + x^uW^o[:7u] \\
&+ x^{2u}W^o[:6u] + x^{3u}W^o[:5u] + x^{6u}W^o[:2u] + x^{3u}W^o[:7u] \\
&+ x^{4u}W^o[:6u] + x^{5u}W^o[:5u] + x^{8u}W^o[:2u] + x^{4u}W^o[:7u] \\
&+ x^{5u}W^o[:6u] + x^{6u}W^o[:5u] + x^{9u}W^o[:2u] + x^{5u}W^o[:7u] \\
&+ x^{6u}W^o[:6u] + x^{7u}W^o[:5u] + x^{10u}W^o[:2u] + x^{6u}W^o[:7u] \\
&+ x^{7u}W^o[:6u] + x^{8u}W^o[:5u] + x^{11u}W^o[:2u] + x^{8u}W^o[:6u] \\
&+ x^{9u}W^o[:5u] + x^{11u}W^o[:3u] + x^{12u}W^o[:2u] \\
&+ (x^{7u} + x^{8u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$
\begin{aligned}
T[:14u] &= T[:7u] + x^uT[:6u] + x^{2u}T[:5u] + x^{5u}T[:2u] + x^uT[:7u] + x^{2u}T[:6u] \\
&+ x^{3u}T[:5u] + x^{6u}T[:2u] + x^{3u}T[:7u] + x^{4u}T[:6u] + x^{5u}T[:5u] \\
&+ x^{8u}T[:2u] + x^{4u}T[:7u] + x^{5u}T[:6u] + x^{6u}T[:5u] + x^{9u}T[:2u] \\
&+ x^{5u}T[:7u] + x^{6u}T[:6u] + x^{7u}T[:5u] + x^{10u}T[:2u] + x^{6u}T[:7u] \\
&+ x^{7u}T[:6u] + x^{8u}T[:5u] + x^{11u}T[:2u] + x^{8u}T[:6u] + x^{9u}T[:5u]
\end{aligned}
$$

$$+ x^{11u}T[:3u] + x^{12u}T[:2u]$$
$$+ (x^{7u} + x^{8u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}).$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
x^7u &= x^{5u}T[2u:3u] + x^{2u}T[5u:6u] + x^uT[6u:7u] + T[7u:8u],\\
x^8u &= x^{8u}T[:u] + x^{6u}T[2u:3u] + x^{5u}T[3u:4u] + x^{3u}T[5u:6u]\\
&\quad + T[8u:9u],\\
0 &= x^{8u}T[u:2u] + x^{6u}T[3u:4u] + x^{5u}T[4u:5u] + x^{3u}T[6u:7u]\\
&\quad + T[9u:10u],\\
x^10u &= x^{10u}T[:u] + x^{6u}T[4u:5u] + x^{4u}T[6u:7u] + T[10u:11u],\\
x^11u &= x^{10u}T[u:2u] + x^{9u}T[2u:3u] + x^{5u}T[6u:7u] + T[11u:12u],\\
x^12u &= x^{12u}T[:u] + x^{9u}T[3u:4u] + x^{7u}T[5u:6u] + x^{6u}T[6u:7u]\\
&\quad + T[12u:13u],\\
x^13u &= x^{12u}T[u:2u] + x^{11u}T[2u:3u] + x^{9u}T[4u:5u] + x^{8u}T[5u:6u]\\
&\quad + T[13u:14u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.7}$$
$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.8}$$
$$0 = T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.9}$$
$$0 = T[[w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$
$$0 = T[[1w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$
$$0 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$
$$0 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.14}$$
$$1 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.15}$$
$$0 = T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$
$$1 = T[[w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$
$$1 = T[[1w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$
$$1 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$
$$1 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2015 states; its transition function and output function can be found in the annex. Let $A(s,w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i,w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,111)), \tag{2.21}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,1000)), \tag{2.22}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1001)), \tag{2.23}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.24}$$

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1011)), \tag{2.25}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.26}$$

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$1 = \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,111)), \tag{2.28}$$

$$1 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,1000)), \tag{2.29}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1001)), \tag{2.30}$$

$$1 = \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.31}$$

$$1 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1011)), \tag{2.32}$$

$$1 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.33}$$

$$1 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{44}), E_{44}) = (A(i, 0^{14}), E_{14})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 22$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:7u] + x^u M^e[:6u] + x^{2u} M^e[:5u] + x^{5u} M^e[:2u] + x^{7u} R^e,$$
$$W_{2k} = W^e[:7u] + x^u W^e[:6u] + x^{2u} W^e[:5u] + x^{5u} W^e[:2u] + x^{7u} R^e.$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:7u] + x^u M^o[:6u] + x^{2u} M^o[:5u] + x^{5u} M^o[:2u] + x^{7u} R^o,$$
$$W_{2k+1} = W^o[:7u] + x^u W^o[:6u] + x^{2u} W^o[:5u] + x^{5u} W^o[:2u] + x^{7u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$W_{2k}M_{2k} = \big(W^e[:7v] + x^v W^e[:6v] + x^{2v} W^e[:5v] + x^{5v} W^e[:2v] + x^{7u} R^e\big) \times \big(M^e[:7v] + x^v M^e[:6v] + x^{2v} M^e[:5v] + x^{5v} M^e[:2v] + x^{7u} R^e\big); \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$W^e[:14v]M^e[:14v] + x^{2v}W^e[:12v]M^e[:12v] + x^{4v}W^e[:10v]M^e[:10v] + x^{10v}W^e[:4v]M^e[:4v].$$

For all $n \le 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j, k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t \left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{20} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^8 + x^7,\\
q_1(x) &= x^{18} + x^{13} + x^{12} + x^{11} + x^9 + x^7 + x^3 + x^2,\\
q_2(x) &= x^{20} + x^{12} + x^6 + x^4 + 1,\\
q_3(x) &= x^{18} + x^{13} + x^{12} + x^{11} + x^9 + x^7 + x^3 + x^2,\\
q_4(x) &= x^{20} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^7 + x^6 + x^4.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\mathrm{CF}(a, b) = f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{98} + x^{90} + x^{86} + x^{84} + x^{82} + x^{78} + x^{70} + x^{62} + x^{60} + x^{58}\\
&\quad + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30}\\
&\quad + x^{28} + x^{26} + x^{24},\\
p_3(x) &= x^{98} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{76} + x^{72} + x^{70} + x^{68}\\
&\quad + x^{66} + x^{64} + x^{60} + x^{56} + x^{54} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{32}\\
&\quad + x^{30} + x^{16},\\
p_6(x) &= x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{84} + x^{82} + x^{78} + x^{76} + x^{68} + x^{66}\\
&\quad + x^{64} + x^{62} + x^{60} + x^{48} + x^{42} + x^{40} + x^{38} + x^{34} + x^{30} + x^{28} + x^{26}\\
&\quad + x^{22} + x^{18} + x^{16} + x^{14} + x^{10} + x^8 + x^6 + x^4 + x^2 + 1,\\
p_9(x) &= x^{98} + x^{92} + x^{90} + x^{86} + x^{82} + x^{80} + x^{76} + x^{74} + x^{68} + x^{66} + x^{64}\\
&\quad + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{46} + x^{38} + x^{34} + x^{32} + x^{30} + x^{26}\\
&\quad + x^{24} + x^{22} + x^{18} + x^{14} + x^{12} + x^4,
\end{aligned}$$

$$\begin{aligned}p_{12}(x) = {} & x^{102} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{78} + x^{74} + x^{72} + x^{62} + x^{58} \\ & + x^{56} + x^{46} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{22} + x^{18} + x^{16} + x^{14} \\ & + x^{10} + x^8 + x^6 + x^2 + 1,\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}p_0(x) = {} & x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{96} + x^{95} + x^{91} \\ & + x^{86} + x^{84} + x^{83} + x^{82} + x^{79} + x^{77} + x^{73} + x^{70} + x^{69} + x^{66} + x^{65} \\ & + x^{64} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{50} + x^{46} + x^{45} + x^{41} + x^{39} \\ & + x^{36} + x^{35} + x^{33}, \\ p_3(x) = {} & x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} \\ & + x^{79} + x^{78} + x^{76} + x^{71} + x^{69} + x^{68} + x^{66} + x^{61} + x^{59} + x^{58} + x^{57} \\ & + x^{56} + x^{54} + x^{47} + x^{44} + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23} \\ & + x^{21} + x^{20} + x^{18}, \\ p_6(x) = {} & x^{99} + x^{97} + x^{96} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{81} + x^{80} \\ & + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{61} + x^{58} \\ & + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{37} \\ & + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{24} + x^{23} + x^{22} \\ & + x^{17} + x^{14} + x^{13} + x^{12}, \\ p_9(x) = {} & x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{92} + x^{85} + x^{82} + x^{81} \\ & + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{64} + x^{61} + x^{59} \\ & + x^{58} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} \\ & + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} \\ & + x^{23} + x^{21} + x^{20} + x^{18} + x^{13} + x^{10} + x^9 + x^6, \\ p_{12}(x) = {} & x^{111} + x^{109} + x^{108} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{89} \\ & + x^{88} + x^{87} + x^{85} + x^{84} + x^{79} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} \\ & + x^{69} + x^{68} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} \\ & + x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} \\ & + x^{28} + x^{19} + x^{17} + x^{16} + x^{11} + x^9 + x^8 + x^7 + x^5 + x^4 + x^3 + x + 1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}p_0(x) = {} & x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{96} + x^{95} + x^{91} \\ & + x^{86} + x^{84} + x^{83} + x^{82} + x^{79} + x^{77} + x^{73} + x^{70} + x^{69} + x^{66} + x^{65} \\ & + x^{64} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{50} + x^{46} + x^{45} + x^{41} + x^{39} \\ & + x^{36} + x^{35} + x^{33}, \\ p_3(x) = {} & x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80}\end{aligned}$$

$$+ x^{79} + x^{78} + x^{76} + x^{71} + x^{69} + x^{68} + x^{66} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{47} + x^{44} + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23} + x^{21} + x^{20} + x^{18},$$

$$p_6(x) = x^{99} + x^{97} + x^{96} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{81} + x^{80} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{24} + x^{23} + x^{22} + x^{17} + x^{14} + x^{13} + x^{12},$$

$$p_9(x) = x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{92} + x^{85} + x^{82} + x^{81} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{64} + x^{61} + x^{59} + x^{58} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{23} + x^{21} + x^{20} + x^{18} + x^{13} + x^{10} + x^{9} + x^{6},$$

$$p_{12}(x) = x^{111} + x^{109} + x^{108} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{79} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{19} + x^{17} + x^{16} + x^{11} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0].

For $\phi(M^e, 1, 1)$:

$$p_0(x) = x^{120} + x^{114} + x^{112} + x^{110} + x^{106} + x^{100} + x^{88} + x^{82} + x^{78} + x^{76} + x^{74} + x^{68} + x^{58} + x^{56} + x^{54} + x^{50} + x^{46} + x^{44} + x^{42},$$

$$p_3(x) = x^{116} + x^{110} + x^{104} + x^{98} + x^{92} + x^{90} + x^{82} + x^{78} + x^{74} + x^{72} + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{54} + x^{52} + x^{50} + x^{46} + x^{42} + x^{28} + x^{26} + x^{22} + x^{20} + x^{14} + x^{12},$$

$$p_6(x) = x^{114} + x^{104} + x^{98} + x^{94} + x^{90} + x^{86} + x^{80} + x^{74} + x^{72} + x^{66} + x^{64} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{48} + x^{42} + x^{38} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{16} + x^{6} + x^{4} + 1,$$

$$p_9(x) = x^{116} + x^{112} + x^{106} + x^{104} + x^{102} + x^{98} + x^{96} + x^{94} + x^{92} + x^{88} + x^{86} + x^{80} + x^{74} + x^{72} + x^{70} + x^{66} + x^{64} + x^{62} + x^{54} + x^{50} + x^{46} + x^{34} + x^{32} + x^{26} + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^{8} + x^{6} + x^{4},$$

$$p_{12}(x) = x^{120} + x^{112} + x^{104} + x^{96} + x^{88} + x^{72} + x^{48} + x^{40} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$p_0(x) = x^{120} + x^{114} + x^{110} + x^{106} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84}$$

$$
\begin{aligned}
&+ x^{78} + x^{76} + x^{72} + x^{70} + x^{68} + x^{62} + x^{50} + x^{48} + x^{46} + x^{40} + x^{36}\\
&+ x^{30} + x^{28} + x^{24},\\
p_3(x) &= x^{116} + x^{110} + x^{108} + x^{104} + x^{100} + x^{96} + x^{94} + x^{90} + x^{82} + x^{78} + x^{76}\\
&+ x^{74} + x^{72} + x^{68} + x^{56} + x^{54} + x^{50} + x^{46} + x^{40} + x^{38} + x^{34} + x^{32}\\
&+ x^{30} + x^{28} + x^{26} + x^{22} + x^{18} + x^{16},\\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{100} + x^{96} + x^{90} + x^{88} + x^{86} + x^{76} + x^{74} + x^{68}\\
&+ x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{48} + x^{36} + x^{32} + x^{26} + x^{24} + x^{20}\\
&+ x^{18} + x^{14} + x^{8} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{116} + x^{112} + x^{106} + x^{104} + x^{102} + x^{98} + x^{96} + x^{94} + x^{92} + x^{88} + x^{86}\\
&+ x^{80} + x^{74} + x^{72} + x^{70} + x^{66} + x^{64} + x^{62} + x^{54} + x^{50} + x^{46} + x^{34}\\
&+ x^{32} + x^{26} + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^{8} + x^{6} + x^{4},\\
p_{12}(x) &= x^{120} + x^{112} + x^{104} + x^{96} + x^{88} + x^{72} + x^{48} + x^{40} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{99} + x^{97} + x^{95}\\
&+ x^{88} + x^{87} + x^{84} + x^{83} + x^{82} + x^{80} + x^{76} + x^{74} + x^{73} + x^{72} + x^{69}\\
&+ x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{58} + x^{57} + x^{55} + x^{53} + x^{51}\\
&+ x^{46} + x^{45} + x^{43} + x^{39} + x^{35} + x^{34} + x^{33},\\
p_3(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80}\\
&+ x^{79} + x^{78} + x^{76} + x^{71} + x^{69} + x^{68} + x^{66} + x^{61} + x^{59} + x^{58} + x^{57}\\
&+ x^{56} + x^{54} + x^{47} + x^{44} + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23}\\
&+ x^{21} + x^{20} + x^{18},\\
p_6(x) &= x^{99} + x^{97} + x^{96} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{81} + x^{80}\\
&+ x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{61} + x^{58}\\
&+ x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{37}\\
&+ x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{24} + x^{23} + x^{22}\\
&+ x^{17} + x^{14} + x^{13} + x^{12},\\
p_9(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{92} + x^{85} + x^{82} + x^{81}\\
&+ x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{64} + x^{61} + x^{59}\\
&+ x^{58} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44}\\
&+ x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26}\\
&+ x^{23} + x^{21} + x^{20} + x^{18} + x^{13} + x^{10} + x^{9} + x^{6},\\
p_{12}(x) &= x^{111} + x^{109} + x^{108} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{89}\\
&+ x^{88} + x^{87} + x^{85} + x^{84} + x^{79} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71}\\
&+ x^{69} + x^{68} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52}
\end{aligned}
$$

$$+ x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29}$$
$$+ x^{28} + x^{19} + x^{17} + x^{16} + x^{11} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{99} + x^{97} + x^{95} \\
&\quad + x^{88} + x^{87} + x^{84} + x^{83} + x^{82} + x^{80} + x^{76} + x^{74} + x^{73} + x^{72} + x^{69} \\
&\quad + x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{58} + x^{57} + x^{55} + x^{53} + x^{51} \\
&\quad + x^{46} + x^{45} + x^{43} + x^{39} + x^{35} + x^{34} + x^{33}, \\
p_3(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} \\
&\quad + x^{79} + x^{78} + x^{76} + x^{71} + x^{69} + x^{68} + x^{66} + x^{61} + x^{59} + x^{58} + x^{57} \\
&\quad + x^{56} + x^{54} + x^{47} + x^{44} + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23} \\
&\quad + x^{21} + x^{20} + x^{18}, \\
p_6(x) &= x^{99} + x^{97} + x^{96} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{81} + x^{80} \\
&\quad + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{61} + x^{58} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{47} + x^{44} + x^{42} + x^{40} + x^{38} + x^{37} \\
&\quad + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{24} + x^{23} + x^{22} \\
&\quad + x^{17} + x^{14} + x^{13} + x^{12}, \\
p_9(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{92} + x^{85} + x^{82} + x^{81} \\
&\quad + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{64} + x^{61} + x^{59} \\
&\quad + x^{58} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} \\
&\quad + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} \\
&\quad + x^{23} + x^{21} + x^{20} + x^{18} + x^{13} + x^{10} + x^{9} + x^{6}, \\
p_{12}(x) &= x^{111} + x^{109} + x^{108} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{89} \\
&\quad + x^{88} + x^{87} + x^{85} + x^{84} + x^{79} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} \\
&\quad + x^{69} + x^{68} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} \\
&\quad + x^{43} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} \\
&\quad + x^{28} + x^{19} + x^{17} + x^{16} + x^{11} + x^{9} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{98} + x^{94} + x^{88} + x^{86} + x^{84} + x^{80} + x^{76} + x^{74} + x^{66} + x^{64} \\
&\quad + x^{54} + x^{50} + x^{44} + x^{42}, \\
p_3(x) &= x^{98} + x^{94} + x^{88} + x^{82} + x^{72} + x^{68} + x^{66} + x^{62} + x^{60} + x^{54} + x^{50} \\
&\quad + x^{46} + x^{40} + x^{36} + x^{24} + x^{18} + x^{16} + x^{12}, \\
p_6(x) &= x^{96} + x^{92} + x^{90} + x^{82} + x^{72} + x^{70} + x^{64} + x^{62} + x^{60} + x^{52} + x^{50} \\
&\quad + x^{46} + x^{40} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{22} + x^{20} + x^{14}
\end{aligned}$$

$$\begin{aligned}
&+ x^6 + x^4 + x^2 + 1,\\
p_9(x) = {}& x^{98} + x^{92} + x^{90} + x^{86} + x^{82} + x^{80} + x^{76} + x^{74} + x^{68} + x^{66} + x^{64}\\
&+ x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{46} + x^{38} + x^{34} + x^{32} + x^{30} + x^{26}\\
&+ x^{24} + x^{22} + x^{18} + x^{14} + x^{12} + x^4,\\
p_{12}(x) = {}& x^{102} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{78} + x^{74} + x^{72} + x^{62} + x^{58}\\
&+ x^{56} + x^{46} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{22} + x^{18} + x^{16} + x^{14}\\
&+ x^{10} + x^8 + x^6 + x^2 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) = {}& x^{111} + x^{109} + x^{108} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98}\\
&+ x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84}\\
&+ x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} + x^{70} + x^{65} + x^{64} + x^{62} + x^{61}\\
&+ x^{59} + x^{58} + x^{57} + x^{56} + x^{53} + x^{51} + x^{50} + x^{44} + x^{40} + x^{35} + x^{34}\\
&+ x^{32} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24},\\
p_3(x) = {}& x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{87}\\
&+ x^{86} + x^{84} + x^{83} + x^{78} + x^{77} + x^{76} + x^{71} + x^{70} + x^{68} + x^{66} + x^{65}\\
&+ x^{64} + x^{63} + x^{61} + x^{59} + x^{57} + x^{54} + x^{53} + x^{52} + x^{49} + x^{47} + x^{45}\\
&+ x^{44} + x^{37} + x^{33} + x^{32} + x^{28} + x^{26} + x^{24} + x^{23} + x^{19} + x^{17} + x^{16},\\
p_6(x) = {}& x^{105} + x^{102} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{89}\\
&+ x^{87} + x^{83} + x^{82} + x^{81} + x^{76} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65}\\
&+ x^{64} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{52} + x^{44} + x^{42} + x^{41}\\
&+ x^{40} + x^{38} + x^{36} + x^{35} + x^{33} + x^{32} + x^{30} + x^{27} + x^{25} + x^{23} + x^{22}\\
&+ x^{21} + x^{20} + x^{18} + x^{17} + x^{13} + x^{12} + x^7 + x^5 + x^4 + x^3 + x + 1,\\
p_9(x) = {}& x^{103} + x^{101} + x^{100} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{75} + x^{73}\\
&+ x^{72} + x^{71} + x^{69} + x^{68} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{43}\\
&+ x^{41} + x^{40} + x^{23} + x^{21} + x^{20} + x^{11} + x^9 + x^8,\\
p_{12}(x) = {}& x^{111} + x^{109} + x^{108} + x^{103} + x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95}\\
&+ x^{93} + x^{92} + x^{79} + x^{77} + x^{76} + x^{63} + x^{61} + x^{60} + x^{39} + x^{37} + x^{36}\\
&+ x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17}\\
&+ x^{16} + x^7 + x^5 + x^4 + x^3 + x + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) = {}& x^{120} + x^{116} + x^{114} + x^{113} + x^{111} + x^{108} + x^{104} + x^{101} + x^{99} + x^{98}\\
&+ x^{97} + x^{96} + x^{93} + x^{92} + x^{89} + x^{88} + x^{87} + x^{85} + x^{81} + x^{80} + x^{79}\\
&+ x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{65} + x^{64} + x^{63}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{61} + x^{60} + x^{55} + x^{52} + x^{50} + x^{46} + x^{42} + x^{40} + x^{39} + x^{38} + x^{37}\\
&+ x^{34} + x^{33},\\
p_3(x) &= x^{102} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{88} + x^{87} + x^{83} + x^{78} + x^{77}\\
&+ x^{76} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61}\\
&+ x^{59} + x^{57} + x^{54} + x^{53} + x^{49} + x^{46} + x^{44} + x^{43} + x^{42} + x^{39} + x^{37}\\
&+ x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{24} + x^{22} + x^{19} + x^{18},\\
p_6(x) &= x^{108} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{70} + x^{68}\\
&+ x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{50} + x^{46} + x^{44} + x^{40}\\
&+ x^{38} + x^{36} + x^{34} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12},\\
p_9(x) &= x^{114} + x^{109} + x^{108} + x^{107} + x^{105} + x^{104} + x^{103} + x^{102} + x^{98} + x^{94}\\
&+ x^{93} + x^{92} + x^{91} + x^{90} + x^{82} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{66}\\
&+ x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{46} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38}\\
&+ x^{34} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{14} + x^{10} + x^{9}\\
&+ x^{8} + x^{7} + x^{6},\\
p_{12}(x) &= x^{120} + x^{92} + x^{88} + x^{80} + x^{76} + x^{72} + x^{64} + x^{60} + x^{48} + x^{44} + x^{32}\\
&+ x^{24} + x^{12} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88}\\
&+ x^{86} + x^{83} + x^{80} + x^{79} + x^{75} + x^{74} + x^{73} + x^{69} + x^{68} + x^{67} + x^{66}\\
&+ x^{64} + x^{63} + x^{60} + x^{59} + x^{58} + x^{55} + x^{51} + x^{45} + x^{44} + x^{42} + x^{40}\\
&+ x^{38} + x^{37} + x^{35} + x^{33},\\
p_3(x) &= x^{90} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{74} + x^{72}\\
&+ x^{71} + x^{67} + x^{66} + x^{58} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{46} + x^{44}\\
&+ x^{43} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29}\\
&+ x^{27} + x^{24} + x^{23} + x^{19} + x^{18},\\
p_6(x) &= x^{96} + x^{88} + x^{86} + x^{82} + x^{80} + x^{74} + x^{70} + x^{66} + x^{58} + x^{56} + x^{52}\\
&+ x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{32} + x^{28} + x^{24} + x^{22} + x^{14} + x^{12},\\
p_9(x) &= x^{102} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{87}\\
&+ x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{65}\\
&+ x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{49} + x^{48}\\
&+ x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{38} + x^{22} + x^{17}\\
&+ x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_{12}(x) &= x^{108} + x^{100} + x^{96} + x^{92} + x^{76} + x^{60} + x^{36} + x^{32} + x^{28} + x^{20} + x^{16}\\
&+ x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{109} + x^{108} + x^{105} + x^{102} + x^{98} + x^{97} + x^{94} + x^{90} + x^{89} + x^{88}\\
&\quad + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{75} + x^{74} + x^{73} + x^{71} + x^{67}\\
&\quad + x^{64} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{51} + x^{49} + x^{48} + x^{45} + x^{42},\\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{97} + x^{94} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84}\\
&\quad + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{73} + x^{68} + x^{67} + x^{66} + x^{60} + x^{56}\\
&\quad + x^{55} + x^{54} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{35} + x^{34} + x^{33} + x^{32}\\
&\quad + x^{29} + x^{28} + x^{22},\\
p_6(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{93} + x^{91} + x^{90} + x^{86} + x^{85}\\
&\quad + x^{81} + x^{77} + x^{76} + x^{74} + x^{70} + x^{69} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60}\\
&\quad + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{41} + x^{37} + x^{30}\\
&\quad + x^{27} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{11} + x^{9} + x^{8} + x^{7}\\
&\quad + x^{5} + x^{4} + x^{3} + x + 1,\\
p_9(x) &= x^{103} + x^{101} + x^{100} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{75} + x^{73}\\
&\quad + x^{72} + x^{71} + x^{69} + x^{68} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{43}\\
&\quad + x^{41} + x^{40} + x^{23} + x^{21} + x^{20} + x^{11} + x^{9} + x^{8},\\
p_{12}(x) &= x^{111} + x^{109} + x^{108} + x^{103} + x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95}\\
&\quad + x^{93} + x^{92} + x^{79} + x^{77} + x^{76} + x^{63} + x^{61} + x^{60} + x^{39} + x^{37} + x^{36}\\
&\quad + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17}\\
&\quad + x^{16} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{109} + x^{108} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98}\\
&\quad + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84}\\
&\quad + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} + x^{70} + x^{65} + x^{64} + x^{62} + x^{61}\\
&\quad + x^{59} + x^{58} + x^{57} + x^{56} + x^{53} + x^{51} + x^{50} + x^{44} + x^{40} + x^{35} + x^{34}\\
&\quad + x^{32} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24},\\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{87}\\
&\quad + x^{86} + x^{84} + x^{83} + x^{78} + x^{77} + x^{76} + x^{71} + x^{70} + x^{68} + x^{66} + x^{65}\\
&\quad + x^{64} + x^{63} + x^{61} + x^{59} + x^{57} + x^{54} + x^{53} + x^{52} + x^{49} + x^{47} + x^{45}\\
&\quad + x^{44} + x^{37} + x^{33} + x^{32} + x^{28} + x^{26} + x^{24} + x^{23} + x^{19} + x^{17} + x^{16},\\
p_6(x) &= x^{105} + x^{102} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{89}\\
&\quad + x^{87} + x^{83} + x^{82} + x^{81} + x^{76} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65}\\
&\quad + x^{64} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{52} + x^{44} + x^{42} + x^{41}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{40} + x^{38} + x^{36} + x^{35} + x^{33} + x^{32} + x^{30} + x^{27} + x^{25} + x^{23} + x^{22}\\
&+ x^{21} + x^{20} + x^{18} + x^{17} + x^{13} + x^{12} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,\\
p_9(x) &= x^{103} + x^{101} + x^{100} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{75} + x^{73}\\
&+ x^{72} + x^{71} + x^{69} + x^{68} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{43}\\
&+ x^{41} + x^{40} + x^{23} + x^{21} + x^{20} + x^{11} + x^{9} + x^{8},\\
p_{12}(x) &= x^{111} + x^{109} + x^{108} + x^{103} + x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95}\\
&+ x^{93} + x^{92} + x^{79} + x^{77} + x^{76} + x^{63} + x^{61} + x^{60} + x^{39} + x^{37} + x^{36}\\
&+ x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17}\\
&+ x^{16} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88}\\
&+ x^{86} + x^{83} + x^{80} + x^{79} + x^{75} + x^{74} + x^{73} + x^{69} + x^{68} + x^{67} + x^{66}\\
&+ x^{64} + x^{63} + x^{60} + x^{59} + x^{58} + x^{55} + x^{51} + x^{45} + x^{44} + x^{42} + x^{40}\\
&+ x^{38} + x^{37} + x^{35} + x^{33},\\
p_3(x) &= x^{90} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{74} + x^{72}\\
&+ x^{71} + x^{67} + x^{66} + x^{58} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{46} + x^{44}\\
&+ x^{43} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29}\\
&+ x^{27} + x^{24} + x^{23} + x^{19} + x^{18},\\
p_6(x) &= x^{96} + x^{88} + x^{86} + x^{82} + x^{80} + x^{74} + x^{70} + x^{66} + x^{58} + x^{56} + x^{52}\\
&+ x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{32} + x^{28} + x^{24} + x^{22} + x^{14} + x^{12},\\
p_9(x) &= x^{102} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{87}\\
&+ x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{65}\\
&+ x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{49} + x^{48}\\
&+ x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{38} + x^{22} + x^{17}\\
&+ x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_{12}(x) &= x^{108} + x^{100} + x^{96} + x^{92} + x^{76} + x^{60} + x^{36} + x^{32} + x^{28} + x^{20} + x^{16}\\
&+ x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].

For $\phi(W^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{116} + x^{114} + x^{113} + x^{111} + x^{108} + x^{104} + x^{101} + x^{99} + x^{98}\\
&+ x^{97} + x^{96} + x^{93} + x^{92} + x^{89} + x^{88} + x^{87} + x^{85} + x^{81} + x^{80} + x^{79}\\
&+ x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{65} + x^{64} + x^{63}\\
&+ x^{61} + x^{60} + x^{55} + x^{52} + x^{50} + x^{46} + x^{42} + x^{40} + x^{39} + x^{38} + x^{37}\\
&+ x^{34} + x^{33},
\end{aligned}
$$

$$\begin{aligned}
p_3(x) &= x^{102} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{88} + x^{87} + x^{83} + x^{78} + x^{77} \\
&\quad + x^{76} + x^{75} + x^{74} + x^{72} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} \\
&\quad + x^{59} + x^{57} + x^{54} + x^{53} + x^{49} + x^{46} + x^{44} + x^{43} + x^{42} + x^{39} + x^{37} \\
&\quad + x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{24} + x^{22} + x^{19} + x^{18}, \\
p_6(x) &= x^{108} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{70} + x^{68} \\
&\quad + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{50} + x^{46} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12}, \\
p_9(x) &= x^{114} + x^{109} + x^{108} + x^{107} + x^{105} + x^{104} + x^{103} + x^{102} + x^{98} + x^{94} \\
&\quad + x^{93} + x^{92} + x^{91} + x^{90} + x^{82} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{66} \\
&\quad + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{46} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} \\
&\quad + x^{34} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{14} + x^{10} + x^{9} \\
&\quad + x^{8} + x^{7} + x^{6}, \\
p_{12}(x) &= x^{120} + x^{92} + x^{88} + x^{80} + x^{76} + x^{72} + x^{64} + x^{60} + x^{48} + x^{44} + x^{32} \\
&\quad + x^{24} + x^{12} + x^{8} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{111} + x^{109} + x^{108} + x^{105} + x^{102} + x^{98} + x^{97} + x^{94} + x^{90} + x^{89} + x^{88} \\
&\quad + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{75} + x^{74} + x^{73} + x^{71} + x^{67} \\
&\quad + x^{64} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{51} + x^{49} + x^{48} + x^{45} + x^{42}, \\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{97} + x^{94} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} \\
&\quad + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{73} + x^{68} + x^{67} + x^{66} + x^{60} + x^{56} \\
&\quad + x^{55} + x^{54} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{35} + x^{34} + x^{33} + x^{32} \\
&\quad + x^{29} + x^{28} + x^{22}, \\
p_6(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{93} + x^{91} + x^{90} + x^{86} + x^{85} \\
&\quad + x^{81} + x^{77} + x^{76} + x^{74} + x^{70} + x^{69} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60} \\
&\quad + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{41} + x^{37} + x^{30} \\
&\quad + x^{27} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{11} + x^{9} + x^{8} + x^{7} \\
&\quad + x^{5} + x^{4} + x^{3} + x + 1, \\
p_9(x) &= x^{103} + x^{101} + x^{100} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{75} + x^{73} \\
&\quad + x^{72} + x^{71} + x^{69} + x^{68} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{43} \\
&\quad + x^{41} + x^{40} + x^{23} + x^{21} + x^{20} + x^{11} + x^{9} + x^{8}, \\
p_{12}(x) &= x^{111} + x^{109} + x^{108} + x^{103} + x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} \\
&\quad + x^{93} + x^{92} + x^{79} + x^{77} + x^{76} + x^{63} + x^{61} + x^{60} + x^{39} + x^{37} + x^{36} \\
&\quad + x^{35} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} \\
&\quad + x^{16} + x^{7} + x^{5} + x^{4} + x^{3} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	504	[887, 888]	1008	[545, 749]	1512	[1714, 1749]
1	[3, 4]	505	[74, 889]	1009	[1528, 1529]	1513	[68, 1696]
2	[5, 6]	506	[890, 891]	1010	[1530, 1531]	1514	[1887, 1076]
3	[7, 8]	507	[892, 893]	1011	[707, 1532]	1515	[1858, 938]
4	[9, 10]	508	[412, 845]	1012	[1533, 236]	1516	[1811, 1026]
5	[11, 12]	509	[894, 895]	1013	[1534, 60]	1517	[1250, 1220]
6	[13, 14]	510	[248, 896]	1014	[1535, 532]	1518	[82, 392]
7	[15, 16]	511	[897, 898]	1015	[1536, 641]	1519	[1888, 1889]
8	[17, 18]	512	[899, 900]	1016	[1537, 1538]	1520	[1890, 1698]
9	[19, 20]	513	[901, 902]	1017	[533, 1539]	1521	[1854, 1127]
10	[21, 22]	514	[903, 23]	1018	[1322, 1339]	1522	[374, 326]
11	[23, 24]	515	[904, 905]	1019	[469, 1354]	1523	[1045, 856]
12	[25, 26]	516	[906, 907]	1020	[154, 1540]	1524	[1014, 288]
13	[27, 28]	517	[908, 909]	1021	[1452, 1350]	1525	[983, 988]
14	[29, 30]	518	[910, 911]	1022	[1455, 494]	1526	[1891, 1071]
15	[31, 32]	519	[912, 913]	1023	[1353, 1541]	1527	[1766, 1829]
16	[33, 34]	520	[914, 915]	1024	[561, 1542]	1528	[1795, 1892]
17	[35, 36]	521	[916, 917]	1025	[224, 1543]	1529	[1876, 111]
18	[37, 38]	522	[918, 919]	1026	[1544, 1545]	1530	[1765, 1893]
19	[39, 40]	523	[354, 920]	1027	[1545, 151]	1531	[1818, 1056]
20	[41, 42]	524	[921, 922]	1028	[1546, 1547]	1532	[1175, 1810]
21	[43, 44]	525	[923, 924]	1029	[1345, 1548]	1533	[1894, 439]
22	[45, 46]	526	[925, 926]	1030	[1475, 1549]	1534	[1882, 1710]
23	[47, 48]	527	[927, 340]	1031	[1550, 467]	1535	[271, 936]
24	[49, 50]	528	[928, 890]	1032	[1337, 1551]	1536	[1836, 1793]
25	[51, 43]	529	[929, 930]	1033	[616, 1269]	1537	[289, 1807]
26	[52, 53]	530	[931, 117]	1034	[1552, 564]	1538	[1067, 1878]
27	[54, 55]	531	[932, 933]	1035	[1553, 1460]	1539	[1101, 1785]
28	[56, 57]	532	[934, 935]	1036	[1554, 773]	1540	[1895, 1796]
29	[58, 59]	533	[936, 937]	1037	[713, 1555]	1541	[1779, 1715]
30	[60, 61]	534	[938, 914]	1038	[462, 480]	1542	[1831, 1080]
31	[62, 63]	535	[939, 940]	1039	[166, 1556]	1543	[1020, 1805]
32	[64, 65]	536	[941, 942]	1040	[1445, 677]	1544	[1896, 1236]
33	[66, 67]	537	[943, 944]	1041	[1557, 1404]	1545	[1897, 296]
34	[28, 68]	538	[945, 946]	1042	[1558, 47]	1546	[1898, 1899]
35	[69, 70]	539	[947, 948]	1043	[14, 155]	1547	[245, 1182]
36	[71, 72]	540	[949, 950]	1044	[471, 1372]	1548	[1900, 21]
37	[73, 74]	541	[881, 951]	1045	[1559, 1560]	1549	[1194, 1751]
38	[75, 76]	542	[952, 872]	1046	[1561, 1562]	1550	[1099, 827]

39	[77, 78]	543	[953, 954]	1047	[1563, 1474]	1551	[1776, 1865]
40	[79, 80]	544	[955, 956]	1048	[1564, 579]	1552	[1111, 835]
41	[81, 82]	545	[957, 958]	1049	[599, 1432]	1553	[16, 119]
42	[83, 84]	546	[959, 960]	1050	[1565, 1453]	1554	[311, 1890]
43	[85, 86]	547	[88, 961]	1051	[1566, 1567]	1555	[1217, 1833]
44	[87, 88]	548	[962, 963]	1052	[1568, 1569]	1556	[316, 394]
45	[89, 90]	549	[964, 965]	1053	[1570, 700]	1557	[865, 1815]
46	[91, 92]	550	[966, 310]	1054	[1571, 1572]	1558	[1719, 95]
47	[93, 94]	551	[967, 957]	1055	[1573, 1574]	1559	[1901, 1898]
48	[95, 96]	552	[968, 969]	1056	[1575, 1375]	1560	[408, 1820]
49	[97, 98]	553	[970, 949]	1057	[152, 600]	1561	[120, 338]
50	[99, 100]	554	[416, 923]	1058	[1381, 1470]	1562	[1886, 1902]
51	[101, 102]	555	[971, 972]	1059	[1274, 1576]	1563	[907, 380]
52	[103, 104]	556	[973, 16]	1060	[572, 1577]	1564	[295, 1007]
53	[105, 106]	557	[974, 928]	1061	[576, 1578]	1565	[1903, 1904]
54	[107, 108]	558	[975, 976]	1062	[1579, 1580]	1566	[1012, 1053]
55	[109, 110]	559	[977, 344]	1063	[205, 141]	1567	[26, 1004]
56	[111, 112]	560	[978, 979]	1064	[132, 234]	1568	[1873, 1230]
57	[113, 114]	561	[116, 980]	1065	[1376, 198]	1569	[1221, 1832]
58	[115, 116]	562	[981, 405]	1066	[521, 1275]	1570	[329, 1877]
59	[117, 118]	563	[924, 982]	1067	[1581, 1582]	1571	[1867, 1905]
60	[119, 120]	564	[414, 983]	1068	[1517, 747]	1572	[884, 1891]
61	[121, 122]	565	[984, 985]	1069	[770, 1583]	1573	[1871, 1098]
62	[123, 124]	566	[950, 800]	1070	[1386, 1584]	1574	[1906, 1907]
63	[125, 126]	567	[358, 411]	1071	[213, 1585]	1575	[1121, 1170]
64	[127, 128]	568	[986, 987]	1072	[1586, 135]	1576	[1721, 336]
65	[129, 130]	569	[988, 989]	1073	[1587, 499]	1577	[813, 921]
66	[131, 132]	570	[990, 918]	1074	[541, 1588]	1578	[999, 1908]
67	[133, 134]	571	[991, 992]	1075	[142, 1254]	1579	[1158, 1850]
68	[135, 136]	572	[993, 994]	1076	[627, 1589]	1580	[842, 91]
69	[137, 138]	573	[995, 996]	1077	[50, 1535]	1581	[1909, 409]
70	[139, 140]	574	[997, 854]	1078	[1532, 1402]	1582	[1778, 244]
71	[141, 142]	575	[998, 999]	1079	[1590, 734]	1583	[287, 1910]
72	[143, 144]	576	[1000, 1001]	1080	[1591, 1592]	1584	[1188, 1911]
73	[145, 146]	577	[1002, 1003]	1081	[1593, 1397]	1585	[1787, 1013]
74	[147, 148]	578	[1004, 1005]	1082	[1426, 1399]	1586	[1052, 1240]
75	[149, 150]	579	[1006, 1002]	1083	[1547, 710]	1587	[1743, 77]
76	[151, 152]	580	[1007, 1008]	1084	[1594, 1595]	1588	[391, 1912]
77	[153, 154]	581	[1009, 1010]	1085	[492, 1596]	1589	[1069, 415]
78	[155, 156]	582	[1011, 1012]	1086	[48, 1597]	1590	[1064, 1913]
79	[157, 158]	583	[1013, 1014]	1087	[1598, 149]	1591	[895, 1806]
80	[159, 160]	584	[1015, 1016]	1088	[1599, 1258]	1592	[1874, 1791]
81	[161, 162]	585	[1017, 1018]	1089	[1596, 1509]	1593	[1061, 1812]
82	[163, 164]	586	[246, 1019]	1090	[1524, 147]	1594	[1104, 1914]

83	[165, 166]	587	[429, 1020]	1091	[1428, 143]	1595	[1915, 1813]
84	[167, 168]	588	[942, 1021]	1092	[664, 192]	1596	[867, 1206]
85	[124, 169]	589	[944, 1022]	1093	[1280, 212]	1597	[94, 251]
86	[170, 171]	590	[1023, 403]	1094	[1600, 522]	1598	[1905, 1028]
87	[172, 173]	591	[920, 1024]	1095	[781, 1601]	1599	[870, 1703]
88	[174, 175]	592	[788, 804]	1096	[1597, 1602]	1600	[1135, 1091]
89	[176, 177]	593	[840, 1025]	1097	[614, 1603]	1601	[1761, 443]
90	[178, 179]	594	[1026, 1027]	1098	[1495, 1306]	1602	[96, 307]
91	[180, 181]	595	[1028, 87]	1099	[778, 1305]	1603	[396, 1916]
92	[182, 183]	596	[1029, 899]	1100	[1604, 1382]	1604	[65, 1859]
93	[184, 185]	597	[243, 1030]	1101	[1317, 1605]	1605	[1821, 1726]
94	[186, 187]	598	[1031, 1032]	1102	[1606, 1377]	1606	[1019, 378]
95	[188, 189]	599	[1033, 1031]	1103	[1607, 1608]	1607	[853, 1917]
96	[190, 191]	600	[318, 1034]	1104	[1609, 1507]	1608	[1863, 1794]
97	[192, 193]	601	[1035, 1036]	1105	[1471, 1406]	1609	[1173, 802]
98	[194, 195]	602	[381, 1037]	1106	[1478, 1570]	1610	[1804, 1828]
99	[196, 197]	603	[250, 1038]	1107	[475, 785]	1611	[1908, 1918]
100	[198, 199]	604	[958, 349]	1108	[1610, 1611]	1612	[1919, 423]
101	[200, 201]	605	[84, 1039]	1109	[1612, 556]	1613	[1005, 1920]
102	[202, 203]	606	[1040, 978]	1110	[1613, 1614]	1614	[830, 369]
103	[204, 205]	607	[1041, 1042]	1111	[1615, 1333]	1615	[1864, 1774]
104	[206, 207]	608	[383, 395]	1112	[1616, 1387]	1616	[1921, 894]
105	[208, 209]	609	[252, 1043]	1113	[1617, 1484]	1617	[1889, 1903]
106	[210, 211]	610	[1044, 1045]	1114	[1618, 1619]	1618	[98, 1922]
107	[212, 213]	611	[1046, 1047]	1115	[1620, 1621]	1619	[807, 1092]
108	[214, 215]	612	[76, 1048]	1116	[1622, 1557]	1620	[1846, 1853]
109	[216, 217]	613	[1049, 1050]	1117	[1538, 1623]	1621	[356, 1923]
110	[218, 200]	614	[1051, 1052]	1118	[1383, 1624]	1622	[1771, 1814]
111	[219, 220]	615	[1053, 121]	1119	[207, 721]	1623	[1797, 1924]
112	[221, 222]	616	[1054, 421]	1120	[1266, 1283]	1624	[1801, 1103]
113	[223, 224]	617	[372, 1055]	1121	[1427, 125]	1625	[1827, 968]
114	[225, 226]	618	[1056, 1057]	1122	[1304, 1521]	1626	[1830, 255]
115	[227, 13]	619	[888, 1058]	1123	[1625, 1434]	1627	[937, 69]
116	[193, 228]	620	[104, 1059]	1124	[1420, 1626]	1628	[851, 1925]
117	[229, 230]	621	[1060, 1061]	1125	[1627, 1620]	1629	[1926, 811]
118	[231, 232]	622	[940, 1062]	1126	[726, 1628]	1630	[70, 1734]
119	[32, 233]	623	[1063, 1064]	1127	[1629, 457]	1631	[1185, 1006]
120	[234, 235]	624	[1065, 1066]	1128	[1630, 631]	1632	[78, 1915]
121	[236, 237]	625	[855, 1067]	1129	[1318, 1631]	1633	[1866, 880]
122	[238, 239]	626	[1068, 1069]	1130	[1549, 736]	1634	[1927, 931]
123	[240, 241]	627	[891, 1070]	1131	[144, 1632]	1635	[297, 818]
124	[242, 243]	628	[1071, 1072]	1132	[1592, 509]	1636	[320, 419]
125	[244, 245]	629	[385, 887]	1133	[625, 1537]	1637	[1114, 1849]
126	[22, 246]	630	[1073, 1074]	1134	[743, 56]	1638	[24, 1086]

127	[247, 248]	631	[72, 1075]	1135	[1299, 1633]	1639	[946, 1921]
128	[249, 250]	632	[1076, 1077]	1136	[181, 1379]	1640	[1922, 1906]
129	[251, 252]	633	[1078, 1079]	1137	[1634, 208]	1641	[1095, 1881]
130	[253, 254]	634	[1080, 1081]	1138	[737, 1635]	1642	[1750, 1848]
131	[255, 256]	635	[1082, 101]	1139	[1360, 491]	1643	[1704, 1888]
132	[257, 258]	636	[1083, 1084]	1140	[1278, 605]	1644	[1198, 304]
133	[259, 260]	637	[267, 1085]	1141	[1636, 223]	1645	[333, 984]
134	[118, 261]	638	[1086, 1087]	1142	[1291, 1550]	1646	[256, 837]
135	[262, 263]	639	[1088, 789]	1143	[1637, 1449]	1647	[987, 282]
136	[264, 265]	640	[994, 1089]	1144	[1638, 1442]	1648	[795, 1136]
137	[266, 267]	641	[1090, 73]	1145	[1503, 58]	1649	[1237, 885]
138	[268, 269]	642	[1027, 75]	1146	[539, 1639]	1650	[440, 1737]
139	[270, 271]	643	[1091, 71]	1147	[691, 453]	1651	[1924, 1740]
140	[272, 273]	644	[1092, 1093]	1148	[1621, 750]	1652	[1928, 793]
141	[274, 275]	645	[1094, 1095]	1149	[1619, 1640]	1653	[1038, 927]
142	[276, 277]	646	[1096, 1097]	1150	[169, 501]	1654	[1129, 1868]
143	[278, 279]	647	[1098, 1099]	1151	[705, 1641]	1655	[1913, 1894]
144	[280, 281]	648	[1100, 997]	1152	[1642, 176]	1656	[926, 1929]
145	[282, 283]	649	[976, 1101]	1153	[1262, 1643]	1657	[951, 1896]
146	[284, 285]	650	[305, 1102]	1154	[1644, 1400]	1658	[1914, 1897]
147	[286, 287]	651	[1103, 1104]	1155	[1645, 1646]	1659	[960, 97]
148	[288, 289]	652	[792, 1105]	1156	[484, 1498]	1660	[1930, 1837]
149	[290, 291]	653	[1106, 1107]	1157	[1647, 1648]	1661	[898, 1931]
150	[292, 293]	654	[1108, 1109]	1158	[1371, 227]	1662	[1929, 1869]
151	[294, 295]	655	[1110, 1111]	1159	[1265, 1491]	1663	[80, 916]
152	[296, 297]	656	[1112, 1113]	1160	[1302, 1649]	1664	[805, 1140]
153	[298, 299]	657	[922, 27]	1161	[1650, 1634]	1665	[886, 1213]
154	[299, 300]	658	[1097, 1114]	1162	[1326, 145]	1666	[1892, 955]
155	[301, 105]	659	[1034, 1083]	1163	[733, 1651]	1667	[1772, 1094]
156	[302, 303]	660	[1115, 1116]	1164	[1577, 1519]	1668	[1128, 247]
157	[304, 305]	661	[1107, 1078]	1165	[1614, 1652]	1669	[1809, 1932]
158	[306, 242]	662	[1117, 1112]	1166	[199, 1522]	1670	[1037, 932]
159	[261, 292]	663	[1058, 1118]	1167	[1450, 1411]	1671	[1932, 1875]
160	[307, 308]	664	[1059, 1119]	1168	[1388, 1653]	1672	[20, 1728]
161	[309, 270]	665	[1120, 1121]	1169	[1421, 571]	1673	[972, 365]
162	[310, 311]	666	[1122, 1123]	1170	[1654, 1655]	1674	[1933, 1705]
163	[30, 312]	667	[1124, 1125]	1171	[1551, 1593]	1675	[1782, 1117]
164	[313, 314]	668	[1126, 441]	1172	[1562, 1656]	1676	[1904, 952]
165	[315, 316]	669	[1127, 1128]	1173	[1655, 1565]	1677	[1884, 1122]
166	[317, 318]	670	[838, 1129]	1174	[1657, 1658]	1678	[1039, 1756]
167	[319, 320]	671	[1130, 386]	1175	[1527, 635]	1679	[1137, 995]
168	[321, 257]	672	[821, 17]	1176	[1659, 1660]	1680	[1709, 1934]
169	[241, 322]	673	[418, 1131]	1177	[612, 741]	1681	[1001, 366]
170	[323, 324]	674	[1132, 1133]	1178	[1414, 586]	1682	[341, 1887]

171	[308, 325]	675	[1134, 1135]	1179	[1631, 1661]	1683	[325, 1935]
172	[326, 327]	676	[279, 355]	1180	[1662, 725]	1684	[1856, 1760]
173	[328, 329]	677	[1136, 1137]	1181	[1378, 1663]	1685	[410, 1909]
174	[330, 331]	678	[1138, 1139]	1182	[699, 225]	1686	[1241, 1933]
175	[332, 333]	679	[1140, 1141]	1183	[455, 1664]	1687	[1916, 1233]
176	[334, 335]	680	[1142, 1033]	1184	[1436, 1290]	1688	[1177, 1860]
177	[336, 337]	681	[1143, 939]	1185	[1652, 1665]	1689	[1168, 1857]
178	[338, 339]	682	[1144, 1145]	1186	[510, 1490]	1690	[992, 1936]
179	[340, 341]	683	[1146, 934]	1187	[769, 767]	1691	[1937, 1110]
180	[342, 343]	684	[1147, 1100]	1188	[1529, 1666]	1692	[1923, 351]
181	[344, 345]	685	[860, 1148]	1189	[488, 1472]	1693	[1752, 1880]
182	[346, 347]	686	[1149, 1150]	1190	[1667, 1573]	1694	[1759, 1900]
183	[100, 348]	687	[402, 970]	1191	[784, 1668]	1695	[1938, 1628]
184	[349, 350]	688	[273, 1151]	1192	[703, 550]	1696	[57, 755]
185	[351, 352]	689	[954, 1152]	1193	[519, 1669]	1697	[504, 744]
186	[353, 354]	690	[1105, 1153]	1194	[1541, 161]	1698	[529, 696]
187	[355, 356]	691	[1154, 814]	1195	[1555, 1670]	1699	[1560, 1939]
188	[357, 358]	692	[1155, 1156]	1196	[1481, 1448]	1700	[622, 1408]
189	[359, 360]	693	[1157, 1158]	1197	[1633, 1285]	1701	[685, 127]
190	[361, 362]	694	[343, 1159]	1198	[1433, 1594]	1702	[1940, 1650]
191	[363, 364]	695	[1160, 264]	1199	[1671, 624]	1703	[1574, 1941]
192	[365, 266]	696	[1161, 1162]	1200	[1271, 1511]	1704	[1301, 1942]
193	[366, 83]	697	[913, 1163]	1201	[1363, 719]	1705	[1690, 1327]
194	[367, 368]	698	[1164, 1165]	1202	[1672, 1451]	1706	[1943, 1523]
195	[369, 370]	699	[1166, 974]	1203	[1673, 1674]	1707	[1944, 1945]
196	[364, 371]	700	[1167, 904]	1204	[1675, 661]	1708	[673, 8]
197	[106, 372]	701	[362, 1168]	1205	[1440, 1676]	1709	[1946, 1647]
198	[373, 374]	702	[1169, 966]	1206	[1355, 708]	1710	[1947, 1612]
199	[375, 79]	703	[1170, 1171]	1207	[1485, 1308]	1711	[191, 1500]
200	[108, 332]	704	[404, 1172]	1208	[61, 1677]	1712	[772, 49]
201	[376, 377]	705	[1066, 1173]	1209	[606, 1678]	1713	[1626, 1948]
202	[378, 379]	706	[1174, 1175]	1210	[1582, 1332]	1714	[1417, 1949]
203	[380, 381]	707	[1176, 1177]	1211	[1320, 777]	1715	[1584, 1610]
204	[382, 383]	708	[1178, 1088]	1212	[527, 178]	1716	[1602, 204]
205	[384, 385]	709	[1179, 1180]	1213	[523, 761]	1717	[1680, 1950]
206	[386, 387]	710	[285, 1181]	1214	[630, 1679]	1718	[1665, 1951]
207	[388, 389]	711	[1182, 1183]	1215	[1611, 1418]	1719	[1945, 190]
208	[390, 391]	712	[433, 962]	1216	[217, 52]	1720	[756, 1446]
209	[392, 393]	713	[1184, 1029]	1217	[1310, 1680]	1721	[1628, 139]
210	[394, 241]	714	[263, 1185]	1218	[1578, 1681]	1722	[1689, 1659]
211	[395, 396]	715	[1186, 1187]	1219	[46, 764]	1723	[578, 1952]
212	[397, 398]	716	[1010, 1188]	1220	[34, 1682]	1724	[762, 1953]
213	[399, 400]	717	[1189, 1190]	1221	[1683, 447]	1725	[695, 1554]
214	[401, 402]	718	[849, 847]	1222	[1467, 1684]	1726	[128, 1368]

215	[403, 404]	719	[825, 359]	1223	[1635, 1536]	1727	[1939, 1409]
216	[405, 103]	720	[1191, 878]	1224	[1640, 644]	1728	[748, 1568]
217	[406, 4]	721	[387, 1192]	1225	[1454, 1263]	1729	[675, 562]
218	[407, 408]	722	[389, 1011]	1226	[1489, 1465]	1730	[671, 1654]
219	[409, 410]	723	[1193, 1194]	1227	[220, 1685]	1731	[1311, 1954]
220	[411, 412]	724	[1195, 367]	1228	[687, 1303]	1732	[1572, 723]
221	[413, 414]	725	[965, 1196]	1229	[1540, 1348]	1733	[1469, 1476]
222	[415, 416]	726	[1197, 1198]	1230	[555, 1686]	1734	[138, 637]
223	[417, 418]	727	[1199, 259]	1231	[742, 1403]	1735	[495, 727]
224	[419, 420]	728	[909, 1200]	1232	[1687, 2]	1736	[1525, 238]
225	[421, 422]	729	[1145, 1195]	1233	[1276, 581]	1737	[779, 1637]
226	[423, 19]	730	[1201, 859]	1234	[1688, 684]	1738	[1443, 1359]
227	[335, 424]	731	[1202, 1203]	1235	[1392, 628]	1739	[1307, 782]
228	[425, 426]	732	[1203, 1204]	1236	[1463, 1346]	1740	[574, 54]
229	[427, 428]	733	[877, 1205]	1237	[233, 1416]	1741	[1656, 1563]
230	[400, 429]	734	[1206, 388]	1238	[634, 1689]	1742	[1398, 593]
231	[430, 431]	735	[1042, 903]	1239	[59, 133]	1743	[1608, 1309]
232	[432, 433]	736	[1207, 1035]	1240	[1567, 615]	1744	[1661, 210]
233	[63, 85]	737	[1152, 1208]	1241	[460, 1690]	1745	[1955, 1956]
234	[314, 434]	738	[331, 1209]	1242	[1691, 654]	1746	[1957, 1692]
235	[435, 436]	739	[312, 1060]	1243	[1692, 180]	1747	[1958, 1957]
236	[437, 438]	740	[1210, 99]	1244	[1458, 1587]	1748	[1497, 1558]
237	[439, 440]	741	[1211, 1212]	1245	[1429, 1479]	1749	[1948, 131]
238	[441, 442]	742	[1213, 1214]	1246	[1693, 1505]	1750	[1676, 1959]
239	[443, 444]	743	[1109, 1215]	1247	[1477, 1607]	1751	[1960, 1279]
240	[445, 446]	744	[110, 1216]	1248	[162, 1644]	1752	[1380, 1671]
241	[447, 448]	745	[1214, 1217]	1249	[515, 1494]	1753	[1315, 1365]
242	[449, 450]	746	[820, 964]	1250	[1431, 1683]	1754	[479, 1566]
243	[451, 452]	747	[1218, 1219]	1251	[201, 1606]	1755	[551, 1272]
244	[453, 454]	748	[1220, 1221]	1252	[498, 1694]	1756	[168, 1636]
245	[454, 455]	749	[956, 1222]	1253	[1695, 1696]	1757	[239, 1312]
246	[38, 456]	750	[844, 1157]	1254	[275, 1697]	1758	[553, 1961]
247	[457, 458]	751	[1223, 839]	1255	[1698, 1242]	1759	[1464, 1526]
248	[459, 460]	752	[1224, 63]	1256	[1699, 1700]	1760	[1694, 167]
249	[461, 462]	753	[1021, 1225]	1257	[911, 912]	1761	[1962, 1590]
250	[450, 463]	754	[1226, 953]	1258	[1701, 1702]	1762	[512, 1693]
251	[185, 464]	755	[112, 1227]	1259	[1703, 945]	1763	[1963, 596]
252	[465, 466]	756	[832, 425]	1260	[1113, 1090]	1764	[570, 1600]
253	[467, 468]	757	[379, 1228]	1261	[1118, 1704]	1765	[1295, 1964]
254	[171, 469]	758	[1229, 901]	1262	[1705, 1046]	1766	[1632, 11]
255	[470, 471]	759	[1230, 1231]	1263	[1706, 1707]	1767	[1369, 774]
256	[472, 473]	760	[90, 824]	1264	[1200, 991]	1768	[1681, 1960]
257	[474, 475]	761	[1183, 1232]	1265	[1085, 298]	1769	[1651, 1591]
258	[476, 477]	762	[1125, 1017]	1266	[1131, 806]	1770	[12, 1638]

259	[478, 479]	763	[1233, 975]	1267	[1708, 1709]	1771	[1499, 1528]
260	[480, 481]	764	[1234, 981]	1268	[1187, 399]	1772	[1588, 1657]
261	[230, 482]	765	[905, 816]	1269	[1710, 1711]	1773	[1643, 639]
262	[483, 484]	766	[1075, 1235]	1270	[1712, 407]	1774	[722, 1284]
263	[485, 486]	767	[980, 328]	1271	[1713, 1714]	1775	[1961, 717]
264	[487, 488]	768	[1236, 1237]	1272	[1715, 1716]	1776	[1385, 165]
265	[489, 490]	769	[422, 294]	1273	[1717, 1040]	1777	[1514, 1518]
266	[491, 492]	770	[1238, 1239]	1274	[1718, 1719]	1778	[1965, 1966]
267	[493, 449]	771	[1165, 1155]	1275	[915, 1124]	1779	[660, 1419]
268	[494, 495]	772	[823, 843]	1276	[917, 1169]	1780	[1435, 1599]
269	[496, 497]	773	[1240, 1241]	1277	[1720, 1251]	1781	[549, 1662]
270	[232, 498]	774	[1225, 864]	1278	[1696, 1721]	1782	[1543, 1616]
271	[499, 500]	775	[1242, 357]	1279	[1119, 397]	1783	[1664, 688]
272	[501, 502]	776	[803, 1243]	1280	[1722, 1723]	1784	[1670, 1335]
273	[503, 194]	777	[1050, 829]	1281	[1030, 1724]	1785	[1967, 1407]
274	[55, 504]	778	[1212, 89]	1282	[1084, 390]	1786	[140, 1618]
275	[505, 506]	779	[1244, 1245]	1283	[260, 857]	1787	[497, 1968]
276	[507, 508]	780	[1003, 1184]	1284	[1022, 1725]	1788	[187, 643]
277	[509, 510]	781	[1246, 986]	1285	[1716, 401]	1789	[175, 738]
278	[511, 512]	782	[919, 301]	1286	[322, 874]	1790	[594, 1370]
279	[158, 513]	783	[1151, 1247]	1287	[1726, 876]	1791	[580, 1946]
280	[514, 515]	784	[1228, 1248]	1288	[1700, 1727]	1792	[543, 1393]
281	[516, 517]	785	[1249, 1250]	1289	[1728, 1051]	1793	[1969, 1501]
282	[518, 519]	786	[1251, 1252]	1290	[1163, 1729]	1794	[1396, 1516]
283	[520, 521]	787	[1253, 42]	1291	[1697, 826]	1795	[150, 1466]
284	[522, 523]	788	[1254, 1255]	1292	[368, 1730]	1796	[1569, 1970]
285	[524, 525]	789	[1256, 1257]	1293	[1731, 1211]	1797	[1684, 656]
286	[526, 527]	790	[1258, 1259]	1294	[1732, 1044]	1798	[1971, 1972]
287	[528, 529]	791	[1260, 461]	1295	[337, 1733]	1799	[1674, 1261]
288	[530, 531]	792	[1261, 1262]	1296	[1734, 1735]	1800	[1973, 694]
289	[532, 533]	793	[1263, 1264]	1297	[858, 1736]	1801	[1641, 1609]
290	[534, 535]	794	[645, 1265]	1298	[436, 1197]	1802	[1623, 1974]
291	[536, 537]	795	[557, 1266]	1299	[258, 971]	1803	[1975, 514]
292	[538, 539]	796	[1267, 493]	1300	[1724, 1130]	1804	[1976, 1688]
293	[540, 541]	797	[595, 1268]	1301	[1737, 1174]	1805	[1953, 1672]
294	[542, 543]	798	[1269, 771]	1302	[1711, 1738]	1806	[1461, 752]
295	[544, 545]	799	[1270, 1271]	1303	[1150, 1224]	1807	[1977, 766]
296	[546, 547]	800	[1272, 554]	1304	[1739, 1126]	1808	[678, 1978]
297	[548, 549]	801	[1273, 1274]	1305	[324, 437]	1809	[1548, 633]
298	[550, 551]	802	[1275, 1276]	1306	[1740, 1741]	1810	[1658, 1544]
299	[552, 553]	803	[601, 1277]	1307	[1239, 66]	1811	[604, 758]
300	[554, 555]	804	[42, 1278]	1308	[1742, 1743]	1812	[1979, 1613]
301	[556, 557]	805	[693, 157]	1309	[1744, 794]	1813	[1663, 1389]
302	[558, 559]	806	[235, 518]	1310	[834, 1744]	1814	[1649, 1546]

303	[560, 561]	807	[1279, 1280]	1311	[1745, 892]	1815	[1441, 730]
304	[562, 563]	808	[1281, 1282]	1312	[1746, 1747]	1816	[1539, 1627]
305	[564, 565]	809	[1283, 776]	1313	[444, 1191]	1817	[1585, 1260]
306	[566, 451]	810	[1284, 702]	1314	[1116, 1748]	1818	[134, 528]
307	[189, 567]	811	[1285, 39]	1315	[1749, 1750]	1819	[666, 1965]
308	[568, 569]	812	[1286, 1287]	1316	[1730, 1708]	1820	[126, 503]
309	[525, 570]	813	[1257, 1288]	1317	[1047, 883]	1821	[739, 1367]
310	[571, 572]	814	[1289, 613]	1318	[1751, 1752]	1822	[547, 1361]
311	[573, 574]	815	[1290, 1291]	1319	[836, 1753]	1823	[508, 1270]
312	[575, 576]	816	[1292, 1293]	1320	[902, 1754]	1824	[1669, 1430]
313	[577, 578]	817	[1294, 1295]	1321	[1755, 947]	1825	[1639, 1575]
314	[579, 580]	818	[1296, 530]	1322	[1756, 1757]	1826	[662, 682]
315	[581, 582]	819	[228, 1297]	1323	[1758, 1759]	1827	[1668, 647]
316	[583, 584]	820	[1288, 1298]	1324	[1760, 1761]	1828	[1331, 1341]
317	[10, 585]	821	[642, 37]	1325	[1036, 1762]	1829	[1980, 470]
318	[586, 174]	822	[490, 1299]	1326	[1763, 330]	1830	[1646, 472]
319	[464, 587]	823	[1300, 1301]	1327	[815, 334]	1831	[164, 1645]
320	[588, 589]	824	[466, 1302]	1328	[438, 871]	1832	[1682, 1513]
321	[590, 591]	825	[559, 184]	1329	[1062, 1167]	1833	[1660, 692]
322	[446, 592]	826	[463, 745]	1330	[1074, 1764]	1834	[1981, 552]
323	[593, 594]	827	[1303, 1304]	1331	[1171, 1765]	1835	[1287, 496]
324	[160, 595]	828	[1305, 45]	1332	[360, 1766]	1836	[1982, 1983]
325	[596, 597]	829	[1306, 573]	1333	[1767, 808]	1837	[1972, 1984]
326	[598, 599]	830	[711, 1307]	1334	[1768, 1161]	1838	[1424, 1579]
327	[600, 601]	831	[1308, 657]	1335	[339, 1769]	1839	[1486, 516]
328	[565, 602]	832	[1309, 1310]	1336	[1735, 1763]	1840	[1483, 1622]
329	[603, 604]	833	[786, 1311]	1337	[6, 1770]	1841	[237, 1940]
330	[605, 606]	834	[1312, 1313]	1338	[377, 1238]	1842	[1686, 1673]
331	[607, 608]	835	[1314, 1315]	1339	[1235, 1771]	1843	[1941, 1973]
332	[609, 610]	836	[1316, 1317]	1340	[1043, 831]	1844	[1959, 1985]
333	[611, 612]	837	[714, 1318]	1341	[1764, 1772]	1845	[1685, 732]
334	[613, 614]	838	[473, 1319]	1342	[1769, 1773]	1846	[1268, 1986]
335	[615, 616]	839	[211, 1320]	1343	[1048, 313]	1847	[1282, 1300]
336	[617, 618]	840	[1321, 1322]	1344	[797, 321]	1848	[1336, 1530]
337	[619, 620]	841	[1323, 1324]	1345	[1089, 1144]	1849	[1603, 1586]
338	[621, 622]	842	[468, 221]	1346	[348, 1166]	1850	[1974, 1944]
339	[623, 617]	843	[1325, 1326]	1347	[1774, 1226]	1851	[1340, 1943]
340	[624, 625]	844	[1327, 1328]	1348	[300, 1758]	1852	[8, 1553]
341	[626, 627]	845	[1329, 1289]	1349	[868, 1722]	1853	[1512, 1349]
342	[628, 629]	846	[1330, 1331]	1350	[1775, 1776]	1854	[36, 1630]
343	[531, 630]	847	[1332, 474]	1351	[1777, 1778]	1855	[1580, 1533]
344	[631, 632]	848	[1333, 1334]	1352	[1070, 1779]	1856	[563, 520]
345	[633, 634]	849	[1293, 1316]	1353	[893, 998]	1857	[1589, 1395]
346	[635, 636]	850	[1335, 1336]	1354	[1780, 1178]	1858	[1508, 1534]

347	[637, 638]	851	[763, 1337]	1355	[1781, 1179]	1859	[146, 1987]
348	[639, 640]	852	[1338, 686]	1356	[1216, 1782]	1860	[1679, 1629]
349	[641, 642]	853	[1339, 1340]	1357	[352, 1783]	1861	[709, 611]
350	[643, 35]	854	[632, 511]	1358	[1196, 1784]	1862	[1985, 1988]
351	[644, 645]	855	[1341, 1342]	1359	[1785, 973]	1863	[1556, 1564]
352	[477, 646]	856	[1343, 163]	1360	[1786, 1787]	1864	[1970, 1462]
353	[647, 648]	857	[1344, 1345]	1361	[1081, 1788]	1865	[1413, 1321]
354	[649, 650]	858	[1346, 698]	1362	[961, 1789]	1866	[1951, 1343]
355	[651, 129]	859	[1347, 485]	1363	[1024, 1790]	1867	[1313, 1314]
356	[513, 124]	860	[697, 465]	1364	[1133, 1015]	1868	[1319, 670]
357	[652, 653]	861	[1348, 1338]	1365	[1748, 1041]	1869	[1952, 1412]
358	[654, 655]	862	[537, 588]	1366	[799, 274]	1870	[1956, 1971]
359	[656, 229]	863	[1349, 720]	1367	[1209, 406]	1871	[1666, 1552]
360	[657, 658]	864	[1350, 1351]	1368	[1791, 863]	1872	[1678, 1384]
361	[659, 660]	865	[1352, 1353]	1369	[1192, 1201]	1873	[222, 544]
362	[661, 662]	866	[1354, 1355]	1370	[1792, 1143]	1874	[1515, 1294]
363	[663, 664]	867	[1356, 536]	1371	[1793, 1244]	1875	[567, 219]
364	[665, 666]	868	[1357, 1358]	1372	[114, 430]	1876	[597, 1459]
365	[667, 590]	869	[1359, 1360]	1373	[1794, 1795]	1877	[602, 1963]
366	[668, 669]	870	[1361, 1362]	1374	[1222, 323]	1878	[1342, 1667]
367	[670, 671]	871	[718, 706]	1375	[420, 375]	1879	[1422, 1955]
368	[672, 673]	872	[768, 1363]	1376	[1725, 373]	1880	[1989, 1990]
369	[674, 675]	873	[1364, 583]	1377	[1796, 353]	1881	[1986, 1947]
370	[676, 186]	874	[1365, 1366]	1378	[303, 1797]	1882	[1601, 568]
371	[677, 678]	875	[592, 459]	1379	[1798, 1799]	1883	[1576, 1444]
372	[679, 609]	876	[1367, 621]	1380	[283, 1065]	1884	[1988, 1625]
373	[680, 598]	877	[1368, 603]	1381	[1008, 1800]	1885	[1964, 1991]
374	[177, 681]	878	[753, 1369]	1382	[1247, 1801]	1886	[1990, 1675]
375	[682, 683]	879	[636, 1281]	1383	[1800, 1802]	1887	[1506, 1598]
376	[684, 685]	880	[1370, 1371]	1384	[873, 315]	1888	[1624, 1992]
377	[458, 575]	881	[1372, 1373]	1385	[1803, 342]	1889	[1984, 153]
378	[686, 687]	882	[1374, 188]	1386	[846, 1804]	1890	[648, 1993]
379	[688, 689]	883	[731, 507]	1387	[1805, 1229]	1891	[1542, 9]
380	[690, 691]	884	[1375, 1376]	1388	[1806, 262]	1892	[1994, 196]
381	[692, 693]	885	[1377, 1378]	1389	[1807, 1713]	1893	[1390, 478]
382	[694, 695]	886	[1379, 1380]	1390	[982, 1176]	1894	[1950, 780]
383	[696, 697]	887	[569, 1381]	1391	[1208, 1082]	1895	[701, 1487]
384	[698, 699]	888	[1382, 1383]	1392	[1790, 276]	1896	[1373, 231]
385	[700, 701]	889	[1384, 1385]	1393	[1808, 1210]	1897	[746, 548]
386	[702, 703]	890	[716, 1386]	1394	[1079, 1809]	1898	[40, 676]
387	[704, 705]	891	[1387, 1388]	1395	[1810, 1811]	1899	[1949, 1691]
388	[706, 707]	892	[1389, 1390]	1396	[828, 1745]	1900	[148, 704]
389	[708, 709]	893	[1391, 1392]	1397	[1783, 1249]	1901	[452, 1642]
390	[710, 711]	894	[1393, 679]	1398	[1812, 1218]	1902	[1991, 1969]

391	[712, 713]	895	[1394, 1256]	1399	[1227, 1186]	1903	[1992, 651]
392	[689, 714]	896	[1395, 1396]	1400	[1159, 1699]	1904	[618, 1979]
393	[715, 170]	897	[1397, 1398]	1401	[1156, 5]	1905	[1942, 1958]
394	[582, 716]	898	[1399, 1391]	1402	[1813, 1712]	1906	[195, 1292]
395	[717, 718]	899	[1400, 1401]	1403	[1754, 788]	1907	[1595, 1976]
396	[719, 715]	900	[1402, 1352]	1404	[1814, 1009]	1908	[1520, 1617]
397	[720, 674]	901	[1403, 1344]	1405	[1815, 1023]	1909	[740, 476]
398	[721, 722]	902	[535, 216]	1406	[1816, 1755]	1910	[1954, 487]
399	[723, 724]	903	[1404, 1405]	1407	[1817, 1818]	1911	[1531, 1962]
400	[725, 726]	904	[1406, 659]	1408	[1770, 1819]	1912	[783, 1604]
401	[727, 560]	905	[1407, 1325]	1409	[1820, 1821]	1913	[610, 1980]
402	[728, 729]	906	[1408, 1329]	1410	[1822, 869]	1914	[44, 546]
403	[730, 731]	907	[1409, 1410]	1411	[1823, 427]	1915	[1993, 1480]
404	[732, 733]	908	[1411, 1323]	1412	[1057, 1115]	1916	[585, 1493]
405	[734, 206]	909	[1412, 1413]	1413	[1824, 1803]	1917	[456, 1504]
406	[608, 735]	910	[724, 1414]	1414	[1825, 284]	1918	[1648, 1967]
407	[736, 737]	911	[1415, 577]	1415	[935, 1054]	1919	[1987, 1981]
408	[738, 739]	912	[1416, 1417]	1416	[265, 798]	1920	[1410, 1267]
409	[130, 740]	913	[1418, 759]	1417	[1826, 1827]	1921	[1457, 1977]
410	[741, 742]	914	[1419, 1420]	1418	[1828, 1234]	1922	[1324, 712]
411	[653, 690]	915	[683, 182]	1419	[1829, 1830]	1923	[1978, 1559]
412	[655, 743]	916	[1264, 1421]	1420	[1747, 1831]	1924	[650, 1415]
413	[744, 619]	917	[179, 1422]	1421	[1832, 993]	1925	[1983, 1468]
414	[745, 746]	918	[1401, 526]	1422	[1833, 1792]	1926	[1605, 1286]
415	[747, 748]	919	[1423, 1424]	1423	[1093, 306]	1927	[1583, 1989]
416	[749, 489]	920	[1425, 1426]	1424	[1087, 1096]	1928	[1968, 1581]
417	[750, 751]	921	[751, 1427]	1425	[1834, 1835]	1929	[1492, 1982]
418	[752, 623]	922	[481, 1428]	1426	[442, 1836]	1930	[517, 1994]
419	[587, 672]	923	[448, 1429]	1427	[1837, 1838]	1931	[591, 172]
420	[589, 753]	924	[1430, 1431]	1428	[1839, 280]	1932	[1351, 558]
421	[754, 542]	925	[197, 32]	1429	[1702, 1189]	1933	[1510, 1561]
422	[755, 756]	926	[1432, 1433]	1430	[930, 1780]	1934	[735, 1975]
423	[203, 757]	927	[506, 626]	1431	[1799, 791]	1935	[1966, 1488]
424	[758, 649]	928	[1434, 1435]	1432	[1032, 253]	1936	[1473, 1571]
425	[759, 760]	929	[760, 1436]	1433	[1016, 25]	1937	[1447, 1615]
426	[761, 762]	930	[1437, 1438]	1434	[1840, 796]	1938	[1995, 270]
427	[2, 763]	931	[1439, 1440]	1435	[1841, 1701]	1939	[1738, 1822]
428	[764, 765]	932	[1255, 1441]	1436	[1231, 1842]	1940	[1936, 1717]
429	[766, 538]	933	[1442, 1443]	1437	[1018, 1816]	1941	[1248, 1798]
430	[767, 768]	934	[1444, 1445]	1438	[1219, 1063]	1942	[1753, 1901]
431	[226, 769]	935	[53, 1356]	1439	[996, 1843]	1943	[1784, 1142]
432	[215, 770]	936	[1446, 137]	1440	[1844, 1845]	1944	[1204, 361]
433	[771, 772]	937	[500, 1447]	1441	[1842, 1202]	1945	[1996, 363]
434	[773, 524]	938	[1448, 159]	1442	[848, 1846]	1946	[1102, 819]

435	[774, 775]	939	[765, 1364]	1443	[269, 1786]	1947	[1723, 866]
436	[776, 754]	940	[1449, 1450]	1444	[1847, 1817]	1948	[979, 417]
437	[777, 778]	941	[1451, 1452]	1445	[1848, 1164]	1949	[393, 1861]
438	[681, 534]	942	[1453, 1454]	1446	[1835, 268]	1950	[1917, 1246]
439	[658, 779]	943	[1328, 1455]	1447	[879, 1767]	1951	[1935, 302]
440	[780, 781]	944	[156, 1456]	1448	[1736, 384]	1952	[1934, 1824]
441	[782, 33]	945	[1362, 1457]	1449	[1232, 1823]	1953	[1245, 1068]
442	[136, 783]	946	[646, 1458]	1450	[1849, 1149]	1954	[291, 861]
443	[669, 784]	947	[1459, 51]	1451	[889, 1775]	1955	[67, 822]
444	[785, 786]	948	[209, 566]	1452	[1819, 1777]	1956	[1025, 959]
445	[787, 788]	949	[1460, 1461]	1453	[1850, 1706]	1957	[1139, 317]
446	[789, 790]	950	[1462, 1273]	1454	[1843, 1851]	1958	[1190, 1223]
447	[791, 792]	951	[183, 1463]	1455	[801, 1808]	1959	[347, 1928]
448	[793, 278]	952	[729, 1464]	1456	[4, 1852]	1960	[1911, 1132]
449	[794, 795]	953	[629, 218]	1457	[1788, 897]	1961	[969, 1839]
450	[796, 797]	954	[1465, 1425]	1458	[1853, 1134]	1962	[1893, 1885]
451	[798, 799]	955	[1466, 1467]	1459	[1077, 1826]	1963	[985, 882]
452	[800, 801]	956	[1468, 1469]	1460	[18, 1854]	1964	[1141, 1997]
453	[802, 803]	957	[1470, 1471]	1461	[1855, 1732]	1965	[948, 249]
454	[804, 805]	958	[1472, 1473]	1462	[1707, 1718]	1966	[1123, 1000]
455	[806, 807]	959	[1474, 1475]	1463	[1845, 1138]	1967	[1762, 1879]
456	[808, 809]	960	[1476, 1477]	1464	[1851, 841]	1968	[875, 977]
457	[810, 309]	961	[173, 1478]	1465	[1773, 1746]	1969	[1997, 113]
458	[811, 812]	962	[1479, 540]	1466	[1252, 1856]	1970	[1918, 1927]
459	[790, 813]	963	[1480, 1330]	1467	[293, 1146]	1971	[1162, 1120]
460	[814, 815]	964	[1297, 1481]	1468	[1857, 1108]	1972	[86, 1872]
461	[816, 817]	965	[1298, 1482]	1469	[371, 852]	1973	[1910, 1160]
462	[818, 29]	966	[1358, 1483]	1470	[989, 1858]	1974	[1998, 1996]
463	[819, 820]	967	[1484, 1485]	1471	[1859, 1720]	1975	[1931, 1870]
464	[350, 821]	968	[1482, 1486]	1472	[1860, 1742]	1976	[1912, 1147]
465	[822, 823]	969	[1487, 1394]	1473	[431, 1844]	1977	[1789, 1106]
466	[824, 825]	970	[1488, 1489]	1474	[963, 1825]	1978	[933, 376]
467	[826, 413]	971	[1405, 667]	1475	[1852, 1207]	1979	[900, 1781]
468	[827, 828]	972	[1490, 483]	1476	[1861, 1862]	1980	[1757, 833]
469	[829, 830]	973	[1491, 1492]	1477	[896, 1863]	1981	[1899, 1739]
470	[831, 832]	974	[640, 446]	1478	[327, 1049]	1982	[434, 1930]
471	[833, 834]	975	[1493, 1437]	1479	[850, 1864]	1983	[1925, 64]
472	[835, 836]	976	[502, 1439]	1480	[1741, 967]	1984	[122, 1895]
473	[837, 838]	977	[1366, 607]	1481	[426, 929]	1985	[1243, 1937]
474	[839, 840]	978	[1494, 1495]	1482	[1727, 906]	1986	[102, 1919]
475	[841, 115]	979	[1496, 1497]	1483	[1865, 1866]	1987	[345, 1841]
476	[254, 842]	980	[1498, 1499]	1484	[277, 1867]	1988	[1920, 1840]
477	[843, 844]	981	[1500, 728]	1485	[1868, 990]	1989	[1181, 943]
478	[845, 846]	982	[1501, 1502]	1486	[1869, 908]	1990	[1907, 1154]

479	[847, 848]	983	[1277, 1503]	1487	[1870, 925]	1991	[1733, 1998]
480	[817, 849]	984	[775, 1374]	1488	[1148, 1871]	1992	[1802, 382]
481	[424, 850]	985	[1502, 1347]	1489	[812, 1834]	1993	[1729, 1926]
482	[428, 851]	986	[1504, 1496]	1490	[1205, 109]	1994	[1902, 941]
483	[852, 853]	987	[1505, 1506]	1491	[1172, 910]	1995	[1999, 499]
484	[854, 855]	988	[1507, 1508]	1492	[1872, 272]	1996	[1677, 665]
485	[856, 81]	989	[1509, 1510]	1493	[1838, 1873]	1997	[1653, 1687]
486	[857, 858]	990	[638, 1423]	1494	[281, 1874]	1998	[620, 663]
487	[859, 860]	991	[1456, 202]	1495	[92, 810]	1999	[2000, 880]
488	[861, 862]	992	[1511, 1512]	1496	[809, 1847]	2000	[2001, 1372]
489	[863, 435]	993	[584, 680]	1497	[370, 93]	2001	[2002, 1794]
490	[864, 865]	994	[1513, 1514]	1498	[1862, 432]	2002	[2003, 150]
491	[866, 867]	995	[1438, 1515]	1499	[1875, 1876]	2003	[2004, 1252]
492	[398, 868]	996	[1516, 1517]	1500	[1877, 346]	2004	[2005, 563]
493	[869, 870]	997	[1518, 1296]	1501	[1878, 1768]	2005	[2006, 914]
494	[871, 290]	998	[1519, 1520]	1502	[1215, 286]	2006	[2007, 683]
495	[872, 873]	999	[1259, 652]	1503	[1879, 1073]	2007	[2008, 346]
496	[874, 875]	1000	[1521, 668]	1504	[1180, 1731]	2008	[2009, 637]
497	[876, 877]	1001	[1522, 214]	1505	[1055, 1193]	2009	[2010, 1086]
498	[862, 319]	1002	[1523, 1524]	1506	[1880, 1199]	2010	[2011, 1598]
499	[878, 879]	1003	[757, 1525]	1507	[1881, 1882]	2011	[2012, 290]
500	[880, 881]	1004	[1526, 1527]	1508	[1153, 1855]	2012	[2013, 536]
501	[882, 107]	1005	[482, 1357]	1509	[1883, 1884]	2013	[2014, 943]
502	[883, 884]	1006	[486, 41]	1510	[1885, 1886]	2014	[1, 156]
503	[885, 886]	1007	[1334, 505]	1511	[1072, 1883]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	336	0	672	0	1008	0	1344	1	1680	0
1	0	337	0	673	1	1009	0	1345	1	1681	0
2	0	338	1	674	0	1010	0	1346	0	1682	0
3	0	339	1	675	1	1011	1	1347	0	1683	1
4	1	340	0	676	0	1012	0	1348	1	1684	0
5	0	341	0	677	1	1013	1	1349	1	1685	1
6	1	342	1	678	0	1014	0	1350	0	1686	0
7	0	343	1	679	0	1015	1	1351	1	1687	0
8	0	344	1	680	0	1016	1	1352	0	1688	0
9	1	345	1	681	0	1017	0	1353	0	1689	1
10	0	346	0	682	1	1018	0	1354	0	1690	0
11	0	347	0	683	0	1019	0	1355	0	1691	0
12	1	348	0	684	1	1020	0	1356	1	1692	1
13	1	349	0	685	1	1021	0	1357	1	1693	0
14	1	350	0	686	0	1022	0	1358	1	1694	1
15	0	351	0	687	1	1023	0	1359	0	1695	0

16	0	352	1	688	1	1024	1	1360	1	1696	0
17	0	353	0	689	1	1025	0	1361	0	1697	0
18	1	354	1	690	1	1026	1	1362	0	1698	1
19	1	355	1	691	1	1027	0	1363	1	1699	0
20	1	356	1	692	1	1028	1	1364	0	1700	0
21	0	357	0	693	1	1029	1	1365	0	1701	1
22	1	358	0	694	1	1030	0	1366	1	1702	1
23	0	359	1	695	0	1031	0	1367	1	1703	0
24	1	360	1	696	1	1032	1	1368	0	1704	1
25	1	361	1	697	1	1033	1	1369	1	1705	0
26	1	362	1	698	1	1034	1	1370	1	1706	1
27	1	363	0	699	1	1035	0	1371	1	1707	0
28	0	364	0	700	1	1036	1	1372	0	1708	1
29	1	365	1	701	1	1037	1	1373	1	1709	0
30	0	366	1	702	0	1038	1	1374	1	1710	0
31	0	367	0	703	1	1039	0	1375	0	1711	0
32	0	368	0	704	0	1040	1	1376	0	1712	1
33	0	369	0	705	0	1041	0	1377	1	1713	0
34	0	370	0	706	0	1042	0	1378	1	1714	1
35	0	371	1	707	1	1043	1	1379	0	1715	0
36	0	372	0	708	0	1044	1	1380	0	1716	1
37	1	373	0	709	1	1045	0	1381	0	1717	0
38	1	374	0	710	1	1046	1	1382	0	1718	0
39	1	375	1	711	1	1047	1	1383	0	1719	0
40	1	376	1	712	1	1048	0	1384	0	1720	1
41	1	377	1	713	1	1049	1	1385	1	1721	0
42	0	378	0	714	0	1050	0	1386	0	1722	1
43	0	379	1	715	1	1051	0	1387	1	1723	0
44	0	380	1	716	0	1052	0	1388	1	1724	1
45	1	381	1	717	1	1053	1	1389	0	1725	0
46	1	382	1	718	1	1054	1	1390	1	1726	0
47	0	383	1	719	1	1055	1	1391	0	1727	0
48	0	384	1	720	1	1056	0	1392	1	1728	0
49	1	385	1	721	0	1057	1	1393	0	1729	1
50	0	386	0	722	0	1058	0	1394	1	1730	1
51	1	387	0	723	1	1059	0	1395	0	1731	1
52	1	388	0	724	1	1060	1	1396	1	1732	0
53	1	389	0	725	0	1061	0	1397	1	1733	1
54	1	390	1	726	1	1062	1	1398	0	1734	1
55	0	391	1	727	1	1063	1	1399	1	1735	1
56	0	392	1	728	1	1064	1	1400	0	1736	1
57	0	393	1	729	0	1065	0	1401	1	1737	1
58	1	394	1	730	1	1066	0	1402	0	1738	0
59	0	395	1	731	1	1067	1	1403	1	1739	0

60	0	396	1	732	0	1068	0	1404	0	1740	0
61	0	397	1	733	0	1069	1	1405	0	1741	1
62	0	398	0	734	0	1070	0	1406	1	1742	0
63	0	399	1	735	0	1071	1	1407	1	1743	1
64	0	400	0	736	1	1072	0	1408	1	1744	1
65	0	401	1	737	0	1073	1	1409	1	1745	1
66	0	402	1	738	0	1074	0	1410	0	1746	1
67	1	403	1	739	0	1075	0	1411	0	1747	1
68	1	404	0	740	1	1076	1	1412	1	1748	0
69	0	405	0	741	1	1077	0	1413	0	1749	0
70	1	406	1	742	1	1078	1	1414	0	1750	0
71	0	407	1	743	1	1079	1	1415	1	1751	1
72	1	408	0	744	1	1080	1	1416	1	1752	0
73	1	409	0	745	1	1081	0	1417	1	1753	0
74	1	410	1	746	0	1082	0	1418	1	1754	1
75	1	411	1	747	0	1083	0	1419	0	1755	0
76	0	412	0	748	0	1084	1	1420	1	1756	0
77	1	413	1	749	1	1085	0	1421	0	1757	0
78	1	414	1	750	0	1086	0	1422	0	1758	1
79	1	415	0	751	1	1087	0	1423	1	1759	1
80	0	416	1	752	1	1088	0	1424	0	1760	1
81	1	417	0	753	0	1089	1	1425	1	1761	1
82	0	418	1	754	0	1090	0	1426	0	1762	0
83	0	419	0	755	1	1091	0	1427	0	1763	0
84	0	420	0	756	1	1092	0	1428	0	1764	0
85	0	421	0	757	1	1093	1	1429	1	1765	0
86	0	422	1	758	1	1094	0	1430	0	1766	1
87	0	423	1	759	0	1095	0	1431	0	1767	1
88	0	424	1	760	1	1096	0	1432	1	1768	0
89	1	425	0	761	1	1097	0	1433	1	1769	1
90	1	426	1	762	1	1098	0	1434	0	1770	1
91	1	427	0	763	0	1099	0	1435	0	1771	0
92	0	428	0	764	0	1100	0	1436	1	1772	1
93	0	429	0	765	1	1101	1	1437	0	1773	1
94	0	430	1	766	0	1102	0	1438	1	1774	0
95	0	431	1	767	1	1103	1	1439	0	1775	0
96	1	432	1	768	1	1104	1	1440	0	1776	1
97	1	433	1	769	1	1105	1	1441	0	1777	1
98	0	434	1	770	1	1106	1	1442	1	1778	1
99	0	435	1	771	1	1107	1	1443	0	1779	1
100	0	436	0	772	1	1108	0	1444	1	1780	0
101	1	437	0	773	1	1109	1	1445	1	1781	0
102	0	438	0	774	1	1110	0	1446	0	1782	0
103	1	439	0	775	0	1111	1	1447	0	1783	1

104	0	440	1	776	0	1112	1	1448	1	1784	1
105	1	441	1	777	0	1113	1	1449	0	1785	0
106	1	442	0	778	0	1114	0	1450	1	1786	1
107	1	443	0	779	1	1115	1	1451	0	1787	1
108	1	444	1	780	1	1116	0	1452	0	1788	1
109	0	445	0	781	0	1117	1	1453	0	1789	0
110	1	446	0	782	1	1118	0	1454	1	1790	1
111	0	447	1	783	0	1119	0	1455	0	1791	0
112	1	448	1	784	1	1120	0	1456	1	1792	1
113	0	449	0	785	1	1121	0	1457	1	1793	1
114	0	450	1	786	1	1122	0	1458	1	1794	1
115	1	451	0	787	0	1123	1	1459	0	1795	0
116	1	452	0	788	1	1124	1	1460	1	1796	1
117	0	453	0	789	0	1125	1	1461	1	1797	0
118	1	454	0	790	1	1126	1	1462	0	1798	0
119	0	455	1	791	1	1127	0	1463	1	1799	0
120	1	456	0	792	0	1128	1	1464	1	1800	0
121	0	457	0	793	1	1129	1	1465	1	1801	0
122	1	458	1	794	0	1130	1	1466	0	1802	0
123	0	459	1	795	1	1131	0	1467	1	1803	1
124	0	460	0	796	1	1132	0	1468	1	1804	0
125	0	461	0	797	1	1133	0	1469	1	1805	1
126	1	462	1	798	0	1134	1	1470	0	1806	1
127	0	463	0	799	1	1135	0	1471	1	1807	0
128	0	464	0	800	0	1136	1	1472	0	1808	0
129	0	465	0	801	0	1137	0	1473	1	1809	0
130	0	466	1	802	0	1138	0	1474	1	1810	0
131	0	467	0	803	0	1139	1	1475	0	1811	0
132	1	468	1	804	0	1140	0	1476	1	1812	0
133	1	469	0	805	1	1141	0	1477	0	1813	0
134	1	470	0	806	1	1142	0	1478	1	1814	0
135	1	471	1	807	0	1143	0	1479	1	1815	0
136	0	472	0	808	0	1144	1	1480	1	1816	1
137	0	473	0	809	0	1145	0	1481	1	1817	1
138	1	474	1	810	0	1146	0	1482	0	1818	1
139	1	475	1	811	1	1147	1	1483	0	1819	0
140	1	476	0	812	0	1148	1	1484	0	1820	1
141	0	477	1	813	1	1149	0	1485	1	1821	0
142	0	478	1	814	0	1150	1	1486	0	1822	0
143	1	479	1	815	0	1151	0	1487	0	1823	0
144	0	480	0	816	0	1152	0	1488	1	1824	0
145	1	481	1	817	0	1153	0	1489	0	1825	0
146	1	482	0	818	1	1154	1	1490	0	1826	1
147	1	483	1	819	0	1155	1	1491	1	1827	1

148	0	484	1	820	0	1156	1	1492	0	1828	1
149	1	485	0	821	0	1157	1	1493	0	1829	0
150	0	486	1	822	0	1158	1	1494	1	1830	0
151	0	487	0	823	1	1159	0	1495	0	1831	0
152	1	488	1	824	1	1160	0	1496	0	1832	0
153	1	489	1	825	1	1161	1	1497	0	1833	0
154	0	490	0	826	0	1162	0	1498	1	1834	1
155	1	491	0	827	1	1163	0	1499	0	1835	0
156	0	492	0	828	1	1164	1	1500	1	1836	1
157	1	493	0	829	0	1165	1	1501	1	1837	0
158	0	494	1	830	1	1166	1	1502	0	1838	0
159	0	495	1	831	0	1167	1	1503	0	1839	0
160	1	496	0	832	1	1168	1	1504	0	1840	0
161	1	497	1	833	1	1169	0	1505	1	1841	0
162	1	498	0	834	1	1170	1	1506	1	1842	0
163	0	499	0	835	0	1171	1	1507	0	1843	1
164	0	500	1	836	1	1172	1	1508	0	1844	0
165	0	501	1	837	0	1173	1	1509	0	1845	1
166	0	502	1	838	0	1174	0	1510	0	1846	1
167	0	503	1	839	1	1175	1	1511	0	1847	1
168	0	504	0	840	0	1176	1	1512	1	1848	1
169	1	505	1	841	1	1177	0	1513	1	1849	1
170	0	506	0	842	1	1178	0	1514	1	1850	0
171	0	507	0	843	1	1179	1	1515	0	1851	1
172	0	508	0	844	0	1180	0	1516	0	1852	0
173	0	509	0	845	1	1181	1	1517	0	1853	1
174	0	510	1	846	0	1182	1	1518	0	1854	0
175	0	511	1	847	1	1183	1	1519	0	1855	1
176	1	512	0	848	1	1184	1	1520	0	1856	0
177	0	513	1	849	1	1185	1	1521	0	1857	1
178	1	514	0	850	1	1186	1	1522	0	1858	0
179	0	515	1	851	0	1187	1	1523	0	1859	1
180	1	516	1	852	1	1188	0	1524	0	1860	0
181	1	517	0	853	1	1189	1	1525	1	1861	1
182	0	518	1	854	1	1190	1	1526	0	1862	1
183	0	519	1	855	0	1191	1	1527	1	1863	1
184	0	520	0	856	0	1192	1	1528	0	1864	1
185	0	521	0	857	1	1193	1	1529	0	1865	0
186	0	522	1	858	0	1194	1	1530	0	1866	1
187	1	523	1	859	0	1195	1	1531	1	1867	1
188	0	524	1	860	1	1196	1	1532	1	1868	1
189	1	525	1	861	1	1197	1	1533	0	1869	0
190	1	526	1	862	0	1198	1	1534	1	1870	0
191	0	527	0	863	1	1199	1	1535	0	1871	1

192	1	528	0	864	0	1200	0	1536	1	1872	0
193	1	529	1	865	0	1201	1	1537	1	1873	0
194	0	530	0	866	0	1202	1	1538	1	1874	0
195	0	531	1	867	1	1203	0	1539	1	1875	0
196	0	532	1	868	1	1204	0	1540	1	1876	0
197	1	533	0	869	0	1205	0	1541	1	1877	1
198	0	534	1	870	0	1206	0	1542	0	1878	1
199	1	535	1	871	1	1207	1	1543	0	1879	0
200	1	536	0	872	1	1208	0	1544	1	1880	1
201	1	537	0	873	0	1209	1	1545	0	1881	0
202	0	538	0	874	0	1210	1	1546	1	1882	1
203	1	539	0	875	1	1211	1	1547	0	1883	0
204	1	540	1	876	1	1212	0	1548	0	1884	0
205	1	541	0	877	0	1213	1	1549	1	1885	0
206	0	542	0	878	0	1214	1	1550	0	1886	1
207	0	543	1	879	0	1215	0	1551	1	1887	1
208	1	544	0	880	1	1216	1	1552	1	1888	0
209	1	545	0	881	0	1217	1	1553	0	1889	1
210	1	546	1	882	1	1218	0	1554	1	1890	0
211	1	547	0	883	1	1219	1	1555	1	1891	0
212	1	548	1	884	0	1220	0	1556	1	1892	1
213	1	549	0	885	1	1221	1	1557	0	1893	1
214	1	550	1	886	0	1222	1	1558	0	1894	0
215	1	551	0	887	0	1223	1	1559	0	1895	1
216	0	552	0	888	0	1224	1	1560	0	1896	1
217	1	553	1	889	0	1225	1	1561	1	1897	0
218	1	554	1	890	0	1226	0	1562	1	1898	1
219	0	555	0	891	1	1227	1	1563	1	1899	1
220	1	556	1	892	0	1228	1	1564	0	1900	0
221	1	557	1	893	0	1229	1	1565	0	1901	0
222	0	558	0	894	0	1230	0	1566	0	1902	1
223	0	559	1	895	1	1231	1	1567	1	1903	0
224	0	560	1	896	0	1232	0	1568	0	1904	0
225	0	561	1	897	1	1233	0	1569	1	1905	0
226	1	562	1	898	1	1234	0	1570	1	1906	0
227	1	563	0	899	0	1235	1	1571	1	1907	1
228	0	564	1	900	0	1236	1	1572	0	1908	0
229	0	565	0	901	1	1237	0	1573	1	1909	1
230	0	566	0	902	1	1238	1	1574	0	1910	0
231	1	567	0	903	0	1239	0	1575	0	1911	1
232	1	568	0	904	1	1240	1	1576	0	1912	0
233	0	569	0	905	1	1241	0	1577	1	1913	1
234	1	570	0	906	1	1242	0	1578	0	1914	0
235	1	571	1	907	1	1243	1	1579	1	1915	1

236	0	572	1	908	0	1244	1	1580	1	1916	0
237	0	573	1	909	1	1245	1	1581	1	1917	0
238	1	574	0	910	1	1246	0	1582	1	1918	1
239	0	575	0	911	1	1247	0	1583	0	1919	1
240	0	576	0	912	1	1248	1	1584	0	1920	0
241	1	577	0	913	1	1249	1	1585	1	1921	1
242	0	578	0	914	0	1250	0	1586	0	1922	1
243	0	579	1	915	0	1251	1	1587	1	1923	1
244	0	580	0	916	0	1252	0	1588	1	1924	1
245	0	581	0	917	0	1253	0	1589	1	1925	0
246	1	582	1	918	1	1254	1	1590	1	1926	0
247	0	583	1	919	1	1255	1	1591	1	1927	0
248	1	584	1	920	1	1256	0	1592	0	1928	1
249	0	585	0	921	1	1257	1	1593	0	1929	0
250	1	586	1	922	1	1258	1	1594	1	1930	0
251	0	587	0	923	1	1259	0	1595	1	1931	1
252	0	588	0	924	0	1260	1	1596	1	1932	1
253	0	589	0	925	1	1261	0	1597	0	1933	0
254	0	590	0	926	1	1262	0	1598	0	1934	0
255	0	591	1	927	0	1263	1	1599	0	1935	1
256	0	592	1	928	0	1264	0	1600	0	1936	1
257	1	593	0	929	1	1265	0	1601	1	1937	0
258	0	594	1	930	0	1266	0	1602	1	1938	0
259	1	595	1	931	0	1267	1	1603	1	1939	0
260	0	596	1	932	1	1268	1	1604	0	1940	1
261	0	597	0	933	1	1269	0	1605	0	1941	1
262	1	598	0	934	1	1270	1	1606	0	1942	0
263	0	599	1	935	1	1271	0	1607	1	1943	1
264	0	600	1	936	0	1272	0	1608	1	1944	0
265	1	601	0	937	1	1273	0	1609	1	1945	0
266	0	602	1	938	1	1274	0	1610	0	1946	0
267	0	603	1	939	1	1275	0	1611	0	1947	0
268	1	604	0	940	0	1276	0	1612	1	1948	0
269	0	605	0	941	0	1277	1	1613	0	1949	1
270	1	606	1	942	0	1278	0	1614	1	1950	0
271	0	607	0	943	0	1279	0	1615	1	1951	1
272	1	608	1	944	0	1280	1	1616	1	1952	0
273	1	609	0	945	0	1281	0	1617	1	1953	1
274	0	610	1	946	0	1282	1	1618	0	1954	0
275	1	611	1	947	0	1283	0	1619	0	1955	1
276	0	612	0	948	1	1284	0	1620	1	1956	0
277	0	613	1	949	1	1285	1	1621	1	1957	1
278	1	614	0	950	0	1286	0	1622	0	1958	1
279	0	615	1	951	0	1287	0	1623	0	1959	0

280	0	616	1	952	0	1288	0	1624	0	1960	1
281	1	617	0	953	1	1289	0	1625	1	1961	0
282	1	618	0	954	1	1290	0	1626	0	1962	1
283	0	619	0	955	0	1291	0	1627	1	1963	0
284	1	620	0	956	1	1292	0	1628	0	1964	0
285	1	621	1	957	0	1293	1	1629	0	1965	1
286	1	622	0	958	0	1294	0	1630	1	1966	1
287	0	623	1	959	1	1295	0	1631	1	1967	0
288	0	624	0	960	1	1296	1	1632	1	1968	1
289	1	625	0	961	0	1297	0	1633	1	1969	1
290	1	626	0	962	1	1298	0	1634	0	1970	1
291	0	627	1	963	1	1299	0	1635	1	1971	0
292	0	628	1	964	0	1300	1	1636	0	1972	0
293	1	629	1	965	0	1301	1	1637	0	1973	0
294	0	630	1	966	1	1302	0	1638	1	1974	0
295	0	631	1	967	0	1303	1	1639	0	1975	1
296	1	632	1	968	0	1304	0	1640	1	1976	0
297	1	633	1	969	0	1305	1	1641	0	1977	0
298	1	634	1	970	1	1306	0	1642	0	1978	1
299	0	635	0	971	0	1307	0	1643	1	1979	0
300	1	636	0	972	0	1308	0	1644	1	1980	0
301	1	637	0	973	1	1309	1	1645	1	1981	1
302	0	638	0	974	1	1310	1	1646	0	1982	1
303	1	639	0	975	0	1311	1	1647	1	1983	0
304	1	640	1	976	1	1312	1	1648	1	1984	1
305	1	641	0	977	1	1313	1	1649	0	1985	1
306	0	642	0	978	1	1314	0	1650	1	1986	0
307	1	643	0	979	0	1315	0	1651	1	1987	1
308	0	644	0	980	1	1316	1	1652	1	1988	0
309	1	645	0	981	1	1317	1	1653	1	1989	1
310	1	646	0	982	1	1318	1	1654	1	1990	1
311	1	647	0	983	1	1319	1	1655	1	1991	1
312	0	648	0	984	0	1320	1	1656	1	1992	0
313	0	649	1	985	0	1321	0	1657	0	1993	1
314	1	650	1	986	0	1322	0	1658	0	1994	1
315	0	651	1	987	1	1323	1	1659	1	1995	0
316	1	652	0	988	0	1324	1	1660	0	1996	0
317	0	653	1	989	0	1325	1	1661	1	1997	1
318	1	654	0	990	0	1326	0	1662	0	1998	0
319	0	655	0	991	1	1327	0	1663	0	1999	0
320	0	656	1	992	0	1328	0	1664	1	2000	0
321	0	657	1	993	1	1329	1	1665	0	2001	0
322	0	658	0	994	1	1330	0	1666	1	2002	0
323	0	659	1	995	1	1331	1	1667	1	2003	0

324	1	660	1	996	0	1332	1	1668	1	2004	0
325	1	661	1	997	0	1333	1	1669	0	2005	0
326	0	662	1	998	0	1334	0	1670	1	2006	0
327	1	663	0	999	0	1335	1	1671	1	2007	0
328	0	664	0	1000	0	1336	1	1672	1	2008	0
329	1	665	0	1001	0	1337	1	1673	0	2009	0
330	0	666	0	1002	0	1338	1	1674	0	2010	0
331	0	667	1	1003	1	1339	1	1675	0	2011	0
332	0	668	1	1004	0	1340	1	1676	0	2012	0
333	1	669	0	1005	0	1341	0	1677	0	2013	0
334	1	670	0	1006	1	1342	1	1678	0	2014	0
335	1	671	1	1007	0	1343	0	1679	0		

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