PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z^4 + z^2 + z + 1, z^3 + z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a, b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z^4 + z^2 + z + 1, z^3 + z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

Date: runone at : 30/04/2020 17:28:40 .

2010 *Mathematics Subject Classification.* 11B85, 11J70, 11B50, 11Y65, 05A15.

Key words and phrases. automatic sequence, continued fraction, Thue-Morse sequence.

* This paper was automatically generated from a computer program developed by the authors, with 36.1 minutes CPU time used.

Theorem 1.1. *When $(a, b) = (z^4 + z^2 + z + 1, z^3 + z^2 + z + 1)$, the two power series* $\mathrm{CF}(a, b)$ *and* $\mathrm{CF}(b, a)$ *are algebraic over $\mathbb{F}_2(z)$, with minimal polynomials of the form*

$$p_4(z)y^4 + p_3(z)y^3 + p_2(z)y^2 + p_1(z)y + p_0(z) = 0$$

where, for $\mathrm{CF}(a, b)$,

$$\begin{aligned}
p_0(z) &= z^{17} + z^{16} + z^{15} + z^{12} + z^{10} + z^7 + z^4 + z^3 + z^2 + 1,\\
p_1(z) &= z^{21} + z^{18} + z^{16} + z^{15} + z^{11} + z^9 + z^8 + z^5 + z^4 + z^3,\\
p_2(z) &= z^{24} + z^{18} + z^8 + z^6 + z^4 + 1,\\
p_3(z) &= z^{21} + z^{18} + z^{16} + z^{15} + z^{11} + z^9 + z^8 + z^5 + z^4 + z^3,\\
p_4(z) &= z^{18} + z^{17} + z^{16} + z^{15} + z^{12} + z^{10} + z^8 + z^7 + z^6 + z^4 + z^3 + z^2 + 1,
\end{aligned}$$

and for $\mathrm{CF}(b, a)$,

$$\begin{aligned}
p_0(z) &= z^{17} + z^{16} + z^{15} + z^{10} + z^7 + z^4 + z^3 + z^2 + 1,\\
p_1(z) &= z^{21} + z^{18} + z^{16} + z^{15} + z^{11} + z^9 + z^8 + z^5 + z^4 + z^3,\\
p_2(z) &= z^{24} + z^{18} + z^8 + z^6 + z^4 + 1,\\
p_3(z) &= z^{21} + z^{18} + z^{16} + z^{15} + z^{11} + z^9 + z^8 + z^5 + z^4 + z^3,\\
p_4(z) &= z^{18} + z^{17} + z^{16} + z^{15} + z^{10} + z^8 + z^7 + z^6 + z^4 + z^3 + z^2 + 1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z) \\ P_{n-1}(z) & Q_{n-1}(z) \end{pmatrix} := \begin{pmatrix} t_n(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t_{n-1}(z) & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} t_0(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a, b) = \frac{P_n(z)}{Q_n(z)} \in \mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a, b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x) = x^{\deg(t_{2^n-1})} &\begin{pmatrix} t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(t_{2^n-2})} \begin{pmatrix} t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\
&\cdots \times x^{\deg(t_0)} \begin{pmatrix} t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},\\
W_n(x) = x^{\deg(\bar{t}_{2^n-1})} &\begin{pmatrix} \bar{t}_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(\bar{t}_{2^n-2})} \begin{pmatrix} \bar{t}_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\
&\cdots \times x^{\deg(\bar{t}_0)} \begin{pmatrix} \bar{t}_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b, a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned}
M_{n+1}(x) &= W_n(x) \cdot M_n(x),\\
W_{n+1}(x) &= M_n(x) \cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. ☐

Define

$$R^e = \begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned}
M^e[:14u] = {} & M^e[:7u] + x^u M^e[:6u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{5u}M^e[:2u]\\
& + x^{6u}M^e[:u] + x^u M^e[:7u] + x^{2u}M^e[:6u] + x^{4u}M^e[:4u]\\
& + x^{5u}M^e[:3u] + x^{6u}M^e[:2u] + x^{7u}M^e[:u] + x^{2u}M^e[:7u]\\
& + x^{3u}M^e[:6u] + x^{5u}M^e[:4u] + x^{6u}M^e[:3u] + x^{7u}M^e[:2u]\\
& + x^{8u}M^e[:u] + x^{5u}M^e[:7u] + x^{6u}M^e[:6u] + x^{8u}M^e[:4u]\\
& + x^{9u}M^e[:3u] + x^{10u}M^e[:2u] + x^{11u}M^e[:u] + x^{10u}M^e[:4u]
\end{aligned}$$

$$
\begin{aligned}
&+ (x^{7u} + x^{8u} + x^{9u} + x^{12u})R^e,\\
W^e[:14u] &= W^e[:7u] + x^u W^e[:6u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u]\\
&+ x^{6u}W^e[:u] + x^u W^e[:7u] + x^{2u}W^e[:6u] + x^{4u}W^e[:4u]\\
&+ x^{5u}W^e[:3u] + x^{6u}W^e[:2u] + x^{7u}W^e[:u] + x^{2u}W^e[:7u]\\
&+ x^{3u}W^e[:6u] + x^{5u}W^e[:4u] + x^{6u}W^e[:3u] + x^{7u}W^e[:2u]\\
&+ x^{8u}W^e[:u] + x^{5u}W^e[:7u] + x^{6u}W^e[:6u] + x^{8u}W^e[:4u]\\
&+ x^{9u}W^e[:3u] + x^{10u}W^e[:2u] + x^{11u}W^e[:u] + x^{10u}W^e[:4u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{12u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] &= M^o[:7u] + x^u M^o[:6u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u]\\
&+ x^{6u}M^o[:u] + x^u M^o[:7u] + x^{2u}M^o[:6u] + x^{4u}M^o[:4u]\\
&+ x^{5u}M^o[:3u] + x^{6u}M^o[:2u] + x^{7u}M^o[:u] + x^{2u}M^o[:7u]\\
&+ x^{3u}M^o[:6u] + x^{5u}M^o[:4u] + x^{6u}M^o[:3u] + x^{7u}M^o[:2u]\\
&+ x^{8u}M^o[:u] + x^{5u}M^o[:7u] + x^{6u}M^o[:6u] + x^{8u}M^o[:4u]\\
&+ x^{9u}M^o[:3u] + x^{10u}M^o[:2u] + x^{11u}M^o[:u] + x^{10u}M^o[:4u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{12u})R^o,\\
W^o[:14u] &= W^o[:7u] + x^u W^o[:6u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u]\\
&+ x^{6u}W^o[:u] + x^u W^o[:7u] + x^{2u}W^o[:6u] + x^{4u}W^o[:4u]\\
&+ x^{5u}W^o[:3u] + x^{6u}W^o[:2u] + x^{7u}W^o[:u] + x^{2u}W^o[:7u]\\
&+ x^{3u}W^o[:6u] + x^{5u}W^o[:4u] + x^{6u}W^o[:3u] + x^{7u}W^o[:2u]\\
&+ x^{8u}W^o[:u] + x^{5u}W^o[:7u] + x^{6u}W^o[:6u] + x^{8u}W^o[:4u]\\
&+ x^{9u}W^o[:3u] + x^{10u}W^o[:2u] + x^{11u}W^o[:u] + x^{10u}W^o[:4u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{12u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$
\begin{aligned}
T[:14u] &= T[:7u] + x^u T[:6u] + x^{3u}T[:4u] + x^{4u}T[:3u] + x^{5u}T[:2u] + x^{6u}T[:u]\\
&+ x^u T[:7u] + x^{2u}T[:6u] + x^{4u}T[:4u] + x^{5u}T[:3u] + x^{6u}T[:2u]\\
&+ x^{7u}T[:u] + x^{2u}T[:7u] + x^{3u}T[:6u] + x^{5u}T[:4u] + x^{6u}T[:3u]\\
&+ x^{7u}T[:2u] + x^{8u}T[:u] + x^{5u}T[:7u] + x^{6u}T[:6u] + x^{8u}T[:4u]\\
&+ x^{9u}T[:3u] + x^{10u}T[:2u] + x^{11u}T[:u] + x^{10u}T[:4u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{12u}).
\end{aligned}
$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$x^7 u = x^{6u}T[u:2u] + x^{5u}T[2u:3u] + x^{4u}T[3u:4u] + x^{3u}T[4u:5u]$$

$$
\begin{aligned}
&+ x^u T[6u\!:\!7u] + T[7u\!:\!8u],\\
x^8u &= x^{7u}T[u\!:\!2u] + x^{3u}T[5u\!:\!6u] + x^{2u}T[6u\!:\!7u] + T[8u\!:\!9u],\\
x^9u &= x^{9u}T[:u] + x^{8u}T[u\!:\!2u] + x^{6u}T[3u\!:\!4u] + x^{5u}T[4u\!:\!5u]\\
&\quad + T[9u\!:\!10u],\\
0 &= x^{9u}T[u\!:\!2u] + x^{8u}T[2u\!:\!3u] + x^{6u}T[4u\!:\!5u] + x^{5u}T[5u\!:\!6u]\\
&\quad + T[10u\!:\!11u],\\
0 &= x^{11u}T[:u] + x^{9u}T[2u\!:\!3u] + x^{8u}T[3u\!:\!4u] + x^{6u}T[5u\!:\!6u]\\
&\quad + x^{5u}T[6u\!:\!7u] + T[11u\!:\!12u],\\
x^1 2u &= x^{10u}T[2u\!:\!3u] + T[12u\!:\!13u],\\
0 &= x^{10u}T[3u\!:\!4u] + T[13u\!:\!14u],
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
0 &= T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2]\\
&\quad + T[[111w]_2], \tag{2.7}\\
0 &= T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.8}\\
0 &= T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.9}\\
0 &= T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1010w]_2], \tag{2.10}\\
0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\
&\quad + T[[1011w]_2], \tag{2.11}\\
0 &= T[[10w]_2] + T[[1100w]_2], \tag{2.12}\\
0 &= T[[11w]_2] + T[[1101w]_2], \tag{2.13}
\end{aligned}
$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$
\begin{aligned}
1 &= T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2]\\
&\quad + T[[111w]_2], \tag{2.14}\\
1 &= T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.15}\\
1 &= T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.16}\\
0 &= T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1010w]_2], \tag{2.17}\\
0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\
&\quad + T[[1011w]_2], \tag{2.18}\\
1 &= T[[10w]_2] + T[[1100w]_2], \tag{2.19}\\
0 &= T[[11w]_2] + T[[1101w]_2], \tag{2.20}
\end{aligned}
$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1690 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$\begin{aligned} 0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,110)) + \tau(A(s,111)), \end{aligned} \tag{2.21}$$

$$0 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.22}$$

$$\begin{aligned} 0 &= \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,1001)), \end{aligned} \tag{2.23}$$

$$\begin{aligned} 0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) \\ &\quad + \tau(A(s,1010)), \end{aligned} \tag{2.24}$$

$$\begin{aligned} 0 &= \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\ &\quad + \tau(A(s,110)) + \tau(A(s,1011)), \end{aligned} \tag{2.25}$$

$$0 = \tau(A(s,10)) + \tau(A(s,1100)), \tag{2.26}$$

$$0 = \tau(A(s,11)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$\begin{aligned} 1 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,110)) + \tau(A(s,111)), \end{aligned} \tag{2.28}$$

$$1 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.29}$$

$$\begin{aligned} 1 &= \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) \\ &\quad + \tau(A(s,1001)), \end{aligned} \tag{2.30}$$

$$\begin{aligned} 0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) \\ &\quad + \tau(A(s,1010)), \end{aligned} \tag{2.31}$$

$$\begin{aligned} 0 &= \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\ &\quad + \tau(A(s,110)) + \tau(A(s,1011)), \end{aligned} \tag{2.32}$$

$$1 = \tau(A(s,10)) + \tau(A(s,1100)), \tag{2.33}$$

$$0 = \tau(A(s,11)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{46}), E_{46}) = (A(i, 0^{18}), E_{18})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 23$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned} M_{2k} &= M^e[:7u] + x^u M^e[:6u] + x^{3u} M^e[:4u] + x^{4u} M^e[:3u] + x^{5u} M^e[:2u] \\ &\quad + x^{6u} M^e[:u] + x^{7u} R^e, \\ W_{2k} &= W^e[:7u] + x^u W^e[:6u] + x^{3u} W^e[:4u] + x^{4u} W^e[:3u] + x^{5u} W^e[:2u] \\ &\quad + x^{6u} W^e[:u] + x^{7u} R^e. \end{aligned}$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$\begin{aligned} M_{2k+1} &= M^o[:7u] + x^u M^o[:6u] + x^{3u} M^o[:4u] + x^{4u} M^o[:3u] + x^{5u} M^o[:2u] \\ &\quad + x^{6u} M^o[:u] + x^{7u} R^o, \end{aligned}$$

$$W_{2k+1} = W^o[:7u] + x^u W^o[:6u] + x^{3u} W^o[:4u] + x^{4u} W^o[:3u] + x^{5u} W^o[:2u] + x^{6u} W^o[:u] + x^{7u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k} M_{2k} = \big(&W^e[:7v] + x^v W^e[:6v] + x^{3v} W^e[:4v] + x^{4v} W^e[:3v] \\ &+ x^{5v} W^e[:2v] + x^{6v} W^e[:v] + x^{7u} R^e\big) \times \big(M^e[:7v] + x^v M^e[:6v] \\ &+ x^{3v} M^e[:4v] + x^{4v} M^e[:3v] + x^{5v} M^e[:2v] + x^{6v} M^e[:v] + x^{7u} R^e \\ &\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v} R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} &W^e[:14v] M^e[:14v] + x^{2v} W^e[:12v] M^e[:12v] + x^{6v} W^e[:8v] M^e[:8v] \\ &\quad + x^{8v} W^e[:6v] M^e[:6v] + x^{10v} W^e[:4v] M^e[:4v] + x^{12v} W^e[:2v] M^e[:2v]. \end{aligned}$$

For all $n \leq 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \end{aligned}$$

$$\lim_{n\to\infty} W_{2n+1,j,k} = W^o_{j,k}.$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{24} + x^{22} + x^{21} + x^{20} + x^{17} + x^{14} + x^{12} + x^9 + x^8 + x^7,\\
q_1(x) &= x^{21} + x^{20} + x^{19} + x^{16} + x^{15} + x^{13} + x^9 + x^8 + x^6 + x^3,\\
q_2(x) &= x^{24} + x^{20} + x^{18} + x^{16} + x^6 + 1,\\
q_3(x) &= x^{21} + x^{20} + x^{19} + x^{16} + x^{15} + x^{13} + x^9 + x^8 + x^6 + x^3,\\
q_4(x) &= x^{24} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{14} + x^{12} + x^9 + x^8 + x^7\\
&\quad + x^6.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{126} + x^{118} + x^{104} + x^{100} + x^{96} + x^{88} + x^{84} + x^{82} + x^{80} + x^{72} + x^{68}\\
&\quad + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{42} + x^{40} + x^{36},\\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{112} + x^{108} + x^{106} + x^{102} + x^{98} + x^{96} + x^{90}\\
&\quad + x^{88} + x^{86} + x^{82} + x^{80} + x^{72} + x^{64} + x^{62} + x^{60} + x^{58} + x^{48} + x^{42}\\
&\quad + x^{38} + x^{32} + x^{30} + x^{24} + x^{20},\\
p_6(x) &= x^{122} + x^{120} + x^{118} + x^{116} + x^{108} + x^{104} + x^{100} + x^{98} + x^{96} + x^{90}\\
&\quad + x^{84} + x^{82} + x^{80} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{62} + x^{60} + x^{56}
\end{aligned}$$

$$
\begin{aligned}
&+x^{46}+x^{44}+x^{38}+x^{30}+x^{22}+x^{16}+x^{14}+x^{12}+x^{10}+x^{2}+1,\\
p_9(x)&=x^{120}+x^{118}+x^{114}+x^{102}+x^{100}+x^{98}+x^{96}+x^{94}+x^{92}+x^{86}+x^{84}\\
&+x^{82}+x^{80}+x^{78}+x^{72}+x^{68}+x^{60}+x^{58}+x^{48}+x^{36}+x^{34}+x^{32}\\
&+x^{30}+x^{28}+x^{26}+x^{24}+x^{14}+x^{12}+x^{8}+x^{6},\\
p_{12}(x)&=x^{126}+x^{122}+x^{120}+x^{118}+x^{114}+x^{112}+x^{94}+x^{90}+x^{88}+x^{86}\\
&+x^{82}+x^{80}+x^{30}+x^{26}+x^{24}+x^{22}+x^{18}+x^{16}+x^{14}+x^{10}+x^{8}\\
&+x^{6}+x^{2}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$
\begin{aligned}
p_0(x)&=x^{129}+x^{126}+x^{124}+x^{122}+x^{121}+x^{118}+x^{115}+x^{114}+x^{111}+x^{107}\\
&+x^{105}+x^{104}+x^{101}+x^{97}+x^{94}+x^{90}+x^{88}+x^{87}+x^{86}+x^{83}+x^{81}\\
&+x^{80}+x^{79}+x^{78}+x^{76}+x^{75}+x^{73}+x^{72}+x^{71}+x^{70}+x^{67}+x^{66}\\
&+x^{65}+x^{63}+x^{61}+x^{60}+x^{58}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{50}\\
&+x^{49}+x^{47}+x^{46}+x^{45}+x^{40}+x^{39},\\
p_3(x)&=x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{94}+x^{93}+x^{90}+x^{89}+x^{82}\\
&+x^{81}+x^{78}+x^{77}+x^{74}+x^{73}+x^{70}+x^{69}+x^{68}+x^{67}+x^{54}+x^{53}\\
&+x^{52}+x^{51}+x^{50}+x^{49}+x^{44}+x^{43}+x^{36}+x^{35}+x^{34}+x^{33}+x^{30}\\
&+x^{29}+x^{28}+x^{27},\\
p_6(x)&=x^{111}+x^{109}+x^{108}+x^{107}+x^{104}+x^{102}+x^{101}+x^{100}+x^{99}+x^{96}\\
&+x^{94}+x^{93}+x^{92}+x^{91}+x^{88}+x^{87}+x^{86}+x^{83}+x^{81}+x^{80}+x^{79}\\
&+x^{76}+x^{75}+x^{74}+x^{71}+x^{69}+x^{68}+x^{65}+x^{63}+x^{62}+x^{61}+x^{58}\\
&+x^{56}+x^{55}+x^{54}+x^{47}+x^{45}+x^{44}+x^{43}+x^{40}+x^{38}+x^{37}+x^{36}\\
&+x^{33}+x^{31}+x^{30}+x^{29}+x^{27}+x^{26}+x^{25}+x^{22}+x^{20}+x^{19}+x^{18},\\
p_9(x)&=x^{120}+x^{119}+x^{116}+x^{115}+x^{106}+x^{105}+x^{104}+x^{103}+x^{100}+x^{99}\\
&+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{88}+x^{87}+x^{86}+x^{85}+x^{82}\\
&+x^{81}+x^{80}+x^{79}+x^{76}+x^{75}+x^{70}+x^{69}+x^{64}+x^{63}+x^{62}+x^{61}\\
&+x^{60}+x^{59}+x^{52}+x^{51}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}\\
&+x^{41}+x^{36}+x^{35}+x^{32}+x^{31}+x^{22}+x^{21}+x^{20}+x^{19}+x^{12}+x^{11}\\
&+x^{10}+x^{9},\\
p_{12}(x)&=x^{129}+x^{126}+x^{125}+x^{124}+x^{121}+x^{118}+x^{116}+x^{114}+x^{113}+x^{112}\\
&+x^{97}+x^{94}+x^{93}+x^{92}+x^{89}+x^{86}+x^{84}+x^{82}+x^{81}+x^{80}+x^{33}\\
&+x^{30}+x^{29}+x^{28}+x^{25}+x^{22}+x^{20}+x^{18}+x^{16}+x^{14}+x^{13}+x^{12}\\
&+x^{9}+x^{6}+x^{4}+x^{2}+x+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{129} + x^{126} + x^{124} + x^{122} + x^{121} + x^{118} + x^{115} + x^{114} + x^{111} + x^{107} \\
& + x^{105} + x^{104} + x^{101} + x^{97} + x^{94} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} \\
& + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{67} + x^{66} \\
& + x^{65} + x^{63} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} \\
& + x^{49} + x^{47} + x^{46} + x^{45} + x^{40} + x^{39}, \\
p_3(x) = {} & x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{89} + x^{82} \\
& + x^{81} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{54} + x^{53} \\
& + x^{52} + x^{51} + x^{50} + x^{49} + x^{44} + x^{43} + x^{36} + x^{35} + x^{34} + x^{33} + x^{30} \\
& + x^{29} + x^{28} + x^{27}, \\
p_6(x) = {} & x^{111} + x^{109} + x^{108} + x^{107} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} \\
& + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} + x^{79} \\
& + x^{76} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{61} + x^{58} \\
& + x^{56} + x^{55} + x^{54} + x^{47} + x^{45} + x^{44} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36} \\
& + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18}, \\
p_9(x) = {} & x^{120} + x^{119} + x^{116} + x^{115} + x^{106} + x^{105} + x^{104} + x^{103} + x^{100} + x^{99} \\
& + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} \\
& + x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{70} + x^{69} + x^{64} + x^{63} + x^{62} + x^{61} \\
& + x^{60} + x^{59} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} \\
& + x^{41} + x^{36} + x^{35} + x^{32} + x^{31} + x^{22} + x^{21} + x^{20} + x^{19} + x^{12} + x^{11} \\
& + x^{10} + x^{9}, \\
p_{12}(x) = {} & x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112} \\
& + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{33} \\
& + x^{30} + x^{29} + x^{28} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} \\
& + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{132} + x^{126} + x^{124} + x^{122} + x^{114} + x^{102} + x^{98} + x^{96} + x^{94} + x^{88} \\
& + x^{86} + x^{82} + x^{80} + x^{76} + x^{74} + x^{72} + x^{62} + x^{60} + x^{54} + x^{52} + x^{46} \\
& + x^{44} + x^{42}, \\
p_3(x) = {} & x^{126} + x^{124} + x^{122} + x^{120} + x^{110} + x^{106} + x^{104} + x^{102} + x^{98} + x^{96} \\
& + x^{88} + x^{80} + x^{78} + x^{76} + x^{68} + x^{64} + x^{58} + x^{54} + x^{52} + x^{50} + x^{48} \\
& + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{20} + x^{18}, \\
p_6(x) = {} & x^{128} + x^{126} + x^{124} + x^{116} + x^{114} + x^{112} + x^{108} + x^{104} + x^{94} + x^{90} \\
& + x^{84} + x^{82} + x^{78} + x^{76} + x^{74} + x^{72} + x^{70} + x^{58} + x^{56} + x^{54} + x^{52}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{46} + x^{34} + x^{18} + x^{14} + x^{12} + x^{10} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{126} + x^{124} + x^{118} + x^{116} + x^{114} + x^{106} + x^{98} + x^{96} + x^{86} + x^{78}\\
&+ x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{64} + x^{60} + x^{56} + x^{54} + x^{48} + x^{46}\\
&+ x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{20} + x^{18} + x^{16} + x^{10} + x^{6},\\
p_{12}(x) &= x^{132} + x^{128} + x^{124} + x^{120} + x^{108} + x^{104} + x^{84} + x^{80} + x^{36} + x^{32}\\
&+ x^{28} + x^{24} + x^{20} + x^{16} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{132} + x^{126} + x^{124} + x^{122} + x^{120} + x^{114} + x^{108} + x^{100} + x^{96} + x^{94}\\
&+ x^{92} + x^{88} + x^{86} + x^{82} + x^{80} + x^{72} + x^{70} + x^{66} + x^{64} + x^{60} + x^{58}\\
&+ x^{56} + x^{52} + x^{42} + x^{40} + x^{38} + x^{36},\\
p_3(x) &= x^{126} + x^{124} + x^{122} + x^{120} + x^{114} + x^{112} + x^{106} + x^{96} + x^{90} + x^{86}\\
&+ x^{82} + x^{80} + x^{78} + x^{76} + x^{68} + x^{64} + x^{58} + x^{54} + x^{52} + x^{50} + x^{48}\\
&+ x^{46} + x^{44} + x^{40} + x^{36} + x^{34} + x^{28} + x^{24} + x^{22} + x^{20},\\
p_6(x) &= x^{128} + x^{126} + x^{124} + x^{120} + x^{116} + x^{112} + x^{110} + x^{104} + x^{102} + x^{98}\\
&+ x^{94} + x^{92} + x^{88} + x^{86} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{58} + x^{56}\\
&+ x^{54} + x^{52} + x^{48} + x^{46} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{22}\\
&+ x^{16} + x^{14} + x^{10} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{126} + x^{124} + x^{118} + x^{116} + x^{114} + x^{106} + x^{98} + x^{96} + x^{86} + x^{78}\\
&+ x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{64} + x^{60} + x^{56} + x^{54} + x^{48} + x^{46}\\
&+ x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{20} + x^{18} + x^{16} + x^{10} + x^{6},\\
p_{12}(x) &= x^{132} + x^{128} + x^{124} + x^{120} + x^{108} + x^{104} + x^{84} + x^{80} + x^{36} + x^{32}\\
&+ x^{28} + x^{24} + x^{20} + x^{16} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{122} + x^{121} + x^{118} + x^{115} + x^{114} + x^{109} + x^{108}\\
&+ x^{105} + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{94} + x^{93} + x^{87} + x^{85}\\
&+ x^{82} + x^{81} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{65} + x^{64}\\
&+ x^{63} + x^{62} + x^{60} + x^{56} + x^{53} + x^{49} + x^{47} + x^{44} + x^{40} + x^{39},\\
p_3(x) &= x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{89} + x^{82}\\
&+ x^{81} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{54} + x^{53}\\
&+ x^{52} + x^{51} + x^{50} + x^{49} + x^{44} + x^{43} + x^{36} + x^{35} + x^{34} + x^{33} + x^{30}\\
&+ x^{29} + x^{28} + x^{27},\\
p_6(x) &= x^{111} + x^{109} + x^{108} + x^{107} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96}\\
&+ x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} + x^{79}
\end{aligned}
$$

$$\begin{aligned}
&+ x^{76} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{61} + x^{58}\\
&+ x^{56} + x^{55} + x^{54} + x^{47} + x^{45} + x^{44} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36}\\
&+ x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18},\\
p_9(x) &= x^{120} + x^{119} + x^{116} + x^{115} + x^{106} + x^{105} + x^{104} + x^{103} + x^{100} + x^{99}\\
&+ x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82}\\
&+ x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{70} + x^{69} + x^{64} + x^{63} + x^{62} + x^{61}\\
&+ x^{60} + x^{59} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42}\\
&+ x^{41} + x^{36} + x^{35} + x^{32} + x^{31} + x^{22} + x^{21} + x^{20} + x^{19} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112}\\
&+ x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{33}\\
&+ x^{30} + x^{29} + x^{28} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12}\\
&+ x^{9} + x^{6} + x^{4} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{122} + x^{121} + x^{118} + x^{115} + x^{114} + x^{109} + x^{108}\\
&+ x^{105} + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{94} + x^{93} + x^{87} + x^{85}\\
&+ x^{82} + x^{81} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{65} + x^{64}\\
&+ x^{63} + x^{62} + x^{60} + x^{56} + x^{53} + x^{49} + x^{47} + x^{44} + x^{40} + x^{39},\\
p_3(x) &= x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{89} + x^{82}\\
&+ x^{81} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{54} + x^{53}\\
&+ x^{52} + x^{51} + x^{50} + x^{49} + x^{44} + x^{43} + x^{36} + x^{35} + x^{34} + x^{33} + x^{30}\\
&+ x^{29} + x^{28} + x^{27},\\
p_6(x) &= x^{111} + x^{109} + x^{108} + x^{107} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96}\\
&+ x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} + x^{79}\\
&+ x^{76} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{61} + x^{58}\\
&+ x^{56} + x^{55} + x^{54} + x^{47} + x^{45} + x^{44} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36}\\
&+ x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18},\\
p_9(x) &= x^{120} + x^{119} + x^{116} + x^{115} + x^{106} + x^{105} + x^{104} + x^{103} + x^{100} + x^{99}\\
&+ x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82}\\
&+ x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{70} + x^{69} + x^{64} + x^{63} + x^{62} + x^{61}\\
&+ x^{60} + x^{59} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42}\\
&+ x^{41} + x^{36} + x^{35} + x^{32} + x^{31} + x^{22} + x^{21} + x^{20} + x^{19} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{116} + x^{114} + x^{113} + x^{112}
\end{aligned}$$

$$+ x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{33} + x^{30} + x^{29} + x^{28} + x^{25} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{126} + x^{118} + x^{114} + x^{108} + x^{102} + x^{94} + x^{92} + x^{86} + x^{84} + x^{82} + x^{76} \\
&\quad + x^{72} + x^{70} + x^{66} + x^{62} + x^{60} + x^{58} + x^{52} + x^{50} + x^{46} + x^{42}, \\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{112} + x^{104} + x^{100} + x^{96} + x^{92} + x^{88} + x^{86} \\
&\quad + x^{82} + x^{80} + x^{76} + x^{74} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} \\
&\quad + x^{56} + x^{54} + x^{46} + x^{44} + x^{40} + x^{36} + x^{34} + x^{24} + x^{18}, \\
p_6(x) &= x^{122} + x^{120} + x^{118} + x^{116} + x^{114} + x^{108} + x^{104} + x^{98} + x^{92} + x^{90} \\
&\quad + x^{86} + x^{80} + x^{74} + x^{72} + x^{70} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{50} \\
&\quad + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{16} + x^{10} \\
&\quad + x^{2} + 1, \\
p_9(x) &= x^{120} + x^{118} + x^{114} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{86} + x^{84} \\
&\quad + x^{82} + x^{80} + x^{78} + x^{72} + x^{68} + x^{60} + x^{58} + x^{48} + x^{36} + x^{34} + x^{32} \\
&\quad + x^{30} + x^{28} + x^{26} + x^{24} + x^{14} + x^{12} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{126} + x^{122} + x^{120} + x^{118} + x^{114} + x^{112} + x^{94} + x^{90} + x^{88} + x^{86} \\
&\quad + x^{82} + x^{80} + x^{30} + x^{26} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^{10} + x^{8} \\
&\quad + x^{6} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{115} \\
&\quad + x^{111} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{97} + x^{96} \\
&\quad + x^{95} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{85} + x^{83} + x^{79} + x^{78} \\
&\quad + x^{77} + x^{76} + x^{75} + x^{74} + x^{71} + x^{68} + x^{67} + x^{64} + x^{62} + x^{61} + x^{60} \\
&\quad + x^{59} + x^{57} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{42} + x^{39} + x^{37} + x^{36}, \\
p_3(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{102} + x^{101} + x^{94} \\
&\quad + x^{92} + x^{89} + x^{87} + x^{86} + x^{85} + x^{83} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} \\
&\quad + x^{73} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{63} + x^{60} + x^{58} + x^{57} \\
&\quad + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{49} + x^{48} + x^{47} + x^{45} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{35} + x^{34} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{24}, \\
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} + x^{112} + x^{110} + x^{109} \\
&\quad + x^{106} + x^{105} + x^{104} + x^{98} + x^{96} + x^{90} + x^{89} + x^{87} + x^{84} + x^{83} + x^{77} \\
&\quad + x^{76} + x^{75} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{65} + x^{64} + x^{63} + x^{62}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{61} + x^{60} + x^{53} + x^{52} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42}\\
&+ x^{37} + x^{36} + x^{33} + x^{32} + x^{29} + x^{26} + x^{25} + x^{23} + x^{20} + x^{19} + x^{17}\\
&+ x^{15} + x^{14} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{112} + x^{110} + x^{109} + x^{108} + x^{101} + x^{98} + x^{96} + x^{94}\\
&+ x^{93} + x^{92} + x^{21} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12},\\
p_{12}(x) &= x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{113} + x^{110}\\
&+ x^{109} + x^{108} + x^{101} + x^{98} + x^{97} + x^{96} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81}\\
&+ x^{80} + x^{33} + x^{30} + x^{29} + x^{28} + x^{25} + x^{22} + x^{21} + x^{20} + x^{9} + x^{6} + x^{4}\\
&+ x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{132} + x^{124} + x^{123} + x^{121} + x^{120} + x^{117} + x^{110} + x^{108} + x^{107} + x^{106}\\
&+ x^{105} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{90}\\
&+ x^{88} + x^{86} + x^{84} + x^{81} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{67}\\
&+ x^{66} + x^{65} + x^{64} + x^{62} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{48} + x^{47}\\
&+ x^{42} + x^{41} + x^{39},\\
p_3(x) &= x^{105} + x^{104} + x^{103} + x^{100} + x^{98} + x^{97} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89}\\
&+ x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{77} + x^{75} + x^{74} + x^{72} + x^{71}\\
&+ x^{70} + x^{67} + x^{57} + x^{56} + x^{55} + x^{52} + x^{50} + x^{47} + x^{45} + x^{44} + x^{43}\\
&+ x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27},\\
p_6(x) &= x^{114} + x^{112} + x^{110} + x^{104} + x^{100} + x^{98} + x^{96} + x^{90} + x^{78} + x^{76} + x^{74}\\
&+ x^{70} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34}\\
&+ x^{32} + x^{24} + x^{22} + x^{18},\\
p_9(x) &= x^{123} + x^{122} + x^{120} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109}\\
&+ x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{101} + x^{99} + x^{98} + x^{96} + x^{95}\\
&+ x^{94} + x^{93} + x^{92} + x^{89} + x^{27} + x^{26} + x^{24} + x^{21} + x^{19} + x^{18} + x^{16}\\
&+ x^{15} + x^{14} + x^{13} + x^{12} + x^{9},\\
p_{12}(x) &= x^{132} + x^{128} + x^{124} + x^{112} + x^{96} + x^{92} + x^{84} + x^{80} + x^{36} + x^{32} + x^{28}\\
&+ x^{20} + x^{12} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{132} + x^{128} + x^{124} + x^{123} + x^{121} + x^{120} + x^{119} + x^{114} + x^{113} + x^{112}\\
&+ x^{111} + x^{110} + x^{109} + x^{108} + x^{105} + x^{104} + x^{99} + x^{97} + x^{95} + x^{94}\\
&+ x^{91} + x^{89} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{77}\\
&+ x^{76} + x^{67} + x^{66} + x^{65} + x^{64} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{44}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{42} + x^{41} + x^{39},\\
p_3(x) &= x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{89}\\
&+ x^{86} + x^{85} + x^{82} + x^{81} + x^{78} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70} + x^{67}\\
&+ x^{57} + x^{56} + x^{55} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{46} + x^{43} + x^{41}\\
&+ x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27},\\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94}\\
&+ x^{92} + x^{84} + x^{80} + x^{72} + x^{70} + x^{66} + x^{62} + x^{60} + x^{58} + x^{54} + x^{50}\\
&+ x^{48} + x^{44} + x^{42} + x^{40} + x^{34} + x^{26} + x^{24} + x^{22} + x^{18},\\
p_9(x) &= x^{123} + x^{122} + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113}\\
&+ x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104}\\
&+ x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94}\\
&+ x^{93} + x^{92} + x^{89} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19}\\
&+ x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{9},\\
p_{12}(x) &= x^{132} + x^{108} + x^{104} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{36} + x^{20} + x^{12}\\
&+ x^{8} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{115} + x^{114}\\
&+ x^{112} + x^{110} + x^{109} + x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96}\\
&+ x^{95} + x^{94} + x^{93} + x^{92} + x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{79}\\
&+ x^{78} + x^{75} + x^{74} + x^{72} + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{61} + x^{60}\\
&+ x^{55} + x^{54} + x^{48} + x^{47} + x^{45} + x^{43} + x^{42},\\
p_3(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{105} + x^{100} + x^{99}\\
&+ x^{97} + x^{96} + x^{94} + x^{92} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73}\\
&+ x^{72} + x^{70} + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{51}\\
&+ x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36}\\
&+ x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{27} + x^{26},\\
p_6(x) &= x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{114} + x^{111} + x^{109} + x^{107}\\
&+ x^{106} + x^{101} + x^{99} + x^{96} + x^{91} + x^{90} + x^{88} + x^{85} + x^{80} + x^{79} + x^{78}\\
&+ x^{77} + x^{76} + x^{73} + x^{71} + x^{68} + x^{66} + x^{65} + x^{60} + x^{58} + x^{57} + x^{56}\\
&+ x^{55} + x^{51} + x^{49} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{33} + x^{32} + x^{31}\\
&+ x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{17} + x^{16} + x^{15}\\
&+ x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,\\
p_9(x) &= x^{117} + x^{114} + x^{112} + x^{110} + x^{109} + x^{108} + x^{101} + x^{98} + x^{96} + x^{94}\\
&+ x^{93} + x^{92} + x^{21} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12},
\end{aligned}
$$

$$\begin{aligned}p_{12}(x) &= x^{129}+x^{126}+x^{125}+x^{124}+x^{121}+x^{118}+x^{117}+x^{116}+x^{113}+x^{110}\\ &\quad+x^{109}+x^{108}+x^{101}+x^{98}+x^{97}+x^{96}+x^{89}+x^{86}+x^{84}+x^{82}+x^{81}\\ &\quad+x^{80}+x^{33}+x^{30}+x^{29}+x^{28}+x^{25}+x^{22}+x^{21}+x^{20}+x^{9}+x^{6}+x^{4}\\ &\quad+x^{2}+x+1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 0].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}p_{0}(x) &= x^{129}+x^{126}+x^{124}+x^{123}+x^{122}+x^{121}+x^{119}+x^{118}+x^{117}+x^{115}\\ &\quad+x^{111}+x^{107}+x^{106}+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{97}+x^{96}\\ &\quad+x^{95}+x^{94}+x^{93}+x^{92}+x^{90}+x^{89}+x^{88}+x^{85}+x^{83}+x^{79}+x^{78}\\ &\quad+x^{77}+x^{76}+x^{75}+x^{74}+x^{71}+x^{68}+x^{67}+x^{64}+x^{62}+x^{61}+x^{60}\\ &\quad+x^{59}+x^{57}+x^{53}+x^{50}+x^{49}+x^{48}+x^{47}+x^{42}+x^{39}+x^{37}+x^{36},\\ p_{3}(x) &= x^{117}+x^{114}+x^{113}+x^{112}+x^{111}+x^{109}+x^{108}+x^{102}+x^{101}+x^{94}\\ &\quad+x^{92}+x^{89}+x^{87}+x^{86}+x^{85}+x^{83}+x^{80}+x^{79}+x^{77}+x^{76}+x^{74}\\ &\quad+x^{73}+x^{72}+x^{71}+x^{70}+x^{68}+x^{67}+x^{66}+x^{63}+x^{60}+x^{58}+x^{57}\\ &\quad+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{50}+x^{49}+x^{48}+x^{47}+x^{45}+x^{42}\\ &\quad+x^{40}+x^{38}+x^{35}+x^{34}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{25}+x^{24},\\ p_{6}(x) &= x^{125}+x^{123}+x^{122}+x^{121}+x^{119}+x^{118}+x^{117}+x^{112}+x^{110}+x^{109}\\ &\quad+x^{106}+x^{105}+x^{104}+x^{98}+x^{96}+x^{90}+x^{89}+x^{87}+x^{84}+x^{83}+x^{77}\\ &\quad+x^{76}+x^{75}+x^{73}+x^{72}+x^{70}+x^{69}+x^{68}+x^{65}+x^{64}+x^{63}+x^{62}\\ &\quad+x^{61}+x^{60}+x^{53}+x^{52}+x^{51}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}\\ &\quad+x^{37}+x^{36}+x^{33}+x^{32}+x^{29}+x^{26}+x^{25}+x^{23}+x^{20}+x^{19}+x^{17}\\ &\quad+x^{15}+x^{14}+x^{9}+x^{6}+x^{4}+x^{2}+x+1,\\ p_{9}(x) &= x^{117}+x^{114}+x^{112}+x^{110}+x^{109}+x^{108}+x^{101}+x^{98}+x^{96}+x^{94}\\ &\quad+x^{93}+x^{92}+x^{21}+x^{18}+x^{16}+x^{14}+x^{13}+x^{12},\\ p_{12}(x) &= x^{129}+x^{126}+x^{125}+x^{124}+x^{121}+x^{118}+x^{117}+x^{116}+x^{113}+x^{110}\\ &\quad+x^{109}+x^{108}+x^{101}+x^{98}+x^{97}+x^{96}+x^{89}+x^{86}+x^{84}+x^{82}+x^{81}\\ &\quad+x^{80}+x^{33}+x^{30}+x^{29}+x^{28}+x^{25}+x^{22}+x^{21}+x^{20}+x^{9}+x^{6}+x^{4}\\ &\quad+x^{2}+x+1,\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}p_{0}(x) &= x^{132}+x^{128}+x^{124}+x^{123}+x^{121}+x^{120}+x^{119}+x^{114}+x^{113}+x^{112}\\ &\quad+x^{111}+x^{110}+x^{109}+x^{108}+x^{105}+x^{104}+x^{99}+x^{97}+x^{95}+x^{94}\\ &\quad+x^{91}+x^{89}+x^{87}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{79}+x^{77}\\ &\quad+x^{76}+x^{67}+x^{66}+x^{65}+x^{64}+x^{56}+x^{53}+x^{52}+x^{51}+x^{48}+x^{44}\\ &\quad+x^{42}+x^{41}+x^{39},\end{aligned}$$

$$\begin{aligned}
p_3(x) &= x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{89} \\
&\quad + x^{86} + x^{85} + x^{82} + x^{81} + x^{78} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70} + x^{67} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{46} + x^{43} + x^{41} \\
&\quad + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} \\
&\quad + x^{92} + x^{84} + x^{80} + x^{72} + x^{70} + x^{66} + x^{62} + x^{60} + x^{58} + x^{54} + x^{50} \\
&\quad + x^{48} + x^{44} + x^{42} + x^{40} + x^{34} + x^{26} + x^{24} + x^{22} + x^{18}, \\
p_9(x) &= x^{123} + x^{122} + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113} \\
&\quad + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} \\
&\quad + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{92} + x^{89} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} \\
&\quad + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{132} + x^{108} + x^{104} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{36} + x^{20} + x^{12} \\
&\quad + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{132} + x^{124} + x^{123} + x^{121} + x^{120} + x^{117} + x^{110} + x^{108} + x^{107} + x^{106} \\
&\quad + x^{105} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{90} \\
&\quad + x^{88} + x^{86} + x^{84} + x^{81} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{67} \\
&\quad + x^{66} + x^{65} + x^{64} + x^{62} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{48} + x^{47} \\
&\quad + x^{42} + x^{41} + x^{39}, \\
p_3(x) &= x^{105} + x^{104} + x^{103} + x^{100} + x^{98} + x^{97} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} \\
&\quad + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{77} + x^{75} + x^{74} + x^{72} + x^{71} \\
&\quad + x^{70} + x^{67} + x^{57} + x^{56} + x^{55} + x^{52} + x^{50} + x^{47} + x^{45} + x^{44} + x^{43} \\
&\quad + x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{114} + x^{112} + x^{110} + x^{104} + x^{100} + x^{98} + x^{96} + x^{90} + x^{78} + x^{76} + x^{74} \\
&\quad + x^{70} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} \\
&\quad + x^{32} + x^{24} + x^{22} + x^{18}, \\
p_9(x) &= x^{123} + x^{122} + x^{120} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} \\
&\quad + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{101} + x^{99} + x^{98} + x^{96} + x^{95} \\
&\quad + x^{94} + x^{93} + x^{92} + x^{89} + x^{27} + x^{26} + x^{24} + x^{21} + x^{19} + x^{18} + x^{16} \\
&\quad + x^{15} + x^{14} + x^{13} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{132} + x^{128} + x^{124} + x^{112} + x^{96} + x^{92} + x^{84} + x^{80} + x^{36} + x^{32} + x^{28} \\
&\quad + x^{20} + x^{12} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{129} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{115} + x^{114} \\
& + x^{112} + x^{110} + x^{109} + x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} \\
& + x^{95} + x^{94} + x^{93} + x^{92} + x^{89} + x^{88} + x^{87} + x^{85} + x^{83} + x^{82} + x^{79} \\
& + x^{78} + x^{75} + x^{74} + x^{72} + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} \\
& + x^{55} + x^{54} + x^{48} + x^{47} + x^{45} + x^{43} + x^{42}, \\
p_3(x) = {} & x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{105} + x^{100} + x^{99} \\
& + x^{97} + x^{96} + x^{94} + x^{92} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} \\
& + x^{72} + x^{70} + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{51} \\
& + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} \\
& + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{27} + x^{26}, \\
p_6(x) = {} & x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{114} + x^{111} + x^{109} + x^{107} \\
& + x^{106} + x^{101} + x^{99} + x^{96} + x^{91} + x^{90} + x^{88} + x^{85} + x^{80} + x^{79} + x^{78} \\
& + x^{77} + x^{76} + x^{73} + x^{71} + x^{68} + x^{66} + x^{65} + x^{60} + x^{58} + x^{57} + x^{56} \\
& + x^{55} + x^{51} + x^{49} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{33} + x^{32} + x^{31} \\
& + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{17} + x^{16} + x^{15} \\
& + x^{13} + x^{12} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1, \\
p_9(x) = {} & x^{117} + x^{114} + x^{112} + x^{110} + x^{109} + x^{108} + x^{101} + x^{98} + x^{96} + x^{94} \\
& + x^{93} + x^{92} + x^{21} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12}, \\
p_{12}(x) = {} & x^{129} + x^{126} + x^{125} + x^{124} + x^{121} + x^{118} + x^{117} + x^{116} + x^{113} + x^{110} \\
& + x^{109} + x^{108} + x^{101} + x^{98} + x^{97} + x^{96} + x^{89} + x^{86} + x^{84} + x^{82} + x^{81} \\
& + x^{80} + x^{33} + x^{30} + x^{29} + x^{28} + x^{25} + x^{22} + x^{21} + x^{20} + x^{9} + x^{6} + x^{4} \\
& + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	423	[666, 667]	846	[1148, 1149]	1269	[1163, 1468]
1	[3, 4]	424	[668, 669]	847	[1096, 551]	1270	[1469, 516]
2	[5, 6]	425	[670, 671]	848	[479, 524]	1271	[1077, 1077]
3	[7, 8]	426	[672, 673]	849	[1150, 478]	1272	[429, 151]
4	[9, 10]	427	[674, 675]	850	[1053, 1017]	1273	[133, 1133]
5	[11, 12]	428	[676, 229]	851	[1151, 1152]	1274	[1470, 1201]
6	[13, 14]	429	[650, 677]	852	[989, 1153]	1275	[1144, 577]
7	[15, 16]	430	[678, 679]	853	[1154, 1029]	1276	[1471, 625]
8	[17, 18]	431	[680, 681]	854	[1155, 1074]	1277	[219, 1047]
9	[19, 20]	432	[682, 683]	855	[171, 540]	1278	[1117, 1136]

10	[21, 22]	433	[684, 685]	856	[1156, 423]	1279	[573, 517]
11	[23, 24]	434	[686, 687]	857	[209, 1157]	1280	[631, 1084]
12	[25, 26]	435	[688, 689]	858	[1158, 465]	1281	[1472, 1473]
13	[27, 28]	436	[690, 648]	859	[607, 1064]	1282	[484, 41]
14	[29, 30]	437	[691, 692]	860	[1140, 1159]	1283	[187, 996]
15	[31, 32]	438	[693, 694]	861	[1160, 134]	1284	[1100, 1474]
16	[33, 34]	439	[695, 696]	862	[632, 434]	1285	[1475, 1457]
17	[35, 36]	440	[697, 698]	863	[1161, 50]	1286	[1462, 1204]
18	[37, 38]	441	[699, 700]	864	[1083, 176]	1287	[443, 1476]
19	[39, 8]	442	[701, 702]	865	[178, 985]	1288	[466, 409]
20	[40, 41]	443	[703, 704]	866	[1140, 1162]	1289	[1477, 1478]
21	[42, 43]	444	[705, 706]	867	[1163, 975]	1290	[181, 1479]
22	[44, 45]	445	[656, 707]	868	[57, 1067]	1291	[1048, 33]
23	[46, 47]	446	[708, 709]	869	[57, 545]	1292	[1480, 1481]
24	[48, 49]	447	[710, 85]	870	[41, 202]	1293	[465, 1482]
25	[50, 51]	448	[711, 651]	871	[1164, 1024]	1294	[1459, 438]
26	[52, 53]	449	[712, 713]	872	[1006, 1000]	1295	[1483, 213]
27	[54, 55]	450	[71, 714]	873	[33, 510]	1296	[1171, 1484]
28	[56, 57]	451	[645, 715]	874	[638, 413]	1297	[1037, 1090]
29	[58, 59]	452	[716, 717]	875	[977, 216]	1298	[416, 47]
30	[60, 61]	453	[375, 334]	876	[1139, 597]	1299	[1485, 470]
31	[62, 63]	454	[718, 719]	877	[1070, 1043]	1300	[1079, 127]
32	[64, 65]	455	[720, 721]	878	[165, 1126]	1301	[209, 1486]
33	[66, 67]	456	[722, 723]	879	[999, 1082]	1302	[1487, 152]
34	[68, 69]	457	[724, 725]	880	[998, 573]	1303	[1480, 1200]
35	[70, 71]	458	[726, 727]	881	[518, 1001]	1304	[59, 1030]
36	[72, 73]	459	[728, 729]	882	[1078, 1077]	1305	[1488, 1034]
37	[74, 75]	460	[730, 731]	883	[150, 214]	1306	[625, 1489]
38	[76, 77]	461	[732, 733]	884	[588, 983]	1307	[1490, 480]
39	[78, 79]	462	[387, 734]	885	[448, 435]	1308	[549, 44]
40	[80, 81]	463	[735, 736]	886	[630, 1165]	1309	[567, 1491]
41	[82, 16]	464	[737, 738]	887	[1091, 189]	1310	[1061, 219]
42	[83, 84]	465	[289, 739]	888	[418, 1022]	1311	[213, 1492]
43	[85, 86]	466	[740, 741]	889	[1166, 454]	1312	[526, 555]
44	[87, 88]	467	[742, 743]	890	[538, 1167]	1313	[1493, 638]
45	[89, 90]	468	[660, 744]	891	[1168, 1069]	1314	[980, 1002]
46	[91, 92]	469	[745, 746]	892	[1169, 1066]	1315	[42, 1494]
47	[93, 94]	470	[694, 688]	893	[978, 1170]	1316	[495, 608]
48	[95, 96]	471	[747, 369]	894	[1171, 1131]	1317	[1174, 1048]
49	[97, 98]	472	[748, 716]	895	[1172, 616]	1318	[1056, 525]
50	[99, 100]	473	[749, 750]	896	[558, 610]	1319	[1495, 1087]
51	[101, 102]	474	[751, 752]	897	[1173, 1174]	1320	[969, 421]
52	[103, 104]	475	[299, 726]	898	[1175, 1176]	1321	[1206, 142]
53	[105, 106]	476	[108, 753]	899	[634, 1112]	1322	[1496, 586]

54	[107, 108]	477	[754, 755]	900	[163, 1177]	1323	[1497, 60]
55	[109, 110]	478	[756, 298]	901	[159, 509]	1324	[626, 422]
56	[111, 112]	479	[757, 758]	902	[1178, 1179]	1325	[583, 1204]
57	[113, 114]	480	[228, 722]	903	[1180, 1003]	1326	[468, 1192]
58	[115, 116]	481	[759, 703]	904	[639, 503]	1327	[144, 51]
59	[117, 118]	482	[760, 64]	905	[1181, 1182]	1328	[612, 1498]
60	[119, 120]	483	[761, 259]	906	[1057, 971]	1329	[1055, 1499]
61	[28, 121]	484	[762, 367]	907	[545, 1068]	1330	[625, 1500]
62	[122, 123]	485	[763, 764]	908	[1183, 1122]	1331	[1501, 1178]
63	[54, 124]	486	[310, 765]	909	[462, 1105]	1332	[492, 1484]
64	[10, 125]	487	[766, 767]	910	[613, 632]	1333	[1502, 1203]
65	[126, 127]	488	[696, 768]	911	[1184, 169]	1334	[1129, 1503]
66	[128, 129]	489	[769, 770]	912	[186, 1049]	1335	[1504, 1058]
67	[130, 131]	490	[771, 772]	913	[1185, 158]	1336	[615, 1195]
68	[132, 133]	491	[773, 774]	914	[590, 1114]	1337	[1029, 1505]
69	[134, 135]	492	[775, 776]	915	[1186, 1187]	1338	[47, 1053]
70	[136, 137]	493	[777, 778]	916	[1188, 492]	1339	[974, 1468]
71	[138, 139]	494	[75, 779]	917	[1015, 1002]	1340	[1506, 1120]
72	[32, 140]	495	[780, 728]	918	[1189, 1190]	1341	[1507, 1059]
73	[141, 142]	496	[729, 781]	919	[1110, 1191]	1342	[1031, 211]
74	[142, 143]	497	[782, 783]	920	[1069, 1192]	1343	[1095, 1001]
75	[144, 145]	498	[287, 358]	921	[1193, 134]	1344	[162, 563]
76	[146, 147]	499	[784, 292]	922	[1194, 1106]	1345	[1150, 1508]
77	[148, 149]	500	[785, 786]	923	[552, 1159]	1346	[1006, 1137]
78	[150, 151]	501	[787, 274]	924	[557, 609]	1347	[443, 199]
79	[152, 153]	502	[788, 789]	925	[1172, 1195]	1348	[435, 167]
80	[154, 155]	503	[790, 791]	926	[465, 1196]	1349	[48, 1063]
81	[40, 130]	504	[792, 793]	927	[1197, 480]	1350	[636, 1124]
82	[156, 43]	505	[794, 397]	928	[1198, 1044]	1351	[1509, 1050]
83	[157, 158]	506	[795, 796]	929	[1199, 1188]	1352	[1182, 1510]
84	[159, 160]	507	[293, 795]	930	[525, 475]	1353	[1511, 1512]
85	[161, 162]	508	[797, 798]	931	[1200, 185]	1354	[1513, 479]
86	[163, 164]	509	[104, 799]	932	[1121, 1201]	1355	[560, 178]
87	[165, 166]	510	[398, 95]	933	[978, 217]	1356	[1173, 1514]
88	[167, 168]	511	[96, 800]	934	[616, 1202]	1357	[1112, 159]
89	[50, 145]	512	[801, 279]	935	[1087, 1203]	1358	[509, 193]
90	[134, 169]	513	[772, 802]	936	[1151, 580]	1359	[1515, 431]
91	[170, 171]	514	[803, 300]	937	[1204, 623]	1360	[1477, 994]
92	[172, 173]	515	[362, 708]	938	[1035, 186]	1361	[523, 1516]
93	[174, 57]	516	[804, 805]	939	[561, 1198]	1362	[1483, 1517]
94	[175, 10]	517	[806, 807]	940	[1201, 14]	1363	[1518, 46]
95	[176, 177]	518	[808, 674]	941	[1205, 613]	1364	[577, 1054]
96	[178, 179]	519	[809, 810]	942	[1206, 559]	1365	[203, 979]
97	[180, 181]	520	[704, 811]	943	[621, 1207]	1366	[156, 1494]

98	[182, 183]	521	[812, 813]	944	[413, 168]	1367	[1519, 425]
99	[184, 150]	522	[807, 814]	945	[216, 1170]	1368	[1160, 1184]
100	[185, 186]	523	[359, 815]	946	[145, 576]	1369	[1472, 1520]
101	[187, 188]	524	[816, 740]	947	[169, 1028]	1370	[490, 1491]
102	[189, 190]	525	[817, 818]	948	[968, 458]	1371	[1065, 1467]
103	[191, 192]	526	[384, 819]	949	[558, 1208]	1372	[1521, 571]
104	[193, 194]	527	[820, 821]	950	[1209, 1182]	1373	[1522, 1513]
105	[56, 195]	528	[822, 823]	951	[481, 456]	1374	[1019, 1445]
106	[196, 197]	529	[4, 748]	952	[527, 556]	1375	[1511, 497]
107	[198, 199]	530	[16, 226]	953	[195, 545]	1376	[535, 535]
108	[200, 201]	531	[824, 825]	954	[1210, 578]	1377	[473, 1171]
109	[130, 202]	532	[826, 827]	955	[1101, 1211]	1378	[555, 1099]
110	[203, 204]	533	[828, 63]	956	[1112, 1114]	1379	[1178, 1523]
111	[205, 206]	534	[829, 830]	957	[1114, 160]	1380	[1524, 1525]
112	[145, 207]	535	[798, 831]	958	[520, 1140]	1381	[1020, 1503]
113	[208, 209]	536	[832, 76]	959	[1212, 1098]	1382	[1442, 1526]
114	[210, 211]	537	[331, 833]	960	[191, 1213]	1383	[542, 198]
115	[212, 213]	538	[834, 82]	961	[601, 526]	1384	[1193, 1184]
116	[214, 215]	539	[352, 835]	962	[1214, 592]	1385	[1186, 1527]
117	[216, 217]	540	[311, 317]	963	[176, 1057]	1386	[501, 488]
118	[218, 219]	541	[836, 346]	964	[1215, 921]	1387	[1090, 1528]
119	[220, 33]	542	[783, 837]	965	[404, 87]	1388	[1166, 977]
120	[221, 222]	543	[838, 839]	966	[1216, 1217]	1389	[553, 1529]
121	[223, 224]	544	[687, 840]	967	[906, 1218]	1390	[1530, 1454]
122	[225, 226]	545	[406, 841]	968	[392, 1219]	1391	[1095, 426]
123	[227, 228]	546	[355, 842]	969	[1220, 664]	1392	[1075, 1531]
124	[229, 230]	547	[843, 844]	970	[226, 1221]	1393	[1532, 1147]
125	[231, 232]	548	[845, 846]	971	[1222, 1223]	1394	[1488, 1533]
126	[233, 234]	549	[847, 848]	972	[258, 1224]	1395	[504, 1477]
127	[235, 74]	550	[849, 850]	973	[1225, 1226]	1396	[594, 970]
128	[236, 237]	551	[738, 851]	974	[1227, 784]	1397	[1195, 1202]
129	[238, 239]	552	[810, 852]	975	[1228, 1229]	1398	[1487, 524]
130	[240, 241]	553	[733, 853]	976	[1230, 930]	1399	[485, 1104]
131	[242, 243]	554	[854, 403]	977	[895, 1231]	1400	[1155, 991]
132	[244, 245]	555	[855, 361]	978	[345, 860]	1401	[450, 1534]
133	[246, 109]	556	[856, 857]	979	[1232, 21]	1402	[1535, 1176]
134	[247, 248]	557	[858, 859]	980	[90, 946]	1403	[193, 1113]
135	[249, 250]	558	[116, 97]	981	[1218, 1233]	1404	[1536, 1444]
136	[251, 252]	559	[860, 354]	982	[1234, 1235]	1405	[1058, 1537]
137	[253, 254]	560	[861, 862]	983	[1236, 1237]	1406	[1084, 177]
138	[255, 256]	561	[863, 864]	984	[1238, 732]	1407	[1538, 1178]
139	[257, 258]	562	[715, 269]	985	[938, 1239]	1408	[616, 1539]
140	[259, 260]	563	[865, 866]	986	[1240, 1241]	1409	[1540, 554]
141	[261, 227]	564	[867, 666]	987	[1242, 1243]	1410	[627, 1153]

142	[262, 263]	565	[94, 868]	988	[20, 1244]	1411	[965, 549]
143	[264, 265]	566	[869, 870]	989	[770, 1245]	1412	[554, 1541]
144	[266, 91]	567	[871, 872]	990	[723, 1246]	1413	[514, 581]
145	[267, 268]	568	[120, 873]	991	[280, 1247]	1414	[1181, 1524]
146	[269, 249]	569	[874, 875]	992	[1248, 1249]	1415	[633, 1542]
147	[270, 271]	570	[876, 877]	993	[1250, 699]	1416	[1543, 1200]
148	[272, 273]	571	[878, 742]	994	[925, 1251]	1417	[1156, 38]
149	[274, 246]	572	[879, 880]	995	[823, 1252]	1418	[1197, 205]
150	[275, 276]	573	[881, 808]	996	[1253, 1254]	1419	[1544, 1090]
151	[277, 278]	574	[86, 312]	997	[1255, 747]	1420	[1045, 1211]
152	[279, 280]	575	[882, 751]	998	[872, 806]	1421	[13, 1447]
153	[281, 282]	576	[100, 883]	999	[302, 1256]	1422	[1545, 1520]
154	[283, 284]	577	[884, 659]	1000	[673, 1257]	1423	[1522, 1546]
155	[285, 257]	578	[118, 885]	1001	[752, 1258]	1424	[1107, 530]
156	[286, 287]	579	[886, 887]	1002	[1259, 1260]	1425	[1092, 223]
157	[288, 289]	580	[888, 251]	1003	[1261, 1262]	1426	[60, 1144]
158	[290, 291]	581	[889, 890]	1004	[1263, 782]	1427	[987, 1466]
159	[292, 293]	582	[382, 891]	1005	[929, 1264]	1428	[537, 527]
160	[294, 295]	583	[892, 893]	1006	[827, 879]	1429	[440, 1547]
161	[282, 296]	584	[894, 895]	1007	[1265, 780]	1430	[180, 1548]
162	[73, 297]	585	[896, 711]	1008	[1266, 1267]	1431	[1458, 1457]
163	[298, 299]	586	[284, 897]	1009	[1239, 255]	1432	[606, 1549]
164	[300, 103]	587	[698, 871]	1010	[744, 1268]	1433	[208, 967]
165	[301, 302]	588	[898, 899]	1011	[774, 1269]	1434	[579, 1152]
166	[303, 304]	589	[739, 900]	1012	[831, 898]	1435	[39, 1107]
167	[305, 306]	590	[901, 294]	1013	[1270, 1271]	1436	[447, 1493]
168	[307, 308]	591	[902, 903]	1014	[818, 336]	1437	[1550, 1190]
169	[309, 310]	592	[904, 763]	1015	[1272, 1273]	1438	[1132, 602]
170	[273, 311]	593	[905, 906]	1016	[1274, 1225]	1439	[624, 1551]
171	[312, 277]	594	[907, 785]	1017	[1275, 809]	1440	[1552, 1196]
172	[313, 314]	595	[908, 909]	1018	[1276, 262]	1441	[1553, 267]
173	[315, 316]	596	[297, 266]	1019	[1277, 242]	1442	[1554, 1555]
174	[317, 318]	597	[379, 910]	1020	[1278, 1279]	1443	[1556, 1248]
175	[319, 320]	598	[911, 912]	1021	[1280, 1281]	1444	[642, 1557]
176	[321, 322]	599	[913, 914]	1022	[1282, 869]	1445	[850, 5]
177	[323, 324]	600	[370, 915]	1023	[935, 385]	1446	[947, 1360]
178	[325, 326]	601	[916, 820]	1024	[1283, 1284]	1447	[1558, 1559]
179	[327, 328]	602	[917, 918]	1025	[1237, 1265]	1448	[1560, 1222]
180	[329, 330]	603	[919, 920]	1026	[1279, 301]	1449	[1561, 393]
181	[234, 331]	604	[921, 89]	1027	[1285, 1286]	1450	[1376, 1562]
182	[332, 333]	605	[922, 275]	1028	[248, 1287]	1451	[1563, 1232]
183	[334, 335]	606	[778, 19]	1029	[377, 693]	1452	[1226, 854]
184	[336, 337]	607	[79, 378]	1030	[1288, 1289]	1453	[1564, 1361]
185	[338, 119]	608	[923, 924]	1031	[1290, 1291]	1454	[1403, 1565]

186	[339, 340]	609	[647, 925]	1032	[912, 1292]	1455	[1566, 1388]
187	[341, 342]	610	[859, 926]	1033	[324, 1293]	1456	[1567, 1568]
188	[343, 344]	611	[927, 270]	1034	[1224, 350]	1457	[1348, 1334]
189	[26, 345]	612	[928, 929]	1035	[1294, 720]	1458	[1569, 773]
190	[346, 347]	613	[930, 686]	1036	[667, 1259]	1459	[875, 944]
191	[348, 349]	614	[931, 845]	1037	[1295, 1296]	1460	[1243, 341]
192	[350, 351]	615	[932, 933]	1038	[663, 1297]	1461	[1570, 319]
193	[314, 352]	616	[821, 642]	1039	[1298, 790]	1462	[1331, 395]
194	[295, 353]	617	[934, 374]	1040	[1219, 657]	1463	[1327, 1561]
195	[354, 355]	618	[764, 935]	1041	[1299, 1300]	1464	[1571, 1381]
196	[356, 357]	619	[381, 832]	1042	[1301, 1302]	1465	[342, 1572]
197	[358, 359]	620	[936, 766]	1043	[658, 1303]	1466	[644, 1573]
198	[330, 360]	621	[937, 938]	1044	[864, 25]	1467	[372, 315]
199	[361, 362]	622	[306, 939]	1045	[1304, 231]	1468	[254, 1385]
200	[363, 364]	623	[741, 940]	1046	[692, 1305]	1469	[1574, 1575]
201	[365, 366]	624	[941, 942]	1047	[394, 1306]	1470	[789, 1558]
202	[367, 368]	625	[943, 757]	1048	[1307, 1308]	1471	[1431, 718]
203	[369, 370]	626	[944, 945]	1049	[1302, 78]	1472	[1350, 695]
204	[245, 371]	627	[946, 947]	1050	[121, 1309]	1473	[880, 1278]
205	[252, 372]	628	[948, 949]	1051	[1310, 23]	1474	[955, 917]
206	[326, 373]	629	[950, 951]	1052	[1296, 1311]	1475	[1576, 99]
207	[374, 375]	630	[952, 777]	1053	[92, 855]	1476	[702, 1577]
208	[376, 377]	631	[953, 323]	1054	[767, 1312]	1477	[1268, 343]
209	[378, 379]	632	[386, 327]	1055	[1313, 1314]	1478	[793, 1411]
210	[380, 381]	633	[954, 955]	1056	[758, 1315]	1479	[373, 1578]
211	[241, 382]	634	[956, 957]	1057	[340, 952]	1480	[1579, 1294]
212	[383, 384]	635	[958, 865]	1058	[18, 921]	1481	[839, 339]
213	[385, 386]	636	[959, 822]	1059	[1316, 1317]	1482	[360, 1580]
214	[337, 387]	637	[957, 960]	1060	[1318, 1228]	1483	[1303, 1581]
215	[276, 388]	638	[961, 307]	1061	[1319, 894]	1484	[349, 1582]
216	[389, 390]	639	[962, 901]	1062	[1320, 233]	1485	[717, 1220]
217	[391, 392]	640	[725, 963]	1063	[1321, 235]	1486	[1217, 1583]
218	[393, 394]	641	[964, 965]	1064	[1322, 1290]	1487	[1584, 281]
219	[395, 396]	642	[966, 967]	1065	[1323, 761]	1488	[1585, 80]
220	[397, 398]	643	[968, 511]	1066	[903, 817]	1489	[942, 1586]
221	[399, 400]	644	[437, 969]	1067	[318, 72]	1490	[1587, 1364]
222	[401, 402]	645	[123, 970]	1068	[669, 934]	1491	[1384, 1588]
223	[403, 404]	646	[439, 455]	1069	[1324, 1325]	1492	[1581, 1589]
224	[405, 406]	647	[177, 971]	1070	[1326, 1327]	1493	[1308, 1590]
225	[407, 408]	648	[972, 622]	1071	[112, 1328]	1494	[1435, 1591]
226	[32, 409]	649	[181, 973]	1072	[776, 288]	1495	[1592, 690]
227	[410, 411]	650	[593, 123]	1073	[351, 923]	1496	[1593, 68]
228	[218, 412]	651	[974, 975]	1074	[870, 1329]	1497	[1594, 1349]
229	[413, 414]	652	[976, 564]	1075	[1330, 1322]	1498	[731, 878]

230	[415, 416]	653	[977, 978]	1076	[679, 1331]	1499	[1398, 365]
231	[417, 418]	654	[979, 175]	1077	[844, 680]	1500	[1430, 1595]
232	[155, 419]	655	[980, 981]	1078	[1078, 1078]	1501	[110, 1419]
233	[420, 34]	656	[223, 418]	1079	[366, 1332]	1502	[1575, 338]
234	[421, 422]	657	[982, 983]	1080	[1333, 391]	1503	[1596, 356]
235	[37, 423]	658	[984, 147]	1081	[1334, 858]	1504	[897, 1316]
236	[424, 425]	659	[434, 985]	1082	[237, 672]	1505	[866, 1330]
237	[426, 427]	660	[169, 986]	1083	[368, 788]	1506	[1597, 1400]
238	[428, 429]	661	[987, 548]	1084	[1297, 936]	1507	[1342, 1598]
239	[430, 431]	662	[489, 988]	1085	[1335, 735]	1508	[1418, 1599]
240	[432, 433]	663	[989, 628]	1086	[727, 1336]	1509	[1595, 756]
241	[434, 179]	664	[990, 991]	1087	[98, 1280]	1510	[1325, 1600]
242	[435, 413]	665	[123, 992]	1088	[781, 1337]	1511	[1601, 27]
243	[436, 410]	666	[51, 207]	1089	[743, 749]	1512	[1315, 1602]
244	[437, 438]	667	[993, 965]	1090	[1300, 29]	1513	[1338, 816]
245	[185, 439]	668	[505, 994]	1091	[24, 1338]	1514	[364, 1603]
246	[440, 441]	669	[617, 995]	1092	[848, 1339]	1515	[316, 1238]
247	[442, 443]	670	[532, 996]	1093	[1340, 797]	1516	[1314, 1604]
248	[444, 124]	671	[997, 470]	1094	[1341, 1342]	1517	[819, 238]
249	[224, 445]	672	[998, 999]	1095	[1343, 882]	1518	[1580, 1236]
250	[126, 446]	673	[427, 1000]	1096	[1344, 888]	1519	[1605, 1417]
251	[447, 448]	674	[1001, 427]	1097	[791, 1345]	1520	[1271, 1606]
252	[449, 450]	675	[518, 426]	1098	[1346, 1242]	1521	[1311, 919]
253	[451, 452]	676	[1002, 989]	1099	[833, 896]	1522	[1607, 1368]
254	[453, 454]	677	[460, 1003]	1100	[840, 1310]	1523	[1608, 1609]
255	[186, 455]	678	[1004, 164]	1101	[1347, 1348]	1524	[114, 244]
256	[206, 456]	679	[128, 1005]	1102	[1349, 1350]	1525	[1433, 1579]
257	[457, 458]	680	[1001, 1006]	1103	[1351, 113]	1526	[851, 762]
258	[36, 459]	681	[515, 1007]	1104	[933, 247]	1527	[1358, 1610]
259	[460, 461]	682	[1008, 192]	1105	[841, 1352]	1528	[256, 1611]
260	[462, 463]	683	[634, 590]	1106	[1291, 829]	1529	[322, 401]
261	[464, 465]	684	[1009, 1010]	1107	[891, 824]	1530	[344, 405]
262	[466, 140]	685	[1011, 1012]	1108	[683, 1353]	1531	[1427, 1612]
263	[467, 468]	686	[1013, 129]	1109	[1354, 847]	1532	[1613, 794]
264	[138, 469]	687	[475, 1014]	1110	[920, 1355]	1533	[862, 240]
265	[470, 471]	688	[1015, 981]	1111	[22, 1356]	1534	[951, 1605]
266	[472, 473]	689	[175, 489]	1112	[796, 1357]	1535	[1614, 1434]
267	[474, 441]	690	[1016, 181]	1113	[914, 956]	1536	[719, 927]
268	[475, 476]	691	[1010, 419]	1114	[899, 1358]	1537	[263, 931]
269	[477, 478]	692	[564, 1017]	1115	[887, 804]	1538	[1615, 1608]
270	[479, 152]	693	[1018, 486]	1116	[1359, 884]	1539	[1410, 1616]
271	[480, 481]	694	[1019, 452]	1117	[1360, 943]	1540	[1617, 1618]
272	[149, 170]	695	[1020, 1021]	1118	[1309, 329]	1541	[734, 1619]
273	[482, 483]	696	[224, 1022]	1119	[926, 253]	1542	[1366, 1395]

274	[484, 130]	697	[1023, 1024]	1120	[1361, 1362]	1543	[1620, 1596]
275	[485, 486]	698	[426, 1006]	1121	[1363, 321]	1544	[1621, 1324]
276	[487, 488]	699	[184, 429]	1122	[1364, 1365]	1545	[1622, 670]
277	[489, 125]	700	[1025, 1026]	1123	[239, 1366]	1546	[1246, 1283]
278	[419, 469]	701	[1027, 408]	1124	[1235, 399]	1547	[1598, 1623]
279	[490, 491]	702	[135, 1028]	1125	[1367, 1346]	1548	[713, 1624]
280	[167, 414]	703	[419, 139]	1126	[1368, 1369]	1549	[396, 1625]
281	[220, 420]	704	[629, 450]	1127	[1221, 643]	1550	[1623, 305]
282	[473, 492]	705	[1029, 1030]	1128	[1370, 1371]	1551	[1260, 303]
283	[493, 32]	706	[9, 489]	1129	[1372, 1343]	1552	[1420, 1230]
284	[152, 494]	707	[981, 627]	1130	[873, 1373]	1553	[1626, 475]
285	[495, 36]	708	[1031, 1032]	1131	[1374, 705]	1554	[1627, 1056]
286	[496, 497]	709	[36, 1033]	1132	[1375, 1376]	1555	[1443, 594]
287	[498, 499]	710	[1034, 40]	1133	[1355, 1377]	1556	[1628, 596]
288	[137, 500]	711	[1035, 439]	1134	[1378, 1379]	1557	[965, 604]
289	[501, 502]	712	[1036, 596]	1135	[1380, 948]	1558	[1085, 1089]
290	[221, 503]	713	[135, 986]	1136	[1381, 1382]	1559	[1106, 1091]
291	[504, 505]	714	[1037, 1038]	1137	[755, 881]	1560	[1526, 500]
292	[506, 499]	715	[417, 224]	1138	[1383, 710]	1561	[172, 1516]
293	[507, 194]	716	[10, 988]	1139	[877, 1384]	1562	[591, 1629]
294	[508, 508]	717	[1039, 1040]	1140	[268, 117]	1563	[967, 1486]
295	[509, 507]	718	[1041, 558]	1141	[1385, 66]	1564	[1630, 513]
296	[420, 510]	719	[1042, 1043]	1142	[357, 712]	1565	[1175, 1631]
297	[457, 511]	720	[1044, 146]	1143	[1386, 1387]	1566	[448, 638]
298	[512, 513]	721	[438, 626]	1144	[1388, 874]	1567	[1168, 468]
299	[514, 53]	722	[1045, 1046]	1145	[893, 828]	1568	[1546, 1487]
300	[160, 193]	723	[412, 1047]	1146	[736, 745]	1569	[1457, 50]
301	[515, 516]	724	[608, 459]	1147	[813, 1389]	1570	[12, 541]
302	[517, 518]	725	[1048, 420]	1148	[1390, 267]	1571	[1632, 539]
303	[519, 520]	726	[439, 1049]	1149	[1391, 1392]	1572	[1532, 1633]
304	[521, 522]	727	[148, 1050]	1150	[1393, 1391]	1573	[1025, 1634]
305	[523, 173]	728	[410, 603]	1151	[388, 1282]	1574	[1517, 1492]
306	[524, 494]	729	[606, 1051]	1152	[102, 1394]	1575	[1496, 1130]
307	[525, 201]	730	[1038, 1052]	1153	[1233, 1301]	1576	[1481, 185]
308	[526, 527]	731	[976, 1053]	1154	[1395, 1288]	1577	[408, 1635]
309	[528, 529]	732	[568, 1054]	1155	[1396, 4]	1578	[1066, 1636]
310	[530, 531]	733	[1055, 1056]	1156	[1397, 1398]	1579	[562, 1044]
311	[170, 133]	734	[442, 198]	1157	[852, 348]	1580	[1106, 541]
312	[532, 188]	735	[1057, 620]	1158	[1247, 1399]	1581	[1016, 1548]
313	[533, 534]	736	[1058, 1059]	1159	[1400, 1347]	1582	[486, 1637]
314	[535, 508]	737	[1060, 1034]	1160	[308, 1261]	1583	[137, 1638]
315	[536, 537]	738	[1061, 412]	1161	[320, 101]	1584	[1199, 473]
316	[538, 539]	739	[1062, 1063]	1162	[811, 1401]	1585	[1533, 40]
317	[483, 32]	740	[1053, 565]	1163	[1402, 1403]	1586	[613, 560]

318	[133, 540]	741	[530, 1064]	1164	[1404, 856]	1587	[454, 216]
319	[161, 436]	742	[1065, 1066]	1165	[1332, 1405]	1588	[512, 1639]
320	[541, 189]	743	[1067, 1068]	1166	[1406, 389]	1589	[1200, 1035]
321	[542, 443]	744	[467, 1069]	1167	[84, 1227]	1590	[982, 589]
322	[543, 544]	745	[1070, 1071]	1168	[1407, 1378]	1591	[1453, 1640]
323	[201, 476]	746	[142, 1072]	1169	[1305, 1383]	1592	[1641, 1174]
324	[545, 546]	747	[449, 630]	1170	[1231, 1408]	1593	[1548, 1479]
325	[547, 548]	748	[8, 530]	1171	[88, 907]	1594	[1463, 61]
326	[189, 529]	749	[1073, 462]	1172	[1409, 1410]	1595	[1546, 479]
327	[549, 550]	750	[990, 1074]	1173	[1411, 1307]	1596	[1490, 205]
328	[551, 552]	751	[1075, 1076]	1174	[706, 661]	1597	[1642, 1179]
329	[553, 554]	752	[1077, 1078]	1175	[1412, 908]	1598	[472, 1188]
330	[555, 556]	753	[49, 1079]	1176	[1413, 1414]	1599	[1097, 1643]
331	[557, 558]	754	[1080, 1081]	1177	[1357, 1415]	1600	[1050, 132]
332	[559, 143]	755	[573, 1082]	1178	[750, 105]	1601	[1529, 1541]
333	[560, 434]	756	[1083, 1084]	1179	[765, 1318]	1602	[1086, 1502]
334	[550, 45]	757	[428, 150]	1180	[909, 1275]	1603	[477, 1508]
335	[561, 562]	758	[1044, 984]	1181	[1416, 285]	1604	[196, 1644]
336	[436, 563]	759	[418, 445]	1182	[1417, 1418]	1605	[1182, 1525]
337	[564, 565]	760	[1063, 126]	1183	[1241, 652]	1606	[424, 1645]
338	[566, 483]	761	[1085, 462]	1184	[1419, 1420]	1607	[1115, 549]
339	[567, 491]	762	[1062, 49]	1185	[1421, 1240]	1608	[1154, 59]
340	[568, 569]	763	[1086, 1087]	1186	[1369, 769]	1609	[1040, 417]
341	[570, 571]	764	[214, 1088]	1187	[815, 83]	1610	[1448, 1646]
342	[572, 573]	765	[1073, 1089]	1188	[853, 1374]	1611	[1087, 1647]
343	[566, 493]	766	[1090, 1052]	1189	[1422, 904]	1612	[997, 1210]
344	[574, 575]	767	[1091, 528]	1190	[799, 1340]	1613	[1551, 1500]
345	[51, 576]	768	[1039, 1092]	1191	[1273, 1423]	1614	[1474, 1051]
346	[61, 577]	769	[1093, 433]	1192	[278, 1412]	1615	[1648, 1040]
347	[155, 138]	770	[984, 619]	1193	[1424, 309]	1616	[554, 1649]
348	[470, 578]	771	[1094, 1076]	1194	[1425, 889]	1617	[1650, 448]
349	[579, 580]	772	[517, 1095]	1195	[837, 17]	1618	[1544, 1038]
350	[52, 581]	773	[1096, 520]	1196	[1399, 1426]	1619	[204, 9]
351	[582, 408]	774	[1097, 1098]	1197	[232, 759]	1620	[1498, 632]
352	[194, 498]	775	[41, 131]	1198	[81, 1427]	1621	[1651, 1477]
353	[508, 535]	776	[527, 1099]	1199	[1428, 775]	1622	[1652, 1527]
354	[583, 584]	777	[58, 1029]	1200	[689, 1429]	1623	[1148, 1653]
355	[585, 586]	778	[487, 502]	1201	[677, 684]	1624	[141, 559]
356	[521, 587]	779	[483, 466]	1202	[250, 902]	1625	[1034, 484]
357	[588, 589]	780	[1100, 606]	1203	[1336, 1250]	1626	[1654, 108]
358	[499, 590]	781	[1101, 1046]	1204	[1245, 1430]	1627	[30, 363]
359	[591, 592]	782	[1093, 1102]	1205	[910, 861]	1628	[1655, 1567]
360	[593, 594]	783	[1103, 1058]	1206	[671, 264]	1629	[1590, 1656]
361	[421, 595]	784	[160, 507]	1207	[296, 1431]	1630	[918, 1657]

362	[596, 597]	785	[1018, 1104]	1208	[945, 1432]	1631	[371, 1295]
363	[598, 8]	786	[1040, 223]	1209	[335, 1433]	1632	[1573, 1438]
364	[61, 568]	787	[1089, 1105]	1210	[1434, 1435]	1633	[1603, 1216]
365	[599, 600]	788	[528, 190]	1211	[662, 1436]	1634	[1256, 1658]
366	[524, 153]	789	[11, 1106]	1212	[1437, 325]	1635	[1286, 1569]
367	[541, 528]	790	[598, 1107]	1213	[1438, 1439]	1636	[714, 1659]
368	[49, 126]	791	[1011, 1108]	1214	[1440, 1386]	1637	[390, 1660]
369	[204, 175]	792	[1109, 965]	1215	[1441, 50]	1638	[1555, 1661]
370	[601, 537]	793	[1041, 609]	1216	[1442, 137]	1639	[675, 236]
371	[599, 602]	794	[1110, 1111]	1217	[1079, 446]	1640	[353, 1375]
372	[444, 55]	795	[499, 1112]	1218	[966, 209]	1641	[1662, 1253]
373	[563, 603]	796	[194, 506]	1219	[611, 984]	1642	[1267, 836]
374	[604, 550]	797	[1113, 506]	1220	[1443, 123]	1643	[1257, 1663]
375	[438, 421]	798	[1114, 509]	1221	[408, 1444]	1644	[400, 1664]
376	[605, 606]	799	[507, 1113]	1222	[451, 1445]	1645	[1429, 1665]
377	[607, 531]	800	[993, 1115]	1223	[412, 1446]	1646	[1269, 1666]
378	[151, 215]	801	[1116, 1108]	1224	[132, 171]	1647	[924, 1667]
379	[35, 608]	802	[1117, 1118]	1225	[1067, 546]	1648	[786, 655]
380	[609, 610]	803	[1119, 1120]	1226	[1201, 1447]	1649	[1618, 1566]
381	[611, 146]	804	[1013, 1005]	1227	[1448, 1449]	1650	[1668, 961]
382	[612, 613]	805	[1121, 13]	1228	[174, 195]	1651	[1423, 1321]
383	[614, 562]	806	[1000, 998]	1229	[1189, 1450]	1652	[1669, 1413]
384	[615, 616]	807	[1000, 572]	1230	[453, 977]	1653	[1258, 1393]
385	[617, 183]	808	[1082, 518]	1231	[1451, 1191]	1654	[1670, 49]
386	[519, 551]	809	[981, 989]	1232	[604, 44]	1655	[1671, 1546]
387	[563, 411]	810	[577, 569]	1233	[1042, 1071]	1656	[1103, 1507]
388	[486, 618]	811	[543, 1122]	1234	[1452, 1449]	1657	[474, 1547]
389	[146, 619]	812	[1123, 1124]	1235	[1113, 498]	1658	[570, 1672]
390	[177, 620]	813	[1125, 1126]	1236	[154, 1010]	1659	[46, 976]
391	[621, 622]	814	[1127, 1128]	1237	[1453, 1454]	1660	[213, 1673]
392	[584, 623]	815	[590, 159]	1238	[1125, 166]	1661	[1056, 200]
393	[624, 625]	816	[1129, 1021]	1239	[480, 206]	1662	[1514, 220]
394	[626, 595]	817	[635, 436]	1240	[1180, 461]	1663	[1164, 1674]
395	[627, 628]	818	[47, 564]	1241	[450, 1165]	1664	[1506, 1675]
396	[629, 630]	819	[585, 1130]	1242	[1127, 1081]	1665	[1501, 1642]
397	[631, 176]	820	[550, 640]	1243	[1137, 572]	1666	[582, 1536]
398	[632, 178]	821	[492, 1131]	1244	[8, 607]	1667	[562, 611]
399	[633, 534]	822	[1132, 600]	1245	[206, 1143]	1668	[1499, 525]
400	[498, 634]	823	[171, 1133]	1246	[200, 475]	1669	[1676, 1653]
401	[635, 162]	824	[182, 995]	1247	[1455, 60]	1670	[1677, 366]
402	[636, 637]	825	[1069, 1134]	1248	[552, 1162]	1671	[1678, 1584]
403	[536, 526]	826	[1135, 1136]	1249	[596, 1456]	1672	[1345, 1679]
404	[638, 167]	827	[1137, 998]	1250	[1050, 170]	1673	[1624, 1678]
405	[639, 222]	828	[1066, 1138]	1251	[1457, 144]	1674	[1657, 932]

406	[44, 640]	829	[432, 1102]	1252	[1458, 1161]	1675	[1339, 905]
407	[641, 642]	830	[1036, 1139]	1253	[1459, 969]	1676	[106, 1370]
408	[643, 644]	831	[506, 634]	1254	[430, 1460]	1677	[1680, 492]
409	[63, 645]	832	[551, 1140]	1255	[1461, 204]	1678	[1174, 220]
410	[646, 647]	833	[210, 1032]	1256	[427, 1137]	1679	[1158, 1552]
411	[648, 649]	834	[1141, 587]	1257	[999, 517]	1680	[1681, 349]
412	[65, 650]	835	[157, 1142]	1258	[1082, 1095]	1681	[1682, 486]
413	[651, 646]	836	[1009, 155]	1259	[1462, 584]	1682	[1683, 390]
414	[652, 653]	837	[481, 1143]	1260	[1451, 1111]	1683	[1684, 213]
415	[654, 93]	838	[1104, 618]	1261	[201, 1014]	1684	[1685, 1624]
416	[333, 290]	839	[1144, 568]	1262	[1455, 1463]	1685	[1686, 1174]
417	[655, 656]	840	[415, 46]	1263	[1464, 575]	1686	[1687, 397]
418	[657, 658]	841	[151, 1088]	1264	[574, 1465]	1687	[1688, 632]
419	[659, 660]	842	[584, 1145]	1265	[547, 1466]	1688	[1689, 325]
420	[661, 662]	843	[1146, 1147]	1266	[1467, 1138]	1689	[1, 189]
421	[67, 663]	844	[572, 999]	1267	[1183, 544]		
422	[664, 665]	845	[482, 493]	1268	[972, 1207]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	282	1	564	0	846	1	1128	0	1410	1
1	0	283	1	565	1	847	1	1129	1	1411	0
2	0	284	0	566	1	848	1	1130	1	1412	1
3	0	285	0	567	1	849	0	1131	1	1413	0
4	0	286	1	568	0	850	1	1132	0	1414	1
5	0	287	1	569	0	851	1	1133	1	1415	0
6	1	288	0	570	0	852	0	1134	1	1416	1
7	0	289	0	571	1	853	0	1135	1	1417	0
8	1	290	0	572	1	854	1	1136	0	1418	0
9	0	291	0	573	0	855	1	1137	0	1419	1
10	0	292	0	574	1	856	0	1138	1	1420	1
11	0	293	0	575	0	857	1	1139	0	1421	1
12	0	294	0	576	1	858	0	1140	0	1422	0
13	1	295	1	577	1	859	0	1141	1	1423	1
14	1	296	0	578	0	860	0	1142	1	1424	0
15	0	297	1	579	0	861	1	1143	0	1425	1
16	1	298	1	580	0	862	0	1144	0	1426	0
17	1	299	0	581	0	863	0	1145	0	1427	0
18	1	300	0	582	1	864	1	1146	1	1428	0
19	0	301	1	583	0	865	1	1147	0	1429	0
20	1	302	1	584	0	866	0	1148	1	1430	1
21	0	303	0	585	0	867	0	1149	1	1431	1
22	1	304	0	586	0	868	1	1150	0	1432	1
23	0	305	0	587	0	869	1	1151	1	1433	1

24	1	306	1	588	1	870	1	1152	1	1434	0
25	0	307	0	589	0	871	1	1153	1	1435	0
26	1	308	1	590	0	872	0	1154	0	1436	1
27	1	309	0	591	0	873	1	1155	1	1437	1
28	1	310	1	592	1	874	0	1156	0	1438	0
29	1	311	0	593	1	875	0	1157	0	1439	0
30	0	312	1	594	1	876	0	1158	0	1440	1
31	0	313	1	595	1	877	0	1159	1	1441	0
32	0	314	1	596	1	878	1	1160	1	1442	0
33	1	315	1	597	1	879	1	1161	0	1443	0
34	1	316	1	598	1	880	0	1162	0	1444	0
35	1	317	0	599	1	881	0	1163	0	1445	1
36	0	318	0	600	0	882	0	1164	1	1446	0
37	1	319	1	601	0	883	0	1165	1	1447	0
38	1	320	0	602	1	884	1	1166	0	1448	1
39	0	321	1	603	1	885	0	1167	0	1449	1
40	1	322	0	604	0	886	0	1168	0	1450	1
41	1	323	1	605	1	887	1	1169	0	1451	0
42	0	324	1	606	1	888	0	1170	0	1452	0
43	1	325	1	607	0	889	0	1171	0	1453	0
44	1	326	1	608	1	890	1	1172	0	1454	1
45	0	327	1	609	1	891	0	1173	0	1455	0
46	0	328	1	610	0	892	0	1174	0	1456	0
47	0	329	1	611	0	893	0	1175	1	1457	1
48	1	330	1	612	1	894	0	1176	0	1458	1
49	1	331	0	613	0	895	0	1177	1	1459	0
50	0	332	0	614	1	896	0	1178	0	1460	0
51	0	333	1	615	1	897	0	1179	1	1461	0
52	1	334	0	616	1	898	1	1180	1	1462	1
53	1	335	0	617	1	899	1	1181	1	1463	1
54	1	336	0	618	0	900	1	1182	0	1464	0
55	0	337	0	619	0	901	0	1183	1	1465	1
56	1	338	1	620	1	902	0	1184	1	1466	1
57	1	339	1	621	1	903	1	1185	1	1467	0
58	1	340	0	622	1	904	1	1186	1	1468	1
59	1	341	0	623	1	905	1	1187	0	1469	0
60	0	342	1	624	0	906	0	1188	0	1470	0
61	1	343	1	625	1	907	1	1189	0	1471	1
62	0	344	1	626	1	908	1	1190	0	1472	1
63	1	345	0	627	1	909	1	1191	0	1473	0
64	0	346	1	628	0	910	0	1192	0	1474	0
65	0	347	0	629	0	911	1	1193	0	1475	0
66	1	348	1	630	0	912	1	1194	1	1476	1
67	0	349	0	631	0	913	1	1195	0	1477	0

68	1	350	1	632	0	914	0	1196	0	1478	1
69	0	351	1	633	0	915	1	1197	0	1479	1
70	1	352	1	634	1	916	0	1198	1	1480	0
71	1	353	0	635	0	917	1	1199	0	1481	0
72	0	354	0	636	1	918	0	1200	1	1482	1
73	1	355	0	637	1	919	1	1201	0	1483	0
74	1	356	0	638	0	920	1	1202	0	1484	0
75	1	357	1	639	1	921	0	1203	1	1485	1
76	1	358	0	640	1	922	1	1204	1	1486	1
77	0	359	0	641	0	923	1	1205	0	1487	0
78	0	360	1	642	0	924	0	1206	0	1488	0
79	0	361	0	643	0	925	0	1207	0	1489	0
80	1	362	1	644	1	926	0	1208	1	1490	1
81	1	363	1	645	0	927	0	1209	0	1491	0
82	1	364	1	646	0	928	1	1210	0	1492	0
83	0	365	1	647	1	929	0	1211	1	1493	1
84	0	366	1	648	0	930	0	1212	1	1494	0
85	1	367	0	649	0	931	1	1213	0	1495	1
86	1	368	1	650	1	932	1	1214	1	1496	1
87	1	369	1	651	1	933	0	1215	0	1497	1
88	0	370	0	652	1	934	1	1216	0	1498	1
89	0	371	1	653	0	935	0	1217	1	1499	0
90	0	372	0	654	0	936	1	1218	0	1500	1
91	0	373	1	655	0	937	1	1219	0	1501	1
92	1	374	0	656	1	938	0	1220	0	1502	1
93	0	375	1	657	0	939	0	1221	0	1503	1
94	1	376	1	658	0	940	0	1222	0	1504	0
95	1	377	0	659	0	941	0	1223	0	1505	0
96	1	378	1	660	0	942	0	1224	1	1506	1
97	1	379	1	661	0	943	1	1225	0	1507	0
98	0	380	1	662	1	944	1	1226	0	1508	0
99	0	381	0	663	0	945	1	1227	1	1509	1
100	1	382	1	664	0	946	1	1228	0	1510	0
101	0	383	1	665	0	947	0	1229	0	1511	0
102	1	384	1	666	0	948	0	1230	1	1512	0
103	1	385	1	667	1	949	0	1231	0	1513	0
104	1	386	0	668	1	950	0	1232	0	1514	1
105	1	387	1	669	1	951	0	1233	1	1515	1
106	0	388	1	670	1	952	0	1234	0	1516	0
107	1	389	1	671	0	953	0	1235	0	1517	0
108	1	390	1	672	0	954	0	1236	1	1518	1
109	0	391	1	673	1	955	0	1237	0	1519	0
110	1	392	0	674	1	956	1	1238	0	1520	1
111	1	393	0	675	0	957	1	1239	0	1521	1

112	1	394	1	676	1	958	0	1240	1	1522	1
113	1	395	1	677	0	959	1	1241	1	1523	0
114	1	396	0	678	0	960	1	1242	0	1524	1
115	1	397	0	679	1	961	0	1243	0	1525	1
116	0	398	0	680	1	962	1	1244	1	1526	1
117	1	399	0	681	1	963	1	1245	1	1527	1
118	0	400	1	682	0	964	0	1246	1	1528	1
119	0	401	0	683	1	965	0	1247	0	1529	0
120	0	402	1	684	0	966	0	1248	1	1530	1
121	1	403	1	685	0	967	0	1249	1	1531	0
122	0	404	0	686	0	968	0	1250	1	1532	0
123	0	405	1	687	0	969	0	1251	1	1533	0
124	1	406	1	688	1	970	0	1252	1	1534	0
125	0	407	0	689	1	971	0	1253	0	1535	0
126	0	408	0	690	0	972	0	1254	0	1536	1
127	1	409	1	691	1	973	0	1255	0	1537	1
128	1	410	0	692	0	974	1	1256	1	1538	0
129	1	411	0	693	1	975	0	1257	1	1539	1
130	0	412	0	694	1	976	1	1258	0	1540	0
131	1	413	1	695	0	977	0	1259	1	1541	0
132	1	414	1	696	1	978	0	1260	0	1542	1
133	0	415	0	697	0	979	0	1261	1	1543	1
134	0	416	1	698	0	980	0	1262	0	1544	1
135	1	417	0	699	0	981	0	1263	0	1545	0
136	1	418	0	700	0	982	0	1264	1	1546	1
137	0	419	0	701	0	983	1	1265	1	1547	1
138	1	420	0	702	1	984	0	1266	0	1548	1
139	1	421	0	703	0	985	0	1267	1	1549	0
140	0	422	0	704	0	986	1	1268	0	1550	1
141	1	423	0	705	0	987	0	1269	0	1551	0
142	1	424	1	706	0	988	1	1270	0	1552	1
143	1	425	1	707	0	989	0	1271	1	1553	0
144	1	426	0	708	0	990	0	1272	1	1554	0
145	1	427	1	709	0	991	0	1273	0	1555	0
146	1	428	1	710	1	992	1	1274	0	1556	0
147	1	429	1	711	0	993	1	1275	0	1557	0
148	0	430	0	712	1	994	0	1276	1	1558	0
149	0	431	1	713	1	995	1	1277	1	1559	1
150	0	432	0	714	0	996	0	1278	0	1560	1
151	1	433	0	715	0	997	0	1279	0	1561	1
152	0	434	0	716	0	998	0	1280	0	1562	0
153	0	435	1	717	1	999	1	1281	1	1563	0
154	1	436	0	718	1	1000	1	1282	0	1564	0
155	0	437	1	719	1	1001	1	1283	0	1565	1

156	1	438	1	720	1	1002	1	1284	0	1566	0
157	0	439	0	721	1	1003	1	1285	0	1567	0
158	0	440	0	722	1	1004	0	1286	1	1568	1
159	0	441	0	723	0	1005	0	1287	0	1569	1
160	0	442	0	724	1	1006	0	1288	1	1570	0
161	1	443	0	725	1	1007	1	1289	0	1571	0
162	1	444	0	726	0	1008	0	1290	0	1572	0
163	1	445	1	727	0	1009	0	1291	1	1573	0
164	0	446	0	728	0	1010	1	1292	0	1574	0
165	1	447	1	729	1	1011	0	1293	0	1575	1
166	0	448	0	730	0	1012	0	1294	0	1576	0
167	0	449	1	731	1	1013	0	1295	0	1577	0
168	0	450	1	732	0	1014	0	1296	0	1578	1
169	0	451	0	733	1	1015	1	1297	0	1579	0
170	0	452	0	734	0	1016	0	1298	1	1580	1
171	1	453	1	735	0	1017	0	1299	1	1581	0
172	1	454	1	736	1	1018	1	1300	1	1582	1
173	1	455	1	737	1	1019	1	1301	1	1583	0
174	0	456	1	738	1	1020	0	1302	0	1584	0
175	1	457	1	739	0	1021	0	1303	0	1585	0
176	1	458	0	740	1	1022	0	1304	1	1586	0
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188	1	470	1	752	1	1034	1	1316	0	1598	1
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191	1	473	1	755	0	1037	0	1319	1	1601	0
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193	1	475	0	757	1	1039	1	1321	0	1603	1
194	1	476	1	758	1	1040	0	1322	1	1604	0
195	0	477	1	759	0	1041	1	1323	1	1605	0
196	0	478	1	760	0	1042	1	1324	1	1606	1
197	0	479	1	761	0	1043	0	1325	0	1607	1
198	1	480	0	762	0	1044	1	1326	0	1608	0
199	0	481	0	763	0	1045	1	1327	1	1609	0

200	1	482	0	764	0	1046	0	1328	1	1610	1
201	1	483	0	765	1	1047	1	1329	1	1611	0
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218	0	500	1	782	1	1064	1	1346	0	1628	0
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221	0	503	1	785	1	1067	0	1349	1	1631	1
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235	1	517	1	799	0	1081	1	1363	1	1645	0
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237	0	519	0	801	1	1083	1	1365	1	1647	0
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239	0	521	0	803	0	1085	0	1367	0	1649	1
240	0	522	1	804	0	1086	0	1368	1	1650	0
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251	1	533	1	815	0	1097	0	1379	0	1661	1
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254	1	536	1	818	0	1100	0	1382	0	1664	1
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256	1	538	1	820	0	1102	1	1384	0	1666	1
257	1	539	1	821	1	1103	1	1385	1	1667	0
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261	1	543	0	825	1	1107	0	1389	1	1671	0
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264	1	546	0	828	1	1110	1	1392	1	1674	1
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266	1	548	0	830	1	1112	1	1394	0	1676	0
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268	0	550	0	832	1	1114	1	1396	1	1678	0
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271	0	553	1	835	0	1117	0	1399	0	1681	0
272	0	554	1	836	0	1118	1	1400	1	1682	0
273	0	555	1	837	0	1119	0	1401	1	1683	0
274	0	556	0	838	0	1120	0	1402	0	1684	0
275	0	557	0	839	0	1121	1	1403	1	1685	0
276	1	558	0	840	0	1122	1	1404	1	1686	0
277	1	559	0	841	1	1123	0	1405	1	1687	0
278	0	560	1	842	0	1124	0	1406	0	1688	0
279	0	561	0	843	1	1125	0	1407	0	1689	0
280	0	562	0	844	1	1126	1	1408	1		
281	0	563	1	845	0	1127	0	1409	0		

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