APPENDIX: EXPLICIT CONTINUED FRACTION EXPANSION FOR $(1+x)^{1/d}$ IN $\mathbb{F}_2$

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Corollary 1. *The continued fraction expansion of* $(1+x^{-1})^{1/3}$ *is*

$$
\begin{aligned}
1 &+ \frac{1\,|}{|\,x+1} + \frac{1\,|}{|\,[3]_x} + \frac{1\,|}{|\,(x+x^2)^1} + \frac{1\,|}{|\,(x+x^2)^3} + \frac{1\,|}{|\,(x+x^2)^5} \\
&+ \frac{1\,|}{|\,(x+x^2)^{11}} + \frac{1\,|}{|\,(x+x^2)^{21}} + \frac{1\,|}{|\,(x+x^2)^{43}} + \frac{1\,|}{|\,(x+x^2)^{85}} \\
&+ \frac{1\,|}{|\,(x+x^2)^{171}} + \frac{1\,|}{|\,(x+x^2)^{341}} + \cdots
\end{aligned}
$$

where the partial quotients a_n *are given by*

$$
\begin{aligned}
a_1 &= 1+x, \quad a_2 = [3]_x \\
a_n &= (x+x^2)^{(2^{n-1}+(-1)^n)/3}
\end{aligned}
$$

Corollary 2. *The continued fraction expansion of* $(1+x^{-1})^{1/5}$ *is*

$$
\begin{aligned}
1 &+ \frac{1\,|}{|\,x} + \frac{1\,|}{|\,[3]_x} + \frac{1\,|}{|\,(x+x^2)^1[3]_x{}^{4}} + \frac{1\,|}{|\,(x+x^2)^3[3]_x{}^{16}} \\
&+ \frac{1\,|}{|\,(x+x^2)^{13}[3]_x{}^{64}} + \frac{1\,|}{|\,(x+x^2)^{51}[3]_x{}^{256}} + \frac{1\,|}{|\,(x+x^2)^{205}[3]_x{}^{1024}} \\
&+ \frac{1\,|}{|\,(x+x^2)^{819}[3]_x{}^{4096}} + \frac{1\,|}{|\,(x+x^2)^{3277}[3]_x{}^{16384}} \\
&+ \frac{1\,|}{|\,(x+x^2)^{13107}[3]_x{}^{65536}} + \frac{1\,|}{|\,(x+x^2)^{52429}[3]_x{}^{262144}} + \cdots
\end{aligned}
$$

where the partial quotients a_n *are given by*

$$
\begin{aligned}
a_1 &= x \\
a_n &= (x+x^2)^{(2^{2n-4}-(-1)^n)/5} \cdot [3]_x{}^{2^{2n-4}}
\end{aligned}
$$

Corollary 3. *The continued fraction expansion of* $(1+x^{-1})^{1/7}$ *is*

$$
\begin{aligned}
1 &+ \frac{1\,|}{|\,x+1} + \frac{1\,|}{|\,[7]_x} + \frac{1\,|}{|\,(x+x^2)^1} + \frac{1\,|}{|\,(x+x^2)^7[3]_x{}^{16}} + \frac{1\,|}{|\,(x+x^2)^9} \\
&+ \frac{1\,|}{|\,(x+x^2)^{55}[3]_x{}^{128}} + \frac{1\,|}{|\,(x+x^2)^{73}} + \frac{1\,|}{|\,(x+x^2)^{439}[3]_x{}^{1024}}
\end{aligned}
$$

$$+\frac{1}{\lfloor (x+x^2)^{585}} + \frac{1}{\lfloor (x+x^2)^{3511}[3]_x{}^{8192}} + \frac{1}{\lfloor (x+x^2)^{4681}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = 1+x, \quad a_2 = [7]_x$$
$$a_{2n} = (x+x^2)^{(3\cdot 2^{3n-2}+1)/7} \cdot [3]_x{}^{2^{3n-2}}$$
$$a_{2n+1} = (x+x^2)^{(2^{3n}-1)/7}$$

Corollary 4. *The continued fraction expansion of $(1+x^{-1})^{1/9}$ is*

$$1+\frac{1}{\lfloor x} + \frac{1}{\lfloor [7]_x} + \frac{1}{\lfloor (x+x^2)^1[7]_x{}^8} + \frac{1}{\lfloor (x+x^2)^7[7]_x{}^{64}}$$
$$+\frac{1}{\lfloor (x+x^2)^{57}[7]_x{}^{512}} + \frac{1}{\lfloor (x+x^2)^{455}[7]_x{}^{4096}}$$
$$+\frac{1}{\lfloor (x+x^2)^{3641}[7]_x{}^{32768}} + \frac{1}{\lfloor (x+x^2)^{29127}[7]_x{}^{262144}}$$
$$+\frac{1}{\lfloor (x+x^2)^{233017}[7]_x{}^{2097152}} + \frac{1}{\lfloor (x+x^2)^{1864135}[7]_x{}^{16777216}}$$
$$+\frac{1}{\lfloor (x+x^2)^{14913081}[7]_x{}^{134217728}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = x$$
$$a_n = (x+x^2)^{(2^{3n-6}-(-1)^n)/9} \cdot [7]_x{}^{2^{3n-6}}$$

Corollary 5. *The continued fraction expansion of $(1+x^{-1})^{1/11}$ is*

$$1+\frac{1}{\lfloor x+1} + \frac{1}{\lfloor [3]_x} + \frac{1}{\lfloor (x+x^2)^1[7]_x{}^4} + \frac{1}{\lfloor (x+x^2)^3} + \frac{1}{\lfloor (x+x^2)^{29}}$$
$$+\frac{1}{\lfloor (x+x^2)^{35}[7]_x{}^{128}} + \frac{1}{\lfloor (x+x^2)^{93}} + \frac{1}{\lfloor (x+x^2)^{931}}$$
$$+\frac{1}{\lfloor (x+x^2)^{1117}[7]_x{}^{4096}} + \frac{1}{\lfloor (x+x^2)^{2979}} + \frac{1}{\lfloor (x+x^2)^{29789}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{3n} = (x+x^2)^{(3\cdot 2^{5n-3}+(-1)^n)/11} \cdot [7]_x{}^{2^{5n-3}}$$
$$a_{3n+1} = (x+x^2)^{(2^{5n}-(-1)^n)/11}$$
$$a_{3n+2} = (x+x^2)^{(5\cdot 2^{5n+1}+(-1)^n)/11}$$

Corollary 6. *The continued fraction expansion of* $(1+x^{-1})^{1/13}$ *is*

$$1+\frac{1}{\lfloor x}+\frac{1}{\lfloor [3]_x}+\frac{1}{\lfloor (x+x^2)^1}+\frac{1}{\lfloor (x+x^2)^3[7]_x{}^8}+\frac{1}{\lfloor (x+x^2)^5[3]_x{}^{64}}$$
$$+\frac{1}{\lfloor (x+x^2)^{59}}+\frac{1}{\lfloor (x+x^2)^{197}[7]_x{}^{512}}+\frac{1}{\lfloor (x+x^2)^{315}[3]_x{}^{4096}}$$
$$+\frac{1}{\lfloor (x+x^2)^{3781}}+\frac{1}{\lfloor (x+x^2)^{12603}[7]_x{}^{32768}}$$
$$+\frac{1}{\lfloor (x+x^2)^{20165}[3]_x{}^{262144}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{3n} = (x+x^2)^{(3\cdot 2^{6n-4}-(-1)^n)/13}$$
$$a_{3n+1} = (x+x^2)^{(5\cdot 2^{6n-3}+(-1)^n)/13}\cdot [7]_x{}^{2^{6n-3}}$$
$$a_{3n+2} = (x+x^2)^{(2^{6n}-(-1)^n)/13}\cdot [3]_x{}^{2^{6n}}$$

Corollary 7. *The continued fraction expansion of* $(1+x^{-1})^{1/15}$ *is*

$$1+\frac{1}{\lfloor x+1}+\frac{1}{\lfloor [15]_x}+\frac{1}{\lfloor (x+x^2)^1}+\frac{1}{\lfloor (x+x^2)^{15}[7]_x{}^{32}}$$
$$+\frac{1}{\lfloor (x+x^2)^{17}}+\frac{1}{\lfloor (x+x^2)^{239}[7]_x{}^{512}}+\frac{1}{\lfloor (x+x^2)^{273}}$$
$$+\frac{1}{\lfloor (x+x^2)^{3823}[7]_x{}^{8192}}+\frac{1}{\lfloor (x+x^2)^{4369}}+\frac{1}{\lfloor (x+x^2)^{61167}[7]_x{}^{131072}}$$
$$+\frac{1}{\lfloor (x+x^2)^{69905}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [15]_x$$
$$a_{2n} = (x+x^2)^{(7\cdot 2^{4n-3}+1)/15}\cdot [7]_x{}^{2^{4n-3}}$$
$$a_{2n+1} = (x+x^2)^{(2^{4n}-1)/15}$$

Corollary 8. *The continued fraction expansion of* $(1+x^{-1})^{1/17}$ *is*

$$1+\frac{1}{\lfloor x}+\frac{1}{\lfloor [15]_x}+\frac{1}{\lfloor (x+x^2)^1[15]_x{}^{16}}+\frac{1}{\lfloor (x+x^2)^{15}[15]_x{}^{256}}$$
$$+\frac{1}{\lfloor (x+x^2)^{241}[15]_x{}^{4096}}+\frac{1}{\lfloor (x+x^2)^{3855}[15]_x{}^{65536}}$$

$$+\frac{1}{|(x+x^2)^{61681}[15]_x{}^{1048576}} + \frac{1}{|(x+x^2)^{986895}[15]_x{}^{16777216}} + \frac{1}{|(x+x^2)^{15790321}[15]_x{}^{268435456}} + \frac{1}{|(x+x^2)^{252645135}[15]_x{}^{4294967296}} + \frac{1}{|(x+x^2)^{4042322161}[15]_x{}^{68719476736}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = x$$
$$a_n = (x+x^2)^{(2^{4n-8}-(-1)^n)/17} \cdot [15]_x{}^{2^{4n-8}}$$

Corollary 9. *The continued fraction expansion of* $(1+x^{-1})^{1/19}$ *is*

$$1+\frac{1}{|x+1}+\frac{1}{|[3]_x}+\frac{1}{|(x+x^2)^1}+\frac{1}{|(x+x^2)^3[3]_x{}^8} + \frac{1}{|(x+x^2)^5[15]_x{}^{32}}+\frac{1}{|(x+x^2)^{27}}+\frac{1}{|(x+x^2)^{485}}+\frac{1}{|(x+x^2)^{539}} + \frac{1}{|(x+x^2)^{1509}[3]_x{}^{4096}}+\frac{1}{|(x+x^2)^{2587}[15]_x{}^{16384}} + \frac{1}{|(x+x^2)^{13797}}+\cdots$$

where the partial quotients a_n are given by

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{5n} = (x+x^2)^{(3\cdot 2^{9n-4}+(-1)^n)/19} \cdot [15]_x{}^{2^{9n-4}}$$
$$a_{5n+1} = (x+x^2)^{(2^{9n}-(-1)^n)/19}$$
$$a_{5n+2} = (x+x^2)^{(9\cdot 2^{9n+1}+(-1)^n)/19}$$
$$a_{5n+3} = (x+x^2)^{(5\cdot 2^{9n+2}-(-1)^n)/19}$$
$$a_{5n+4} = (x+x^2)^{(7\cdot 2^{9n+3}+(-1)^n)/19} \cdot [3]_x{}^{2^{9n+3}}$$

Corollary 10. *The continued fraction expansion of* $(1+x^{-1})^{1/21}$ *is*

$$1+\frac{1}{|x}+\frac{1}{|[3]_x}+\frac{1}{|(x+x^2)^1[15]_x{}^4}+\frac{1}{|(x+x^2)^3[3]_x{}^{64}} + \frac{1}{|(x+x^2)^{61}[15]_x{}^{256}}+\frac{1}{|(x+x^2)^{195}[3]_x{}^{4096}} + \frac{1}{|(x+x^2)^{3901}[15]_x{}^{16384}}+\frac{1}{|(x+x^2)^{12483}[3]_x{}^{262144}}$$

$$+\frac{1\rfloor}{\lceil (x+x^2)^{249661}[15]_x{}^{1048576}} + \frac{1\rfloor}{\lceil (x+x^2)^{798915}[3]_x{}^{16777216}} + \frac{1\rfloor}{\lceil (x+x^2)^{15978301}[15]_x{}^{67108864}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{2n} = (x+x^2)^{(2^{6n-6}-1)/21} \cdot [3]_x{}^{2^{6n-6}}$$
$$a_{2n+1} = (x+x^2)^{(5\cdot 2^{6n-4}+1)/21} \cdot [15]_x{}^{2^{6n-4}}$$

Corollary 11. *The continued fraction expansion of* $(1+x^{-1})^{1/23}$ *is*

$$1+\frac{1\rfloor}{\lceil x+1} + \frac{1\rfloor}{\lceil [7]_x} + \frac{1\rfloor}{\lceil (x+x^2)^1[3]_x{}^8} + \frac{1\rfloor}{\lceil (x+x^2)^7} + \frac{1\rfloor}{\lceil (x+x^2)^{25}} + \frac{1\rfloor}{\lceil (x+x^2)^{39}[15]_x{}^{128}} + \frac{1\rfloor}{\lceil (x+x^2)^{89}} + \frac{1\rfloor}{\lceil (x+x^2)^{1959}[3]_x{}^{4096}} + \frac{1\rfloor}{\lceil (x+x^2)^{2137}[3]_x{}^{16384}} + \frac{1\rfloor}{\lceil (x+x^2)^{14247}} + \frac{1\rfloor}{\lceil (x+x^2)^{51289}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [7]_x$$
$$a_{6n} = (x+x^2)^{(7\cdot 2^{11n-4}+1)/23} \cdot [15]_x{}^{2^{11n-4}}$$
$$a_{6n+1} = (x+x^2)^{(2^{11n}-1)/23}$$
$$a_{6n+2} = (x+x^2)^{(11\cdot 2^{11n+1}+1)/23} \cdot [3]_x{}^{2^{11n+1}}$$
$$a_{6n+3} = (x+x^2)^{(3\cdot 2^{11n+3}-1)/23} \cdot [3]_x{}^{2^{11n+3}}$$
$$a_{6n+4} = (x+x^2)^{(5\cdot 2^{11n+5}+1)/23}$$
$$a_{6n+5} = (x+x^2)^{(9\cdot 2^{11n+6}-1)/23}$$

Corollary 12. *The continued fraction expansion of* $(1+x^{-1})^{1/25}$ *is*

$$1+\frac{1\rfloor}{\lceil x} + \frac{1\rfloor}{\lceil [7]_x} + \frac{1\rfloor}{\lceil (x+x^2)^1} + \frac{1\rfloor}{\lceil (x+x^2)^7} + \frac{1\rfloor}{\lceil (x+x^2)^9} + \frac{1\rfloor}{\lceil (x+x^2)^{23}[15]_x{}^{64}} + \frac{1\rfloor}{\lceil (x+x^2)^{41}[7]_x{}^{1024}} + \frac{1\rfloor}{\lceil (x+x^2)^{983}} + \frac{1\rfloor}{\lceil (x+x^2)^{7209}} + \frac{1\rfloor}{\lceil (x+x^2)^{9175}} + \frac{1\rfloor}{\lceil (x+x^2)^{23593}[15]_x{}^{65536}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$

$$\begin{aligned}
a_{5n} &= (x+x^2)^{(7\cdot 2^{10n-5}-(-1)^n)/25}\\
a_{5n+1} &= (x+x^2)^{(9\cdot 2^{10n-4}+(-1)^n)/25}\cdot [15]_x{}^{2^{10n-4}}\\
a_{5n+2} &= (x+x^2)^{(2^{10n}-(-1)^n)/25}\cdot [7]_x{}^{2^{10n}}\\
a_{5n+3} &= (x+x^2)^{(3\cdot 2^{10n+3}+(-1)^n)/25}\\
a_{5n+4} &= (x+x^2)^{(11\cdot 2^{10n+4}-(-1)^n)/25}
\end{aligned}$$

Corollary 13. *The continued fraction expansion of* $(1+x^{-1})^{1/27}$ *is*

$$1+\frac{1|}{|x+1}+\frac{1|}{|[3]_x}+\frac{1|}{|(x+x^2)^1[3]_x{}^4}+\frac{1|}{|(x+x^2)^3}+\frac{1|}{|(x+x^2)^{13}[15]_x{}^{32}}+\frac{1|}{|(x+x^2)^{19}}+\frac{1|}{|(x+x^2)^{493}}+\frac{1|}{|(x+x^2)^{531}[3]_x{}^{2048}}+\frac{1|}{|(x+x^2)^{1517}}+\frac{1|}{|(x+x^2)^{6675}[15]_x{}^{16384}}+\frac{1|}{|(x+x^2)^{9709}}+\cdots$$

where the partial quotients a_n *are given by*

$$\begin{aligned}
a_1 &= 1+x, \quad a_2=[3]_x\\
a_{5n} &= (x+x^2)^{(11\cdot 2^{9n-4}+(-1)^n)/27}\cdot [15]_x{}^{2^{9n-4}}\\
a_{5n+1} &= (x+x^2)^{(2^{9n}-(-1)^n)/27}\\
a_{5n+2} &= (x+x^2)^{(13\cdot 2^{9n+1}+(-1)^n)/27}\\
a_{5n+3} &= (x+x^2)^{(7\cdot 2^{9n+2}-(-1)^n)/27}\cdot [3]_x{}^{2^{9n+2}}\\
a_{5n+4} &= (x+x^2)^{(5\cdot 2^{9n+4}+(-1)^n)/27}
\end{aligned}$$

Corollary 14. *The continued fraction expansion of* $(1+x^{-1})^{1/29}$ *is*

$$1+\frac{1|}{|x}+\frac{1|}{|[3]_x}+\frac{1|}{|(x+x^2)^1}+\frac{1|}{|(x+x^2)^3}+\frac{1|}{|(x+x^2)^5[3]_x{}^{16}}+\frac{1|}{|(x+x^2)^{11}[7]_x{}^{64}}+\frac{1|}{|(x+x^2)^{53}}+\frac{1|}{|(x+x^2)^{459}[15]_x{}^{1024}}+\frac{1|}{|(x+x^2)^{565}[3]_x{}^{16384}}+\frac{1|}{|(x+x^2)^{15819}}+\frac{1|}{|(x+x^2)^{49717}}+\cdots$$

where the partial quotients a_n *are given by*

$$\begin{aligned}
a_1 &= x\\
a_{7n} &= (x+x^2)^{(3\cdot 2^{14n-5}-(-1)^n)/29}\\
a_{7n+1} &= (x+x^2)^{(13\cdot 2^{14n-4}+(-1)^n)/29}\cdot [15]_x{}^{2^{14n-4}}
\end{aligned}$$

$$a_{7n+2} = (x+x^2)^{(2^{14n}-(-1)^n)/29} \cdot [3]_x{}^{2^{14n}}$$
$$a_{7n+3} = (x+x^2)^{(7\cdot 2^{14n+2}+(-1)^n)/29}$$
$$a_{7n+4} = (x+x^2)^{(11\cdot 2^{14n+3}-(-1)^n)/29}$$
$$a_{7n+5} = (x+x^2)^{(9\cdot 2^{14n+4}+(-1)^n)/29} \cdot [3]_x{}^{2^{14n+4}}$$
$$a_{7n+6} = (x+x^2)^{(5\cdot 2^{14n+6}-(-1)^n)/29} \cdot [7]_x{}^{2^{14n+6}}$$

Corollary 15. *The continued fraction expansion of* $(1+x^{-1})^{1/31}$ *is*

$$1 + \frac{1|}{|x+1} + \frac{1|}{|[31]_x} + \frac{1|}{|(x+x^2)^1} + \frac{1|}{|(x+x^2)^{31}[15]_x{}^{64}} + \frac{1|}{|(x+x^2)^{33}} + \frac{1|}{|(x+x^2)^{991}[15]_x{}^{2048}} + \frac{1|}{|(x+x^2)^{1057}} + \frac{1|}{|(x+x^2)^{31711}[15]_x{}^{65536}} + \frac{1|}{|(x+x^2)^{33825}} + \frac{1|}{|(x+x^2)^{1014751}[15]_x{}^{2097152}} + \frac{1|}{|(x+x^2)^{1082401}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [31]_x$$
$$a_{2n} = (x+x^2)^{(15\cdot 2^{5n-4}+1)/31} \cdot [15]_x{}^{2^{5n-4}}$$
$$a_{2n+1} = (x+x^2)^{(2^{5n}-1)/31}$$

Corollary 16. *The continued fraction expansion of* $(1+x^{-1})^{1/33}$ *is*

$$1 + \frac{1|}{|x} + \frac{1|}{|[31]_x} + \frac{1|}{|(x+x^2)^1[31]_x{}^{32}} + \frac{1|}{|(x+x^2)^{31}[31]_x{}^{1024}} + \frac{1|}{|(x+x^2)^{993}[31]_x{}^{32768}} + \frac{1|}{|(x+x^2)^{31775}[31]_x{}^{1048576}} + \frac{1|}{|(x+x^2)^{1016801}[31]_x{}^{33554432}} + \frac{1|}{|(x+x^2)^{32537631}[31]_x{}^{1073741824}} + \frac{1|}{|(x+x^2)^{1041204193}[31]_x{}^{34359738368}} + \frac{1|}{|(x+x^2)^{33318534175}[31]_x{}^{1099511627776}} + \frac{1|}{|(x+x^2)^{1066193093601}[31]_x{}^{35184372088832}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$

$$a_n = (x+x^2)^{(2^{5n-10}-(-1)^n)/33} \cdot [31]_x{}^{2^{5n-10}}$$

Corollary 17. *The continued fraction expansion of* $(1+x^{-1})^{1/35}$ *is*

$$1+\frac{1}{\lceil x+1}+\frac{1}{\lceil [3]_x}+\frac{1}{\lceil (x+x^2)^1}+\frac{1}{\lceil (x+x^2)^3}+\frac{1}{\lceil (x+x^2)^5[7]_x{}^{16}}$$
$$+\frac{1}{\lceil (x+x^2)^{11}[31]_x{}^{128}}+\frac{1}{\lceil (x+x^2)^{117}}+\frac{1}{\lceil (x+x^2)^{3979}}$$
$$+\frac{1}{\lceil (x+x^2)^{4213}}+\frac{1}{\lceil (x+x^2)^{12171}}+\frac{1}{\lceil (x+x^2)^{20597}[7]_x{}^{65536}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{6n} = (x+x^2)^{(3\cdot 2^{12n-5}+1)/35} \cdot [31]_x{}^{2^{12n-5}}$$
$$a_{6n+1} = (x+x^2)^{(2^{12n}-1)/35}$$
$$a_{6n+2} = (x+x^2)^{(17\cdot 2^{12n+1}+1)/35}$$
$$a_{6n+3} = (x+x^2)^{(9\cdot 2^{12n+2}-1)/35}$$
$$a_{6n+4} = (x+x^2)^{(13\cdot 2^{12n+3}+1)/35}$$
$$a_{6n+5} = (x+x^2)^{(11\cdot 2^{12n+4}-1)/35} \cdot [7]_x{}^{2^{12n+4}}$$

Corollary 18. *The continued fraction expansion of* $(1+x^{-1})^{1/37}$ *is*

$$1+\frac{1}{\lceil x}+\frac{1}{\lceil [3]_x}+\frac{1}{\lceil (x+x^2)^1[3]_x{}^4}+\frac{1}{\lceil (x+x^2)^3}+\frac{1}{\lceil (x+x^2)^{13}}$$
$$+\frac{1}{\lceil (x+x^2)^{19}}+\frac{1}{\lceil (x+x^2)^{45}[7]_x{}^{128}}+\frac{1}{\lceil (x+x^2)^{83}}$$
$$+\frac{1}{\lceil (x+x^2)^{941}[3]_x{}^{2048}}+\frac{1}{\lceil (x+x^2)^{1107}[31]_x{}^{8192}}$$
$$+\frac{1}{\lceil (x+x^2)^{7085}[3]_x{}^{262144}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{9n} = (x+x^2)^{(17\cdot 2^{18n-7}-(-1)^n)/37} \cdot [3]_x{}^{2^{18n-7}}$$
$$a_{9n+1} = (x+x^2)^{(5\cdot 2^{18n-5}+(-1)^n)/37} \cdot [31]_x{}^{2^{18n-5}}$$
$$a_{9n+2} = (x+x^2)^{(2^{18n}-(-1)^n)/37} \cdot [3]_x{}^{2^{18n}}$$
$$a_{9n+3} = (x+x^2)^{(9\cdot 2^{18n+2}+(-1)^n)/37} \cdot [3]_x{}^{2^{18n+2}}$$
$$a_{9n+4} = (x+x^2)^{(7\cdot 2^{18n+4}-(-1)^n)/37}$$

$$a_{9n+5} = (x+x^2)^{(15\cdot 2^{18n+5}+(-1)^n)/37}$$
$$a_{9n+6} = (x+x^2)^{(11\cdot 2^{18n+6}-(-1)^n)/37}$$
$$a_{9n+7} = (x+x^2)^{(13\cdot 2^{18n+7}+(-1)^n)/37} \cdot [7]_x^{2^{18n+7}}$$
$$a_{9n+8} = (x+x^2)^{(3\cdot 2^{18n+10}-(-1)^n)/37}$$

Corollary 19. *The continued fraction expansion of* $(1+x^{-1})^{1/39}$ *is*

$$1+\frac{1}{\lfloor x+1}+\frac{1}{\lfloor [7]_x}+\frac{1}{\lfloor (x+x^2)^1}+\frac{1}{\lfloor (x+x^2)^7}+\frac{1}{\lfloor (x+x^2)^9[3]_x{}^{32}}$$
$$+\frac{1}{\lfloor (x+x^2)^{23}[31]_x{}^{128}}+\frac{1}{\lfloor (x+x^2)^{105}}+\frac{1}{\lfloor (x+x^2)^{3991}[3]_x{}^{8192}}$$
$$+\frac{1}{\lfloor (x+x^2)^{4201}}+\frac{1}{\lfloor (x+x^2)^{28567}}+\frac{1}{\lfloor (x+x^2)^{36969}[3]_x{}^{131072}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [7]_x$$
$$a_{6n} = (x+x^2)^{(7\cdot 2^{12n-5}+1)/39} \cdot [31]_x^{2^{12n-5}}$$
$$a_{6n+1} = (x+x^2)^{(2^{12n}-1)/39}$$
$$a_{6n+2} = (x+x^2)^{(19\cdot 2^{12n+1}+1)/39} \cdot [3]_x^{2^{12n+1}}$$
$$a_{6n+3} = (x+x^2)^{(5\cdot 2^{12n+3}-1)/39}$$
$$a_{6n+4} = (x+x^2)^{(17\cdot 2^{12n+4}+1)/39}$$
$$a_{6n+5} = (x+x^2)^{(11\cdot 2^{12n+5}-1)/39} \cdot [3]_x^{2^{12n+5}}$$

Corollary 20. *The continued fraction expansion of* $(1+x^{-1})^{1/41}$ *is*

$$1+\frac{1}{\lfloor x}+\frac{1}{\lfloor [7]_x}+\frac{1}{\lfloor (x+x^2)^1[3]_x{}^{8}}+\frac{1}{\lfloor (x+x^2)^7[31]_x{}^{32}}$$
$$+\frac{1}{\lfloor (x+x^2)^{25}[7]_x{}^{1024}}+\frac{1}{\lfloor (x+x^2)^{999}[3]_x{}^{8192}}$$
$$+\frac{1}{\lfloor (x+x^2)^{7193}[31]_x{}^{32768}}+\frac{1}{\lfloor (x+x^2)^{25575}[7]_x{}^{1048576}}$$
$$+\frac{1}{\lfloor (x+x^2)^{1023001}[3]_x{}^{8388608}}+\frac{1}{\lfloor (x+x^2)^{7365607}[31]_x{}^{33554432}}$$
$$+\frac{1}{\lfloor (x+x^2)^{26188825}[7]_x{}^{1073741824}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$

$$a_{3n} = (x+x^2)^{(5\cdot 2^{10n-7}-(-1)^n)/41} \cdot [3]_x{}^{2^{10n-7}}$$
$$a_{3n+1} = (x+x^2)^{(9\cdot 2^{10n-5}+(-1)^n)/41} \cdot [31]_x{}^{2^{10n-5}}$$
$$a_{3n+2} = (x+x^2)^{(2^{10n}-(-1)^n)/41} \cdot [7]_x{}^{2^{10n}}$$

Corollary 21. *The continued fraction expansion of* $(1+x^{-1})^{1/43}$ *is*

$$1 + \frac{1}{\lfloor x+1} + \frac{1}{\lfloor [3]_x} + \frac{1}{\lfloor (x+x^2)^1[31]_x{}^4} + \frac{1}{\lfloor (x+x^2)^3} + \frac{1}{\lfloor (x+x^2)^{125}} + \frac{1}{\lfloor (x+x^2)^{131}[31]_x{}^{512}} + \frac{1}{\lfloor (x+x^2)^{381}} + \frac{1}{\lfloor (x+x^2)^{16003}} + \frac{1}{\lfloor (x+x^2)^{16765}[31]_x{}^{65536}} + \frac{1}{\lfloor (x+x^2)^{48771}} + \frac{1}{\lfloor (x+x^2)^{2048381}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{3n} = (x+x^2)^{(11\cdot 2^{7n-5}+(-1)^n)/43} \cdot [31]_x{}^{2^{7n-5}}$$
$$a_{3n+1} = (x+x^2)^{(2^{7n}-(-1)^n)/43}$$
$$a_{3n+2} = (x+x^2)^{(21\cdot 2^{7n+1}+(-1)^n)/43}$$

Corollary 22. *The continued fraction expansion of* $(1+x^{-1})^{1/45}$ *is*

$$1 + \frac{1}{\lfloor x} + \frac{1}{\lfloor [3]_x} + \frac{1}{\lfloor (x+x^2)^1} + \frac{1}{\lfloor (x+x^2)^3[3]_x{}^8} + \frac{1}{\lfloor (x+x^2)^5} + \frac{1}{\lfloor (x+x^2)^{27}} + \frac{1}{\lfloor (x+x^2)^{37}[31]_x{}^{128}} + \frac{1}{\lfloor (x+x^2)^{91}[3]_x{}^{4096}} + \frac{1}{\lfloor (x+x^2)^{4005}} + \frac{1}{\lfloor (x+x^2)^{12379}[3]_x{}^{32768}} + \frac{1}{\lfloor (x+x^2)^{20389}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{6n} = (x+x^2)^{(19\cdot 2^{12n-6}-1)/45}$$
$$a_{6n+1} = (x+x^2)^{(13\cdot 2^{12n-5}+1)/45} \cdot [31]_x{}^{2^{12n-5}}$$
$$a_{6n+2} = (x+x^2)^{(2^{12n}-1)/45} \cdot [3]_x{}^{2^{12n}}$$
$$a_{6n+3} = (x+x^2)^{(11\cdot 2^{12n+2}+1)/45}$$
$$a_{6n+4} = (x+x^2)^{(17\cdot 2^{12n+3}-1)/45} \cdot [3]_x{}^{2^{12n+3}}$$
$$a_{6n+5} = (x+x^2)^{(7\cdot 2^{12n+5}+1)/45}$$

Corollary 23. *The continued fraction expansion of* $(1+x^{-1})^{1/47}$ *is*

$$1+\frac{1}{\lceil x+1}+\frac{1}{\lceil [15]_x}+\frac{1}{\lceil (x+x^2)^1[3]_x{}^{16}}+\frac{1}{\lceil (x+x^2)^{15}[3]_x{}^{64}}+\frac{1}{\lceil (x+x^2)^{49}}+\frac{1}{\lceil (x+x^2)^{207}[3]_x{}^{512}}+\frac{1}{\lceil (x+x^2)^{305}[7]_x{}^{2048}}+\frac{1}{\lceil (x+x^2)^{1743}}+\frac{1}{\lceil (x+x^2)^{14641}}+\frac{1}{\lceil (x+x^2)^{18127}}+\frac{1}{\lceil (x+x^2)^{47409}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [15]_x$$
$$a_{12n} = (x+x^2)^{(15\cdot 2^{23n-5}+1)/47}\cdot [31]_x{}^{2^{23n-5}}$$
$$a_{12n+1} = (x+x^2)^{(2^{23n}-1)/47}$$
$$a_{12n+2} = (x+x^2)^{(23\cdot 2^{23n+1}+1)/47}\cdot [7]_x{}^{2^{23n+1}}$$
$$a_{12n+3} = (x+x^2)^{(3\cdot 2^{23n+4}-1)/47}\cdot [3]_x{}^{2^{23n+4}}$$
$$a_{12n+4} = (x+x^2)^{(11\cdot 2^{23n+6}+1)/47}\cdot [3]_x{}^{2^{23n+6}}$$
$$a_{12n+5} = (x+x^2)^{(9\cdot 2^{23n+8}-1)/47}$$
$$a_{12n+6} = (x+x^2)^{(19\cdot 2^{23n+9}+1)/47}\cdot [3]_x{}^{2^{23n+9}}$$
$$a_{12n+7} = (x+x^2)^{(7\cdot 2^{23n+11}-1)/47}\cdot [7]_x{}^{2^{23n+11}}$$
$$a_{12n+8} = (x+x^2)^{(5\cdot 2^{23n+14}+1)/47}$$
$$a_{12n+9} = (x+x^2)^{(21\cdot 2^{23n+15}-1)/47}$$
$$a_{12n+10} = (x+x^2)^{(13\cdot 2^{23n+16}+1)/47}$$
$$a_{12n+11} = (x+x^2)^{(17\cdot 2^{23n+17}-1)/47}$$

Corollary 24. *The continued fraction expansion of* $(1+x^{-1})^{1/49}$ *is*

$$1+\frac{1}{\lceil x}+\frac{1}{\lceil [15]_x}+\frac{1}{\lceil (x+x^2)^1}+\frac{1}{\lceil (x+x^2)^{15}}+\frac{1}{\lceil (x+x^2)^{17}[3]_x{}^{64}}+\frac{1}{\lceil (x+x^2)^{47}[7]_x{}^{256}}+\frac{1}{\lceil (x+x^2)^{209}[3]_x{}^{2048}}+\frac{1}{\lceil (x+x^2)^{1839}}+\frac{1}{\lceil (x+x^2)^{6353}}+\frac{1}{\lceil (x+x^2)^{10031}}+\frac{1}{\lceil (x+x^2)^{22737}[31]_x{}^{65536}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$

$$\begin{aligned}
a_{10\,n} &= (x+x^2)^{(15\cdot 2^{21\,n-6}-1)/49}\\
a_{10\,n+1} &= (x+x^2)^{(17\cdot 2^{21\,n-5}+1)/49}\cdot [31]_x{}^{2^{21\,n-5}}\\
a_{10\,n+2} &= (x+x^2)^{(2^{21\,n}-1)/49}\cdot [15]_x{}^{2^{21\,n}}\\
a_{10\,n+3} &= (x+x^2)^{(3\cdot 2^{21\,n+4}+1)/49}\\
a_{10\,n+4} &= (x+x^2)^{(23\cdot 2^{21\,n+5}-1)/49}\\
a_{10\,n+5} &= (x+x^2)^{(13\cdot 2^{21\,n+6}+1)/49}\cdot [3]_x{}^{2^{21\,n+6}}\\
a_{10\,n+6} &= (x+x^2)^{(9\cdot 2^{21\,n+8}-1)/49}\cdot [7]_x{}^{2^{21\,n+8}}\\
a_{10\,n+7} &= (x+x^2)^{(5\cdot 2^{21\,n+11}+1)/49}\cdot [3]_x{}^{2^{21\,n+11}}\\
a_{10\,n+8} &= (x+x^2)^{(11\cdot 2^{21\,n+13}-1)/49}\\
a_{10\,n+9} &= (x+x^2)^{(19\cdot 2^{21\,n+14}+1)/49}
\end{aligned}$$

Corollary 25. *The continued fraction expansion of* $(1+x^{-1})^{1/51}$ *is*

$$1+\frac{1|}{|x+1}+\frac{1|}{|[3]_x}+\frac{1|}{|(x+x^2)^1}+\frac{1|}{|(x+x^2)^3[31]_x{}^8}+\frac{1|}{|(x+x^2)^5}+\frac{1|}{|(x+x^2)^{251}}+\frac{1|}{|(x+x^2)^{261}}+\frac{1|}{|(x+x^2)^{763}[31]_x{}^{2048}}+\frac{1|}{|(x+x^2)^{1285}}+\frac{1|}{|(x+x^2)^{64251}}+\frac{1|}{|(x+x^2)^{66821}}+\cdots$$

where the partial quotients a_n *are given by*

$$\begin{aligned}
&a_1 = 1+x,\quad a_2=[3]_x\\
a_{4\,n} &= (x+x^2)^{(19\cdot 2^{8\,n-5}+1)/51}\cdot [31]_x{}^{2^{8\,n-5}}\\
a_{4\,n+1} &= (x+x^2)^{(2^{8\,n}-1)/51}\\
a_{4\,n+2} &= (x+x^2)^{(25\cdot 2^{8\,n+1}+1)/51}\\
a_{4\,n+3} &= (x+x^2)^{(13\cdot 2^{8\,n+2}-1)/51}
\end{aligned}$$

Corollary 26. *The continued fraction expansion of* $(1+x^{-1})^{1/53}$ *is*

$$1+\frac{1|}{|x}+\frac{1|}{|[3]_x}+\frac{1|}{|(x+x^2)^1[7]_x{}^4}+\frac{1|}{|(x+x^2)^3[15]_x{}^{32}}+\frac{1|}{|(x+x^2)^{29}}+\frac{1|}{|(x+x^2)^{483}[3]_x{}^{1024}}+\frac{1|}{|(x+x^2)^{541}}+\frac{1|}{|(x+x^2)^{3555}}+\frac{1|}{|(x+x^2)^{4637}}+\frac{1|}{|(x+x^2)^{11747}}+\frac{1|}{|(x+x^2)^{21021}[3]_x{}^{65536}}+\cdots$$

where the partial quotients a_n are given by

$$
\begin{aligned}
a_1 &= x \\
a_{13\,n} &= (x+x^2)^{(11\cdot 2^{26\,n-6}-(-1)^n)/53} \\
a_{13\,n+1} &= (x+x^2)^{(21\cdot 2^{26\,n-5}+(-1)^n)/53}\cdot [31]_x{}^{2^{26\,n-5}} \\
a_{13\,n+2} &= (x+x^2)^{(2^{26\,n}-(-1)^n)/53}\cdot [3]_x{}^{2^{26\,n}} \\
a_{13\,n+3} &= (x+x^2)^{(13\cdot 2^{26\,n+2}+(-1)^n)/53}\cdot [7]_x{}^{2^{26\,n+2}} \\
a_{13\,n+4} &= (x+x^2)^{(5\cdot 2^{26\,n+5}-(-1)^n)/53}\cdot [15]_x{}^{2^{26\,n+5}} \\
a_{13\,n+5} &= (x+x^2)^{(3\cdot 2^{26\,n+9}+(-1)^n)/53} \\
a_{13\,n+6} &= (x+x^2)^{(25\cdot 2^{26\,n+10}-(-1)^n)/53}\cdot [3]_x{}^{2^{26\,n+10}} \\
a_{13\,n+7} &= (x+x^2)^{(7\cdot 2^{26\,n+12}+(-1)^n)/53} \\
a_{13\,n+8} &= (x+x^2)^{(23\cdot 2^{26\,n+13}-(-1)^n)/53} \\
a_{13\,n+9} &= (x+x^2)^{(15\cdot 2^{26\,n+14}+(-1)^n)/53} \\
a_{13\,n+10} &= (x+x^2)^{(19\cdot 2^{26\,n+15}-(-1)^n)/53} \\
a_{13\,n+11} &= (x+x^2)^{(17\cdot 2^{26\,n+16}+(-1)^n)/53}\cdot [3]_x{}^{2^{26\,n+16}} \\
a_{13\,n+12} &= (x+x^2)^{(9\cdot 2^{26\,n+18}-(-1)^n)/53}\cdot [3]_x{}^{2^{26\,n+18}}
\end{aligned}
$$

Corollary 27. *The continued fraction expansion of $(1+x^{-1})^{1/55}$ is*

$$
1+\cfrac{1}{x+1}+\cfrac{1}{[7]_x}+\cfrac{1}{(x+x^2)^1[15]_x{}^8}+\cfrac{1}{(x+x^2)^7[3]_x{}^{128}}
+\cfrac{1}{(x+x^2)^{121}}+\cfrac{1}{(x+x^2)^{391}}+\cfrac{1}{(x+x^2)^{633}}
+\cfrac{1}{(x+x^2)^{1415}[3]_x{}^{4096}}+\cfrac{1}{(x+x^2)^{2681}}+\cfrac{1}{(x+x^2)^{13703}[31]_x{}^{32768}}
+\cfrac{1}{(x+x^2)^{19065}}+\cdots
$$

where the partial quotients a_n are given by

$$
\begin{aligned}
a_1 &= 1+x, \quad a_2 = [7]_x \\
a_{10\,n} &= (x+x^2)^{(23\cdot 2^{20\,n-5}+1)/55}\cdot [31]_x{}^{2^{20\,n-5}} \\
a_{10\,n+1} &= (x+x^2)^{(2^{20\,n}-1)/55} \\
a_{10\,n+2} &= (x+x^2)^{(27\cdot 2^{20\,n+1}+1)/55}\cdot [3]_x{}^{2^{20\,n+1}} \\
a_{10\,n+3} &= (x+x^2)^{(7\cdot 2^{20\,n+3}-1)/55}\cdot [15]_x{}^{2^{20\,n+3}} \\
a_{10\,n+4} &= (x+x^2)^{(3\cdot 2^{20\,n+7}+1)/55}\cdot [3]_x{}^{2^{20\,n+7}}
\end{aligned}
$$

$$a_{10\,n+5} = (x+x^2)^{(13\cdot 2^{20\,n+9}-1)/55}$$
$$a_{10\,n+6} = (x+x^2)^{(21\cdot 2^{20\,n+10}+1)/55}$$
$$a_{10\,n+7} = (x+x^2)^{(17\cdot 2^{20\,n+11}-1)/55}$$
$$a_{10\,n+8} = (x+x^2)^{(19\cdot 2^{20\,n+12}+1)/55} \cdot [3]_x{}^{2^{20\,n+12}}$$
$$a_{10\,n+9} = (x+x^2)^{(9\cdot 2^{20\,n+14}-1)/55}$$

Corollary 28. *The continued fraction expansion of* $(1+x^{-1})^{1/57}$ *is*

$$1 + \frac{1}{\lfloor x} + \frac{1}{\lfloor [7]_x} + \frac{1}{\lfloor (x+x^2)^1} + \frac{1}{\lfloor (x+x^2)^7 [31]_x{}^{16}} + \frac{1}{\lfloor (x+x^2)^9 [7]_x{}^{512}} + \frac{1}{\lfloor (x+x^2)^{503}} + \frac{1}{\lfloor (x+x^2)^{3593} [31]_x{}^{8192}} + \frac{1}{\lfloor (x+x^2)^{4599} [7]_x{}^{262144}} + \frac{1}{\lfloor (x+x^2)^{257545}} + \frac{1}{\lfloor (x+x^2)^{1839607} [31]_x{}^{4194304}} + \frac{1}{\lfloor (x+x^2)^{2354697} [7]_x{}^{134217728}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{3\,n} = (x+x^2)^{(7\cdot 2^{9\,n-6}-(-1)^n)/57}$$
$$a_{3\,n+1} = (x+x^2)^{(25\cdot 2^{9\,n-5}+(-1)^n)/57} \cdot [31]_x{}^{2^{9\,n-5}}$$
$$a_{3\,n+2} = (x+x^2)^{(2^{9\,n}-(-1)^n)/57} \cdot [7]_x{}^{2^{9\,n}}$$

Corollary 29. *The continued fraction expansion of* $(1+x^{-1})^{1/59}$ *is*

$$1 + \frac{1}{\lfloor x+1} + \frac{1}{\lfloor [3]_x} + \frac{1}{\lfloor (x+x^2)^1 [3]_x{}^4} + \frac{1}{\lfloor (x+x^2)^3 [15]_x{}^{16}} + \frac{1}{\lfloor (x+x^2)^{13} [7]_x{}^{256}} + \frac{1}{\lfloor (x+x^2)^{243} [3]_x{}^{2048}} + \frac{1}{\lfloor (x+x^2)^{1805}} + \frac{1}{\lfloor (x+x^2)^{6387} [3]_x{}^{16384}} + \frac{1}{\lfloor (x+x^2)^{9997}} + \frac{1}{\lfloor (x+x^2)^{55539}} + \frac{1}{\lfloor (x+x^2)^{75533}} + \cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{15\,n} = (x+x^2)^{(27\cdot 2^{29\,n-5}+(-1)^n)/59} \cdot [31]_x{}^{2^{29\,n-5}}$$
$$a_{15\,n+1} = (x+x^2)^{(2^{29\,n}-(-1)^n)/59}$$

$$\begin{aligned}
a_{15\,n+2} &= (x+x^2)^{(29\cdot 2^{29\,n+1}+(-1)^n)/59}\\
a_{15\,n+3} &= (x+x^2)^{(15\cdot 2^{29\,n+2}-(-1)^n)/59}\cdot [3]_x{}^{2^{29\,n+2}}\\
a_{15\,n+4} &= (x+x^2)^{(11\cdot 2^{29\,n+4}+(-1)^n)/59}\cdot [15]_x{}^{2^{29\,n+4}}\\
a_{15\,n+5} &= (x+x^2)^{(3\cdot 2^{29\,n+8}-(-1)^n)/59}\cdot [7]_x{}^{2^{29\,n+8}}\\
a_{15\,n+6} &= (x+x^2)^{(7\cdot 2^{29\,n+11}+(-1)^n)/59}\cdot [3]_x{}^{2^{29\,n+11}}\\
a_{15\,n+7} &= (x+x^2)^{(13\cdot 2^{29\,n+13}-(-1)^n)/59}\\
a_{15\,n+8} &= (x+x^2)^{(23\cdot 2^{29\,n+14}+(-1)^n)/59}\cdot [3]_x{}^{2^{29\,n+14}}\\
a_{15\,n+9} &= (x+x^2)^{(9\cdot 2^{29\,n+16}-(-1)^n)/59}\\
a_{15\,n+10} &= (x+x^2)^{(25\cdot 2^{29\,n+17}+(-1)^n)/59}\\
a_{15\,n+11} &= (x+x^2)^{(17\cdot 2^{29\,n+18}-(-1)^n)/59}\\
a_{15\,n+12} &= (x+x^2)^{(21\cdot 2^{29\,n+19}+(-1)^n)/59}\\
a_{15\,n+13} &= (x+x^2)^{(19\cdot 2^{29\,n+20}-(-1)^n)/59}\cdot [7]_x{}^{2^{29\,n+20}}\\
a_{15\,n+14} &= (x+x^2)^{(5\cdot 2^{29\,n+23}+(-1)^n)/59}
\end{aligned}$$

Corollary 30. *The continued fraction expansion of* $(1+x^{-1})^{1/61}$ *is*

$$\begin{aligned}
1 &+ \frac{1}{x} + \frac{1}{[3]_x} + \frac{1}{(x+x^2)^1} + \frac{1}{(x+x^2)^3} + \frac{1}{(x+x^2)^5}\\
&+ \frac{1}{(x+x^2)^{11}[7]_x{}^{32}} + \frac{1}{(x+x^2)^{21}[7]_x{}^{256}} + \frac{1}{(x+x^2)^{235}}\\
&+ \frac{1}{(x+x^2)^{1813}} + \frac{1}{(x+x^2)^{2283}[3]_x{}^{8192}} + \frac{1}{(x+x^2)^{5909}} + \cdots
\end{aligned}$$

where the partial quotients a_n *are given by*

$$\begin{aligned}
a_1 &= x\\
a_{15\,n} &= (x+x^2)^{(3\cdot 2^{30\,n-6}-(-1)^n)/61}\\
a_{15\,n+1} &= (x+x^2)^{(29\cdot 2^{30\,n-5}+(-1)^n)/61}\cdot [31]_x{}^{2^{30\,n-5}}\\
a_{15\,n+2} &= (x+x^2)^{(2^{30\,n}-(-1)^n)/61}\cdot [3]_x{}^{2^{30\,n}}\\
a_{15\,n+3} &= (x+x^2)^{(15\cdot 2^{30\,n+2}+(-1)^n)/61}\\
a_{15\,n+4} &= (x+x^2)^{(23\cdot 2^{30\,n+3}-(-1)^n)/61}\\
a_{15\,n+5} &= (x+x^2)^{(19\cdot 2^{30\,n+4}+(-1)^n)/61}\\
a_{15\,n+6} &= (x+x^2)^{(21\cdot 2^{30\,n+5}-(-1)^n)/61}\cdot [7]_x{}^{2^{30\,n+5}}\\
a_{15\,n+7} &= (x+x^2)^{(5\cdot 2^{30\,n+8}+(-1)^n)/61}\cdot [7]_x{}^{2^{30\,n+8}}
\end{aligned}$$

$$a_{15\,n+8} = (x+x^2)^{(7\cdot 2^{30\,n+11}-(-1)^n)/61}$$
$$a_{15\,n+9} = (x+x^2)^{(27\cdot 2^{30\,n+12}+(-1)^n)/61}$$
$$a_{15\,n+10} = (x+x^2)^{(17\cdot 2^{30\,n+13}-(-1)^n)/61} \cdot [3]_x{}^{2^{30\,n+13}}$$
$$a_{15\,n+11} = (x+x^2)^{(11\cdot 2^{30\,n+15}+(-1)^n)/61}$$
$$a_{15\,n+12} = (x+x^2)^{(25\cdot 2^{30\,n+16}-(-1)^n)/61} \cdot [3]_x{}^{2^{30\,n+16}}$$
$$a_{15\,n+13} = (x+x^2)^{(9\cdot 2^{30\,n+18}+(-1)^n)/61} \cdot [3]_x{}^{2^{30\,n+18}}$$
$$a_{15\,n+14} = (x+x^2)^{(13\cdot 2^{30\,n+20}-(-1)^n)/61} \cdot [15]_x{}^{2^{30\,n+20}}$$

Corollary 31. *The continued fraction expansion of* $(1+x^{-1})^{1/63}$ *is*

$$1+\frac{1}{\lfloor x+1}+\frac{1}{\lfloor [63]_x}+\frac{1}{\lfloor (x+x^2)^1}+\frac{1}{\lfloor (x+x^2)^{63}[31]_x{}^{128}}$$
$$+\frac{1}{\lfloor (x+x^2)^{65}}+\frac{1}{\lfloor (x+x^2)^{4031}[31]_x{}^{8192}}+\frac{1}{\lfloor (x+x^2)^{4161}}$$
$$+\frac{1}{\lfloor (x+x^2)^{257983}[31]_x{}^{524288}}+\frac{1}{\lfloor (x+x^2)^{266305}}$$
$$+\frac{1}{\lfloor (x+x^2)^{16510911}[31]_x{}^{33554432}}+\frac{1}{\lfloor (x+x^2)^{17043521}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [63]_x$$
$$a_{2\,n} = (x+x^2)^{(31\cdot 2^{6\,n-5}+1)/63} \cdot [31]_x{}^{2^{6\,n-5}}$$
$$a_{2\,n+1} = (x+x^2)^{(2^{6\,n}-1)/63}$$

Corollary 32. *The continued fraction expansion of* $(1+x^{-1})^{1/65}$ *is*

$$1+\frac{1}{\lfloor x}+\frac{1}{\lfloor [63]_x}+\frac{1}{\lfloor (x+x^2)^1[63]_x{}^{64}}+\frac{1}{\lfloor (x+x^2)^{63}[63]_x{}^{4096}}$$
$$+\frac{1}{\lfloor (x+x^2)^{4033}[63]_x{}^{262144}}+\frac{1}{\lfloor (x+x^2)^{258111}[63]_x{}^{16777216}}$$
$$+\frac{1}{\lfloor (x+x^2)^{16519105}[63]_x{}^{1073741824}}$$
$$+\frac{1}{\lfloor (x+x^2)^{1057222719}[63]_x{}^{68719476736}}$$
$$+\frac{1}{\lfloor (x+x^2)^{67662254017}[63]_x{}^{4398046511104}}$$
$$+\frac{1}{\lfloor (x+x^2)^{4330384257087}[63]_x{}^{281474976710656}}$$

$$+\frac{1}{\lfloor (x+x^2)^{277144592453569}[63]_x{}^{18014398509481984}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = x$$
$$a_n = (x+x^2)^{(2^{6n-12}-(-1)^n)/65} \cdot [63]_x{}^{2^{6n-12}}$$

Corollary 33. *The continued fraction expansion of $(1+x^{-1})^{1/67}$ is*

$$1+\frac{1}{\lfloor x+1} + \frac{1}{\lfloor [3]_x} + \frac{1}{\lfloor (x+x^2)^1} + \frac{1}{\lfloor (x+x^2)^3} + \frac{1}{\lfloor (x+x^2)^5}$$
$$+\frac{1}{\lfloor (x+x^2)^{11}[3]_x{}^{32}} + \frac{1}{\lfloor (x+x^2)^{21}[7]_x{}^{128}} + \frac{1}{\lfloor (x+x^2)^{107}[3]_x{}^{1024}}$$
$$+\frac{1}{\lfloor (x+x^2)^{917}[3]_x{}^{4096}} + \frac{1}{\lfloor (x+x^2)^{3179}}$$
$$+\frac{1}{\lfloor (x+x^2)^{13205}[7]_x{}^{32768}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{17n} = (x+x^2)^{(3\cdot 2^{33n-6}+(-1)^n)/67} \cdot [63]_x{}^{2^{33n-6}}$$
$$a_{17n+1} = (x+x^2)^{(2^{33n}-(-1)^n)/67}$$
$$a_{17n+2} = (x+x^2)^{(33\cdot 2^{33n+1}+(-1)^n)/67}$$
$$a_{17n+3} = (x+x^2)^{(17\cdot 2^{33n+2}-(-1)^n)/67}$$
$$a_{17n+4} = (x+x^2)^{(25\cdot 2^{33n+3}+(-1)^n)/67}$$
$$a_{17n+5} = (x+x^2)^{(21\cdot 2^{33n+4}-(-1)^n)/67}$$
$$a_{17n+6} = (x+x^2)^{(23\cdot 2^{33n+5}+(-1)^n)/67} \cdot [3]_x{}^{2^{33n+5}}$$
$$a_{17n+7} = (x+x^2)^{(11\cdot 2^{33n+7}-(-1)^n)/67} \cdot [7]_x{}^{2^{33n+7}}$$
$$a_{17n+8} = (x+x^2)^{(7\cdot 2^{33n+10}+(-1)^n)/67} \cdot [3]_x{}^{2^{33n+10}}$$
$$a_{17n+9} = (x+x^2)^{(15\cdot 2^{33n+12}-(-1)^n)/67} \cdot [3]_x{}^{2^{33n+12}}$$
$$a_{17n+10} = (x+x^2)^{(13\cdot 2^{33n+14}+(-1)^n)/67}$$
$$a_{17n+11} = (x+x^2)^{(27\cdot 2^{33n+15}-(-1)^n)/67} \cdot [7]_x{}^{2^{33n+15}}$$
$$a_{17n+12} = (x+x^2)^{(5\cdot 2^{33n+18}+(-1)^n)/67}$$
$$a_{17n+13} = (x+x^2)^{(31\cdot 2^{33n+19}-(-1)^n)/67} \cdot [3]_x{}^{2^{33n+19}}$$
$$a_{17n+14} = (x+x^2)^{(9\cdot 2^{33n+21}+(-1)^n)/67}$$

$$a_{17\,n+15} = (x+x^2)^{(29\cdot 2^{33\,n+22}-(-1)^n)/67}$$
$$a_{17\,n+16} = (x+x^2)^{(19\cdot 2^{33\,n+23}+(-1)^n)/67}\cdot [15]_x{}^{2^{33\,n+23}}$$

Corollary 34. *The continued fraction expansion of* $(1+x^{-1})^{1/69}$ *is*

$$1+\frac{1}{x}+\frac{1}{[3]_x}+\frac{1}{(x+x^2)^1[3]_x{}^4}+\frac{1}{(x+x^2)^3[7]_x{}^{16}}+\frac{1}{(x+x^2)^{13}}+\frac{1}{(x+x^2)^{115}}+\frac{1}{(x+x^2)^{141}}+\frac{1}{(x+x^2)^{371}[3]_x{}^{1024}}+\frac{1}{(x+x^2)^{653}}+\frac{1}{(x+x^2)^{3443}[7]_x{}^{8192}}+\frac{1}{(x+x^2)^{4749}[63]_x{}^{65536}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = x$$
$$a_{10\,n} = (x+x^2)^{(29\cdot 2^{22\,n-9}-1)/69}\cdot [7]_x{}^{2^{22\,n-9}}$$
$$a_{10\,n+1} = (x+x^2)^{(5\cdot 2^{22\,n-6}+1)/69}\cdot [63]_x{}^{2^{22\,n-6}}$$
$$a_{10\,n+2} = (x+x^2)^{(2^{22\,n}-1)/69}\cdot [3]_x{}^{2^{22\,n}}$$
$$a_{10\,n+3} = (x+x^2)^{(17\cdot 2^{22\,n+2}+1)/69}\cdot [3]_x{}^{2^{22\,n+2}}$$
$$a_{10\,n+4} = (x+x^2)^{(13\cdot 2^{22\,n+4}-1)/69}\cdot [7]_x{}^{2^{22\,n+4}}$$
$$a_{10\,n+5} = (x+x^2)^{(7\cdot 2^{22\,n+7}+1)/69}$$
$$a_{10\,n+6} = (x+x^2)^{(31\cdot 2^{22\,n+8}-1)/69}$$
$$a_{10\,n+7} = (x+x^2)^{(19\cdot 2^{22\,n+9}+1)/69}$$
$$a_{10\,n+8} = (x+x^2)^{(25\cdot 2^{22\,n+10}-1)/69}\cdot [3]_x{}^{2^{22\,n+10}}$$
$$a_{10\,n+9} = (x+x^2)^{(11\cdot 2^{22\,n+12}+1)/69}$$

Corollary 35. *The continued fraction expansion of* $(1+x^{-1})^{1/71}$ *is*

$$1+\frac{1}{x+1}+\frac{1}{[7]_x}+\frac{1}{(x+x^2)^1}+\frac{1}{(x+x^2)^7[7]_x{}^{16}}+\frac{1}{(x+x^2)^9}+\frac{1}{(x+x^2)^{119}}+\frac{1}{(x+x^2)^{137}[3]_x{}^{512}}+\frac{1}{(x+x^2)^{375}}+\frac{1}{(x+x^2)^{1673}}+\frac{1}{(x+x^2)^{2423}}+\frac{1}{(x+x^2)^{5769}}+\cdots$$

where the partial quotients a_n *are given by*

$$a_1 = 1+x, \quad a_2 = [7]_x$$

$$\begin{aligned}
a_{18\,n} &= (x+x^2)^{(7\cdot 2^{35\,n-6}+1)/71} \cdot [63]_x{}^{2^{35\,n-6}} \\
a_{18\,n+1} &= (x+x^2)^{(2^{35\,n}-1)/71} \\
a_{18\,n+2} &= (x+x^2)^{(35\cdot 2^{35\,n+1}+1)/71} \cdot [3]_x{}^{2^{35\,n+1}} \\
a_{18\,n+3} &= (x+x^2)^{(9\cdot 2^{35\,n+3}-1)/71} \\
a_{18\,n+4} &= (x+x^2)^{(31\cdot 2^{35\,n+4}+1)/71} \cdot [7]_x{}^{2^{35\,n+4}} \\
a_{18\,n+5} &= (x+x^2)^{(5\cdot 2^{35\,n+7}-1)/71} \\
a_{18\,n+6} &= (x+x^2)^{(33\cdot 2^{35\,n+8}+1)/71} \\
a_{18\,n+7} &= (x+x^2)^{(19\cdot 2^{35\,n+9}-1)/71} \cdot [3]_x{}^{2^{35\,n+9}} \\
a_{18\,n+8} &= (x+x^2)^{(13\cdot 2^{35\,n+11}+1)/71} \\
a_{18\,n+9} &= (x+x^2)^{(29\cdot 2^{35\,n+12}-1)/71} \\
a_{18\,n+10} &= (x+x^2)^{(21\cdot 2^{35\,n+13}+1)/71} \\
a_{18\,n+11} &= (x+x^2)^{(25\cdot 2^{35\,n+14}-1)/71} \\
a_{18\,n+12} &= (x+x^2)^{(23\cdot 2^{35\,n+15}+1)/71} \cdot [15]_x{}^{2^{35\,n+15}} \\
a_{18\,n+13} &= (x+x^2)^{(3\cdot 2^{35\,n+19}-1)/71} \cdot [3]_x{}^{2^{35\,n+19}} \\
a_{18\,n+14} &= (x+x^2)^{(17\cdot 2^{35\,n+21}+1)/71} \\
a_{18\,n+15} &= (x+x^2)^{(27\cdot 2^{35\,n+22}-1)/71} \cdot [3]_x{}^{2^{35\,n+22}} \\
a_{18\,n+16} &= (x+x^2)^{(11\cdot 2^{35\,n+24}+1)/71} \cdot [3]_x{}^{2^{35\,n+24}} \\
a_{18\,n+17} &= (x+x^2)^{(15\cdot 2^{35\,n+26}-1)/71} \cdot [7]_x{}^{2^{35\,n+26}}
\end{aligned}$$

Corollary 36. *The continued fraction expansion of* $(1+x^{-1})^{1/73}$ *is*

$$1 + \frac{1|}{|x} + \frac{1|}{|[7]_x} + \frac{1|}{|(x+x^2)^1[63]_x{}^8} + \frac{1|}{|(x+x^2)^7[7]_x{}^{512}} + \frac{1|}{|(x+x^2)^{505}[63]_x{}^{4096}} + \frac{1|}{|(x+x^2)^{3591}[7]_x{}^{262144}} + \frac{1|}{|(x+x^2)^{258553}[63]_x{}^{2097152}} + \frac{1|}{|(x+x^2)^{1838599}[7]_x{}^{134217728}} + \frac{1|}{|(x+x^2)^{132379129}[63]_x{}^{1073741824}} + \frac{1|}{|(x+x^2)^{941362695}[7]_x{}^{68719476736}} + \frac{1|}{|(x+x^2)^{67778114041}[63]_x{}^{549755813888}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = x$$
$$a_{2n} = (x+x^2)^{(2^{9n-9}-1)/73} \cdot [7]_x{}^{2^{9n-9}}$$
$$a_{2n+1} = (x+x^2)^{(9\cdot 2^{9n-6}+1)/73} \cdot [63]_x{}^{2^{9n-6}}$$

Corollary 37. *The continued fraction expansion of* $(1+x^{-1})^{1/75}$ *is*

$$1 + \frac{1 |}{| x+1} + \frac{1 |}{| [3]_x} + \frac{1 |}{| (x+x^2)^1 [7]_x{}^4} + \frac{1 |}{| (x+x^2)^3 [3]_x{}^{32}}$$
$$+ \frac{1 |}{| (x+x^2)^{29}} + \frac{1 |}{| (x+x^2)^{99}} + \frac{1 |}{| (x+x^2)^{157} [3]_x{}^{512}} + \frac{1 |}{| (x+x^2)^{355}}$$
$$+ \frac{1 |}{| (x+x^2)^{1693} [3]_x{}^{4096}} + \frac{1 |}{| (x+x^2)^{2403} [63]_x{}^{16384}}$$
$$+ \frac{1 |}{| (x+x^2)^{13981}} + \cdots$$

where the partial quotients a_n are given by

$$a_1 = 1+x, \quad a_2 = [3]_x$$
$$a_{10n} = (x+x^2)^{(11\cdot 2^{20n-6}+1)/75} \cdot [63]_x{}^{2^{20n-6}}$$
$$a_{10n+1} = (x+x^2)^{(2^{20n}-1)/75}$$
$$a_{10n+2} = (x+x^2)^{(37\cdot 2^{20n+1}+1)/75}$$
$$a_{10n+3} = (x+x^2)^{(19\cdot 2^{20n+2}-1)/75} \cdot [7]_x{}^{2^{20n+2}}$$
$$a_{10n+4} = (x+x^2)^{(7\cdot 2^{20n+5}+1)/75} \cdot [3]_x{}^{2^{20n+5}}$$
$$a_{10n+5} = (x+x^2)^{(17\cdot 2^{20n+7}-1)/75}$$
$$a_{10n+6} = (x+x^2)^{(29\cdot 2^{20n+8}+1)/75}$$
$$a_{10n+7} = (x+x^2)^{(23\cdot 2^{20n+9}-1)/75} \cdot [3]_x{}^{2^{20n+9}}$$
$$a_{10n+8} = (x+x^2)^{(13\cdot 2^{20n+11}+1)/75}$$
$$a_{10n+9} = (x+x^2)^{(31\cdot 2^{20n+12}-1)/75} \cdot [3]_x{}^{2^{20n+12}}$$

Corollary 38. *The continued fraction expansion of* $(1+x^{-1})^{1/77}$ *is*

$$1 + \frac{1 |}{| x} + \frac{1 |}{| [3]_x} + \frac{1 |}{| (x+x^2)^1} + \frac{1 |}{| (x+x^2)^3 [15]_x{}^8} + \frac{1 |}{| (x+x^2)^5}$$
$$+ \frac{1 |}{| (x+x^2)^{123} [7]_x{}^{256}} + \frac{1 |}{| (x+x^2)^{133} [7]_x{}^{2048}}$$
$$+ \frac{1 |}{| (x+x^2)^{1915} [3]_x{}^{16384}} + \frac{1 |}{| (x+x^2)^{14469} [3]_x{}^{65536}} + \frac{1 |}{| (x+x^2)^{51067}}$$

$$+\frac{1}{(x+x^2)^{211077}} + \cdots$$

where the partial quotients a_n are given by

$$\begin{aligned}
a_1 &= x \\
a_{14n} &= (x+x^2)^{(25\cdot 2^{30n-8}-1)/77} \cdot [3]_x^{2^{30n-8}} \\
a_{14n+1} &= (x+x^2)^{(13\cdot 2^{30n-6}+1)/77} \cdot [63]_x^{2^{30n-6}} \\
a_{14n+2} &= (x+x^2)^{(2^{30n}-1)/77} \cdot [3]_x^{2^{30n}} \\
a_{14n+3} &= (x+x^2)^{(19\cdot 2^{30n+2}+1)/77} \\
a_{14n+4} &= (x+x^2)^{(29\cdot 2^{30n+3}-1)/77} \cdot [15]_x^{2^{30n+3}} \\
a_{14n+5} &= (x+x^2)^{(3\cdot 2^{30n+7}+1)/77} \\
a_{14n+6} &= (x+x^2)^{(37\cdot 2^{30n+8}-1)/77} \cdot [7]_x^{2^{30n+8}} \\
a_{14n+7} &= (x+x^2)^{(5\cdot 2^{30n+11}+1)/77} \cdot [7]_x^{2^{30n+11}} \\
a_{14n+8} &= (x+x^2)^{(9\cdot 2^{30n+14}-1)/77} \cdot [3]_x^{2^{30n+14}} \\
a_{14n+9} &= (x+x^2)^{(17\cdot 2^{30n+16}+1)/77} \cdot [3]_x^{2^{30n+16}} \\
a_{14n+10} &= (x+x^2)^{(15\cdot 2^{30n+18}-1)/77} \\
a_{14n+11} &= (x+x^2)^{(31\cdot 2^{30n+19}+1)/77} \\
a_{14n+12} &= (x+x^2)^{(23\cdot 2^{30n+20}-1)/77} \\
a_{14n+13} &= (x+x^2)^{(27\cdot 2^{30n+21}+1)/77}
\end{aligned}$$

Corollary 39. *The continued fraction expansion of $(1+x^{-1})^{1/79}$ is*

$$1 + \frac{1}{x+1} + \frac{1}{[15]_x} + \frac{1}{(x+x^2)^1} + \frac{1}{(x+x^2)^{15}} + \frac{1}{(x+x^2)^{17}}$$

$$+ \frac{1}{(x+x^2)^{47}} + \frac{1}{(x+x^2)^{81}} + \frac{1}{(x+x^2)^{175}[3]_x^{512}} + \frac{1}{(x+x^2)^{337}}$$

$$+ \frac{1}{(x+x^2)^{1711}} + \frac{1}{(x+x^2)^{2385}[7]_x^{8192}} + \cdots$$

where the partial quotients a_n are given by

$$\begin{aligned}
a_1 &= 1+x, \quad a_2 = [15]_x \\
a_{20n} &= (x+x^2)^{(15\cdot 2^{39n-6}+1)/79} \cdot [63]_x^{2^{39n-6}} \\
a_{20n+1} &= (x+x^2)^{(2^{39n}-1)/79} \\
a_{20n+2} &= (x+x^2)^{(39\cdot 2^{39n+1}+1)/79} \cdot [7]_x^{2^{39n+1}} \\
a_{20n+3} &= (x+x^2)^{(5\cdot 2^{39n+4}-1)/79}
\end{aligned}$$

$$
\begin{aligned}
a_{20\,n+4} &= (x+x^2)^{(37\cdot 2^{39\,n+5}+1)/79}\\
a_{20\,n+5} &= (x+x^2)^{(21\cdot 2^{39\,n+6}-1)/79}\\
a_{20\,n+6} &= (x+x^2)^{(29\cdot 2^{39\,n+7}+1)/79}\\
a_{20\,n+7} &= (x+x^2)^{(25\cdot 2^{39\,n+8}-1)/79}\\
a_{20\,n+8} &= (x+x^2)^{(27\cdot 2^{39\,n+9}+1)/79}\cdot [3]_x{}^{2^{39\,n+9}}\\
a_{20\,n+9} &= (x+x^2)^{(13\cdot 2^{39\,n+11}-1)/79}\\
a_{20\,n+10} &= (x+x^2)^{(33\cdot 2^{39\,n+12}+1)/79}\\
a_{20\,n+11} &= (x+x^2)^{(23\cdot 2^{39\,n+13}-1)/79}\cdot [7]_x{}^{2^{39\,n+13}}\\
a_{20\,n+12} &= (x+x^2)^{(7\cdot 2^{39\,n+16}+1)/79}\cdot [7]_x{}^{2^{39\,n+16}}\\
a_{20\,n+13} &= (x+x^2)^{(9\cdot 2^{39\,n+19}-1)/79}\\
a_{20\,n+14} &= (x+x^2)^{(35\cdot 2^{39\,n+20}+1)/79}\cdot [3]_x{}^{2^{39\,n+20}}\\
a_{20\,n+15} &= (x+x^2)^{(11\cdot 2^{39\,n+22}-1)/79}\cdot [3]_x{}^{2^{39\,n+22}}\\
a_{20\,n+16} &= (x+x^2)^{(17\cdot 2^{39\,n+24}+1)/79}\\
a_{20\,n+17} &= (x+x^2)^{(31\cdot 2^{39\,n+25}-1)/79}\cdot [15]_x{}^{2^{39\,n+25}}\\
a_{20\,n+18} &= (x+x^2)^{(3\cdot 2^{39\,n+29}+1)/79}\cdot [3]_x{}^{2^{39\,n+29}}\\
a_{20\,n+19} &= (x+x^2)^{(19\cdot 2^{39\,n+31}-1)/79}\cdot [3]_x{}^{2^{39\,n+31}}
\end{aligned}
$$

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