Dominique Foata (1934–2026)

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> *Surprises are a way of learning, of getting to know, of knowing. Not everything is in books. [...] I would not want to say the Pharisee's prayer, but I truly consider myself lucky to have a temperament that never lets me be bored.*
>
> — Letter to his parents, Columbus (Ohio), November 16, 1959 [9]

Our colleague Dominique Foata died in Strasbourg on the morning of September 30, 2026, at the age of ninety-one, less than two weeks before his ninety-second birthday. His funeral Mass was celebrated on October 6 in the church of Saint-Maurice in Strasbourg, followed by burial at Benfeld (Bas-Rhin).

Dominique Foata was born on October 12, 1934, in Damascus, Syria, then under French mandate. After his *licence* in mathematics, he obtained in 1959 a third-cycle doctorate, prepared at the Institut de statistique of the University of Paris under the supervision of Daniel Dugué, and joined the CNRS. Reading H. B. Mann's book on the analysis of variance [18] convinced him "that it was better to go to the source and work with the author of the book": from September 1959 to June 1961 he stayed in the United States, first at Ohio State University (Columbus) with Mann, then at the University of North Carolina at Chapel Hill with R. C. Bose. Working with Bose forced him to describe precisely what he meant by statistics; it was, he later wrote, "really a rediscovery of combinatorics". Back in France in 1961, he did his military service in the French Navy, at the Operations Research Bureau, where he met Paul-André Meyer again; in 1964 he obtained a provisional position at Lille. In 1965 he defended his *doctorat d'État* [3] in Paris, prepared under the guidance of Marcel-Paul Schützenberger. In October 1965, at Meyer's invitation, he obtained a position in Strasbourg, where he spent the rest of his career.

His thesis reread MacMahon's *Combinatory Analysis* in the light of contemporary algebra: MacMahon's Master Theorem was shown to hold in a noncommutative setting, for a new product, the intercalation product, and the equidistribution of statistics on rearrangements of words was proved by explicit bijections. Let $w = x_1 x_2 \cdots x_n$ be a word over a totally ordered alphabet. Its number of inversions $\operatorname{inv} w$ is the number of pairs (i, j) with $i < j$ and $x_i > x_j$, and its major index $\operatorname{maj} w$ is the sum of the positions i, $1 \le i \le n-1$, such that $x_i > x_{i+1}$. The rearrangement class of w is the set of all words obtained by permuting its letters. MacMahon had shown, by computation, that inv and maj have the same generating function, the q-multinomial coefficient, over each rearrangement class. Foata constructed a bijection Φ of each rearrangement class onto itself such that $\operatorname{maj} w = \operatorname{inv} \Phi(w)$ for every w [4]; it was the first bijective proof of MacMahon's theorem. Foata and Schützenberger later showed that, on permutations, Φ also preserves the descent set of the inverse [14]. This "second fundamental transformation" and the first one, already in his thesis, which relates excedances and descents, became the starting point of a vast literature on Mahonian and Euler–Mahonian statistics. With Volker Strehl he introduced a group action on permutations [15], now known as the Foata–Strehl action, which has become a standard tool in the study of Eulerian polynomials.

The thesis also had unexpected consequences in another direction. Jean-Louis Verdier passed a rewritten version of it on to Pierre Cartier, who saw in it an opportunity to apply

his own algebraic techniques. Their joint monograph, *Problèmes combinatoires de commutation et réarrangements* [1] (LNM 85, 1969), introduced what is now called the Cartier–Foata monoid (the free partially commutative monoid, also called the trace monoid), which became, far beyond combinatorics, a basic tool of theoretical computer science for modelling concurrent processes. The following year saw the publication of the *Théorie géométrique des polynômes eulériens* [13], written with Schützenberger, and in 1974 of a third monograph, *La série génératrice exponentielle dans les problèmes d'énumération* [5]. Donald Knuth, who had read the thesis, met him at the International Congress of Mathematicians in Nice in 1970 and invited him to spend a semester at Stanford in 1971; Foata's work on permutations of multisets and the intercalation product is presented in Section 5.1.2 of volume 3 of *The Art of Computer Programming* [16].

All his life he returned to classical texts, from MacMahon to the literature on special functions, to see whether the calculations of their authors in fact reflected relations between combinatorial objects. His combinatorial proof of Mehler's formula for the Hermite polynomials [6], and the combinatorial derivation, with Pierre Leroux, of the generating function for the Jacobi polynomials [12], showed that classical identities of analysis could be read as two ways of counting the same objects. This programme was the subject of his invited lecture at the International Congress of Mathematicians in Warsaw in 1983 [7]. In 1985 he received the Paul Doistau–Émile Blutet Prize of the Académie des sciences. In 1996 the *Electronic Journal of Combinatorics* published *The Foata Festschrift* [19], and on July 7–9, 2000, the conference *Classical Combinatorics* (the "FoataFest") was held in honour of his sixty-fifth birthday at Temple University (Philadelphia), where Herbert S. Wilf hailed him as "a creative, prolific, and influential combinatorialist, of the first rank". He remained mathematically active for more than half a century, notably through long collaborations with Doron Zeilberger and, later, with the present author; his last paper, written with Volker Strehl and the present author, appeared in 2017 [11].

He also served the community at large. From April 26 to 30, 1976, he organized a CNRS Round Table in Strasbourg, *Combinatoire et représentation du groupe symétrique* (Combinatorics and representation of the symmetric group), whose proceedings he edited [10]. Its participants and the authors of its proceedings included many of the leading names in the field — among them George Andrews, Louis Comtet, H. O. Foulkes, Curtis Greene, Gordon James, Adalbert Kerber, Alain Lascoux, Gilbert de B. Robinson, Marcel-Paul Schützenberger, Richard Stanley, Xavier Viennot, Helmut Wielandt and Herbert S. Wilf. The topics discussed there — Young tableaux, the Robinson–Schensted correspondence, symmetric functions, Schubert calculus, permutation groups, partitions — remain, fifty years later, central themes of algebraic combinatorics.

Throughout the 1970s, regular meetings on combinatorics were held at the Mathematisches Forschungsinstitut Oberwolfach under the direction of Konrad Jacobs. As Foata recalled at the fiftieth session of the Séminaire [8], "suddenly, the Oberwolfach Institute had become too small to shelter all the European mathematicians interested in Combinatorics. It became urgent to create a new structure, where everybody could come, bring his contribution and be exposed to the current ideas of that field." Thus in 1980 Foata, Adalbert Kerber, who was then shuttling between Aachen and Bayreuth, and Volker Strehl (Erlangen) founded the Séminaire Lotharingien de Combinatoire. Its name refers to Lotharingia, the kingdom of Lothair, to which Aachen and Strasbourg once belonged — "a kingdom that hardly ever existed", as Foata put it (in translation) in a speech at the 33rd session (Freiberg, 1994) [20]. At first a joint seminar of the universities of Bayreuth, Erlangen and Strasbourg and of those "along the Lotharingian boundaries" [20], it soon met in other countries as well, and it still meets twice a year. Since 1994 it has also been a refereed electronic journal, free of charge for authors and readers alike, now edited by Christian Krattenthaler and Jean-Yves Thibon. At the time of his funeral, the 96th session of the Séminaire was being held in Strobl, Austria.

Foata was also one of the authors behind the collective pseudonym M. Lothaire: he wrote the chapter "Rearrangements of words" of *Combinatorics on Words* [17], a classic of the field. The two Lecture Notes volumes [1] and [13] were reissued in electronic form in the "Books" collection of the Séminaire Lotharingien de Combinatoire, the first one with three new appendices by Foata, Bodo Lass and Christian Krattenthaler. He also took a lasting interest in teaching and exposition: he wrote the article "Combinatoire (analyse)" of the *Encyclopædia Universalis* and, with Aimé Fuchs, two textbooks, *Calcul des probabilités* (German translation by Volker Strehl, *Wahrscheinlichkeitsrechnung*, Birkhäuser, 1999; 3rd edition, also with Jacques Franchi, Dunod, 2012) and *Processus stochastiques* (Dunod, 2002).

He published some 125 papers, with coauthors from several generations and countries, and supervised or co-supervised twelve doctoral students: Volker Strehl, Jean Françon, Don Rawlings, Dominique Dumont, Jacques Désarménien, Jean-Jacques Pansiot, Yves Roy, Laurent Habsieger, Jiang Zeng, Guoniu Han, Arthur Randrianarivony and Bodo Lass. Several of them went on to supervise doctoral students of their own, in combinatorics as well as in computer science. Most of his texts are available on his web page `https://irma.math.unistra.fr/~foata/`.

His last book, *Lettres d'Amérique 1959–61* [9], was completed in February 2026 with the help of Raymond Séroul and Cyril Banderier; a corrected edition followed in April 2026. It gathers the letters he wrote to his parents during his American stay, which his mother had "piously" filed away and which his brother Xavier found in the family archives. The book, in seventy chapters, opens with short biographies of the people he met and closes with an afterword on his later life. The letters form, as he wrote, "a kind of expanded diary", which offers insights into the mathematical and cultural life of the time: the Sputnik-driven revival of American research funding, the segregation he observed at Chapel Hill, where only the university campus was exempt from it, Kennedy's election, the events in Algeria followed from afar. The afterword recalls the program he wrote at Chapel Hill implementing an algorithm of D. K. Ray-Chaudhuri for a minimum cover of an abstract complex, used in particular to construct projective geometries [2]. He finished the book after a minor stroke; his fingers had lost their precision, but typing with all ten had been a lifelong habit, and he went on that way, slowly, patiently, letter by letter, to the end of the book. Its closing pages reproduce the words he spoke at the surprise party for his ninetieth birthday, on October 17, 2024, where he paid tribute to his students with a sentence he had already used at the FoataFest: "I have always wondered who the master was and who the pupil."

Those who knew Dominique Foata remember his kindness, his patience in listening, and his wish to help, from which many of his students and colleagues benefited far beyond mathematics.

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