ARTICLE IN PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Pattern Recognition ▮▮▮ (▮▮▮▮) ▮▮▮–▮▮▮

PATTERN RECOGNITION

THE JOURNAL OF THE PATTERN RECOGNITION SOCIETY

www.elsevier.com/locate/patcog

Efficient Legendre moment computation for grey level images

G.Y. Yang[a], H.Z. Shu[a,*], C. Toumoulin[b], G.N. Han[c], L.M. Luo[a]

[a]Laboratory of Image Science and Technology, Department of Computer Science and Engineering, Southeast University, 210096, Nanjing, People's Republic of China

[b]Laboratoire Traitement du Signal et de l'Image, INSERM U642, Université de Rennes 1, Campus de Beaulieu, 35042 Rennes Cedex, France

[c]IRMA, Université Louis Pasteur et C.N.R.S., 7, rue René-Descartes F, 67084 Strasbourg, France

Received 30 March 2004; received in revised form 24 August 2005; accepted 24 August 2005

Abstract

Legendre orthogonal moments have been widely used in the field of image analysis. Because their computation by a direct method is very time expensive, recent efforts have been devoted to the reduction of computational complexity. Nevertheless, the existing algorithms are mainly focused on binary images. We propose here a new fast method for computing the Legendre moments, which is not only suitable for binary images but also for grey level images. We first establish a recurrence formula of one-dimensional (1D) Legendre moments by using the recursive property of Legendre polynomials. As a result, the 1D Legendre moments of order p, $L_p = L_p(0)$, can be expressed as a linear combination of $L_{p-1}(1)$ and $L_{p-2}(0)$. Based on this relationship, the 1D Legendre moments $L_p(0)$ can thus be obtained from the arrays of $L_1(a)$ and $L_0(a)$, where a is an integer number less than p. To further decrease the computation complexity, an algorithm, in which no multiplication is required, is used to compute these quantities. The method is then extended to the calculation of the two-dimensional Legendre moments L_{pq}. We show that the proposed method is more efficient than the direct method.

Keywords: Legendre moments; Fast algorithm; Recurrence formula; Grey level images

1. Introduction

Since Hu introduced the moment invariants [1], moments and moment functions of image intensity values have been successfully and widely used in the field of image analysis, such as object recognition, object representation, edge detection [2]. Orthogonal moments (e.g. Legendre moment and Zernike moment) can be used to represent an image with the minimum amount of information redundancy [3]. Since the computation of orthogonal moments of a two-dimensional (2D) image by a direct method involves a significant amount of arithmetic operations, some fast algorithms have been developed to reduce the computational complexity. However, the existing methods for fast computation of Legendre moments are mainly focused on binary images [4–6]. Because the moments of a grey level image are also used in many applications, such as texture analysis [7], in this paper we propose a fast algorithm for computing the Legendre moments for grey level images. The principle is as follows. The recurrence formula of one-dimensional (1D) Legendre moments is firstly established by using the recursive property of Legendre polynomials. The 1D Legendre moment of order p, $L_p = L_p(0)$, is expressed as a linear combination of $L_{p-1}(1)$ and $L_{p-2}(0)$. Based on this relationship, the 1D Legendre moments $L_p(0)$ can thus be obtained from the arrays of $L_1(a)$ and $L_0(a)$, where a is an integer number less than p. An algorithm based on a systolic array in which no multiplication is required is used to compute these quantities. We then propose an extension of this method to the 2D Legendre moment L_{pq} computation.

The remainder of this paper is organized as follows. In Section 2, we first describe a new approach for computing the 1D Legendre moments of 1D signal, and then extend this method to the 2D Legendre moment calculation. Section 3 gives the detailed analysis of the computational complexity and some experimental results. Section 4 provides some concluding remarks.

* Corresponding author. Tel.: +86 25 83794249; fax: +86 25 83792698.
E-mail address: shu.list@seu.edu.cn (H.Z. Shu).

0031-3203/$30.00 © 2005 Published by Elsevier Ltd on behalf of Pattern Recognition Society.
doi:10.1016/j.patcog.2005.08.008

PR2393 DTD VER: 5.0.1 pp: 1-7 (col.fig.: nil) PROD. TYPE: COM ED: Babu PAGN: Jagan – SCAN: nil

2. Fast computation of 2D Legendre moments

The $(p+q)$th-order Legendre moment of an image with intensity function $f(x, y)$ is defined by

$$L_{pq}=\frac{(2p+1)(2q+1)}{4}\int_{-1}^{1}\int_{-1}^{1}P_p(x)P_q(y)f(x,y)\,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where $P_p(x)$ is the pth-order Legendre polynomial given by

$$P_p(x)=\frac{1}{2^p}\sum_{k=0}^{p/2}(-1)^k\frac{(2p-2k)!}{k!(p-k)!(p-2k)!}x^{p-2k},\quad x\in[-1,1]. \tag{2}$$

For a digital image of size $N \times N$, Eq. (1) is usually approximated by

$$L_{pq}=\frac{(2p+1)(2q+1)}{(N-1)^2}\sum_{i=1}^{N}\sum_{j=1}^{N}P_p(x_i)P_q(y_j)f(x_i,y_j), \tag{3}$$

with $x_i=(2i-N-1)/(N-1)$, $y_j=(2j-N-1)/(N-1)$.

The Legendre polynomial obeys the following recursive relation

$$P_{p+1}(x)=\frac{2p+1}{p+1}xP_p(x)-\frac{p}{p+1}P_{p-1}(x),\qquad p\geqslant 1, \tag{4}$$

with $P_0(x)=1$, $P_1(x)=x$.

In the following, we present an algorithm for the fast calculation of the 2D Legendre moment for grey level images. For the sake of simplicity, we first consider the computation of 1D Legendre moments.

For a 1D discrete signal $f(x_i)$, $1\leqslant i\leqslant N$, the 1D Legendre moment is given by

$$L_p=\frac{2p+1}{N-1}\sum_{i=1}^{N}P_p(x_i)f(x_i). \tag{5}$$

Let us now introduce the following notation:

$$L_p(a)=\frac{2p+1}{N-1}\sum_{i=1}^{N}x_i^aP_p(x_i)f(x_i). \tag{6}$$

It can be easily seen that $L_p=L_p(0)$. Thus, we turn to the fast computation of $L_p(a)$ in the following:

Substitution of Eq. (4) into Eq. (6) yields

$$\begin{aligned}L_p(a)&=\frac{2p+1}{N-1}\sum_{i=1}^{N}x_i^a\left[\frac{2p-1}{p}x_iP_{p-1}(x_i)-\frac{p-1}{p}P_{p-2}(x_i)\right]f(x_i)\\&=\frac{2p+1}{p}\frac{2p-1}{N-1}\sum_{i=1}^{N}x_i^{a+1}P_{p-1}(x_i)f(x_i)\\&\quad-\frac{p-1}{p}\frac{2p+1}{2p-3}\frac{2p-3}{N-1}\sum_{i=1}^{N}x_i^aP_{p-2}(x_i)f(x_i)\end{aligned} \tag{7}$$

therefore, we have the following recurrence relation for $p\geqslant 2$:

$$L_p(a)=\frac{2p+1}{p}\left[L_{p-1}(a+1)-\frac{p-1}{2p-3}L_{p-2}(a)\right] \tag{8}$$

with

$$L_0(a)=\frac{1}{N-1}\sum_{i=1}^{N}x_i^af(x_i)=\frac{1}{N-1}G_N(a), \tag{9}$$

$$L_1(a)=\frac{3}{N-1}\sum_{i=1}^{N}x_i^{a+1}f(x_i)=\frac{3}{N-1}G_N(a+1), \tag{10}$$

$$G_N(a)=\sum_{i=1}^{N}x_i^af(x_i). \tag{11}$$

The above discussion shows that the 1D Legendre moments $L_p=L_p(0)$, for $p\geqslant 2$, can be deduced from the values of $L_0(a)$ and $L_1(a)$ where a is an integer less than p, $L_0(a)$ and $L_1(a)$ can be obtained by $G_N(a)$. The calculation of Eq. (11) needs to distinguish two different cases: odd N and even N.

(1) $N=2L+1$:

Since $x_i=(2i-N-1)/(N-1)$, we deduce from Eq. (11) that

$$\begin{aligned}G_{2L+1}(a)&=\sum_{i=1}^{2L+1}\left(\frac{2i-2L-2}{2L}\right)^af(x_i)\\&=\frac{1}{L^a}\sum_{i=1}^{2L+1}(i-L-1)^af(x_i),\\&=\begin{cases}\frac{1}{L^a}\big[-L^af(x_1)-(L-1)^af(x_2)\\\quad-\cdots-f(x_L)+f(x_{L+2})+2^af(x_{L+3})\\\quad+\cdots+L^af(x_{2L+1})\big]\quad a\text{ is odd},\\\frac{1}{L^a}\big[L^af(x_1)+(L-1)^af(x_2)\\\quad+\cdots+f(x_L)+f(x_{L+2})+2^af(x_{L+3})\\\quad+\cdots+L^af(x_{2L+1})\big]\quad a\text{ is even}.\end{cases}\end{aligned} \tag{12}$$

Eq. (12) can be rewritten as

$$G_{2L+1}(a)=\begin{cases}\frac{1}{L^a}\sum_{i=1}^{L}i^ag_1(x_i)\quad a\text{ is odd},\\\frac{1}{L^a}\sum_{i=1}^{L}i^ag_2(x_i)\quad a\text{ is even}\end{cases} \tag{13}$$

with

$$g_1(x_i) = f(x_{L+i+1}) - f(x_{L-i+1}), \quad i = 1, 2, \ldots, L, \quad (14)$$

$$g_2(x_i) = f(x_{L+i+1}) + f(x_{L-i+1}), \quad i = 1, 2, \ldots, L. \quad (15)$$

(2) $N = 2L$:
In this case, Eq. (11) becomes

$$\begin{aligned}
G_{2L}(a) &= \sum_{i=1}^{2L} \left(\frac{2i-2L-1}{2L-1}\right)^a f(x_i) \\
&= \frac{1}{(2L-1)^a} \sum_{i=1}^{2L} (2i-2L-1)^a f(x_i), \\
&= \begin{cases} \dfrac{1}{(2L-1)^a} \big[-(2L-1)^a f(x_1) \\ \quad -(2L-3)^a f(x_2) - \cdots - f(x_L) \\ \quad +f(x_{L+1}) + 3^a f(x_{L+2}) \\ \quad + \cdots + (2L-1)^a f(x_{2L})\big] & a \text{ is odd}, \\ \dfrac{1}{(2L-1)^a} \big[(2L-1)^a f(x_1) \\ \quad +(2L-3)^a f(x_2) + \cdots + f(x_L) \\ \quad +f(x_{L+1}) + 3^a f(x_{L+2}) \\ \quad + \cdots + (2L-1)^a f(x_{2L})\big] & a \text{ is even} \end{cases}
\end{aligned} \quad (16)$$

or

$$G_{2L}(a) = \begin{cases} \dfrac{1}{(2L-1)^a} \sum_{i=1}^{L} (2i-1)^a g_3(x_i), & a \text{ is odd}, \\ \dfrac{1}{(2L-1)^a} \sum_{i=1}^{L} (2i-1)^a g_4(x_i), & a \text{ is even} \end{cases} \quad (17)$$

with

$$g_3(x_i) = f(x_{L+i}) - f(x_{L-i+1}), \quad i = 1, 2, \ldots, L, \quad (18)$$

$$g_4(x_i) = f(x_{L+i}) + f(x_{L-i+1}), \quad i = 1, 2, \ldots, L. \quad (19)$$

We discuss, in the following two subsections, the way to efficiently calculate $G_N(a)$ given by Eqs. (13) or (17), according to the different modalities of the 1D signal $f(x_i)$.

2.1. $f(x_i) = 1$ for $i = 1, 2, \ldots, N$

In this case, Eqs. (13) and (17) become

$$G_{2L+1}(a) = \begin{cases} 0, & a \text{ is odd}, \\ \dfrac{2}{L^a} \sum_{i=1}^{L} i^a, & a \text{ is even}, \end{cases} \quad (20)$$

$$G_{2L}(a) = \begin{cases} 0, & a \text{ is odd}, \\ \dfrac{2}{(2L-1)^a} \sum_{i=1}^{L} (2i-1)^a \\ \quad = \dfrac{2}{(2L-1)^a} \\ \quad \times \left(\sum_{i=1}^{2L} i^a - 2^a \sum_{i=1}^{L} i^a\right), & a \text{ is even}. \end{cases} \quad (21)$$

The above equations show that to obtain the values of $G_N(a)$, we only need to calculate the following summation:

$$H_M(a) = \sum_{i=1}^{M} i^a. \quad (22)$$

For the computation of Eq. (22), which is just the 1D geometric moment of order a of a 'binary' signal, we use the formulae proposed by Spiliotis and Mertzios [8]

$$\begin{aligned}
&H_M(1) = \frac{M(M+1)}{2}, \quad H_M(2) = \frac{M(M+1)(2M+1)}{6}, \\
&H_M(3) = \frac{M^2(M+1)^2}{4}, \\
&H_M(4) = \frac{M(M+1)(2M+1)(3M^2+3M+1)}{30},
\end{aligned} \quad (23)$$

and for $a \geqslant 4$, the recurrence formula

$$\begin{aligned}
&\binom{a+1}{1} H_M(1) + \binom{a+1}{2} H_M(2) \\
&\quad + \cdots + \binom{a+1}{a} H_M(a) \\
&= (M+1)^{a+1} - (M+1),
\end{aligned} \quad (24)$$

where

$$\binom{i}{j} = \frac{i!}{j!(i-j)!}$$

is a combination number.

2.2. $f(x_i) \neq f(x_j)$ for some $i \neq j$

Eq. (17) can be written as

$$G_{2L}(a) = \begin{cases} \dfrac{1}{(2L-1)^a} \sum_{i=1}^{2L} i^a h_1(x_i), & a \text{ is odd}, \\ \dfrac{1}{(2L-1)^a} \sum_{i=1}^{2L} i^a h_2(x_i), & a \text{ is even}, \end{cases} \quad (25)$$

where

$$h_1(x_i) = \begin{cases} g_3(x_{(i+1)/2}) & \text{if } i \text{ is odd}, \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

$$h_2(x_i) = \begin{cases} g_4(x_{(i+1)/2}) & \text{if } i \text{ is odd}, \\ 0 & \text{otherwise}. \end{cases} \quad (27)$$

Here $g_3(x_i)$ and $g_4(x_i)$ are given by Eqs. (18) and (19), respectively.

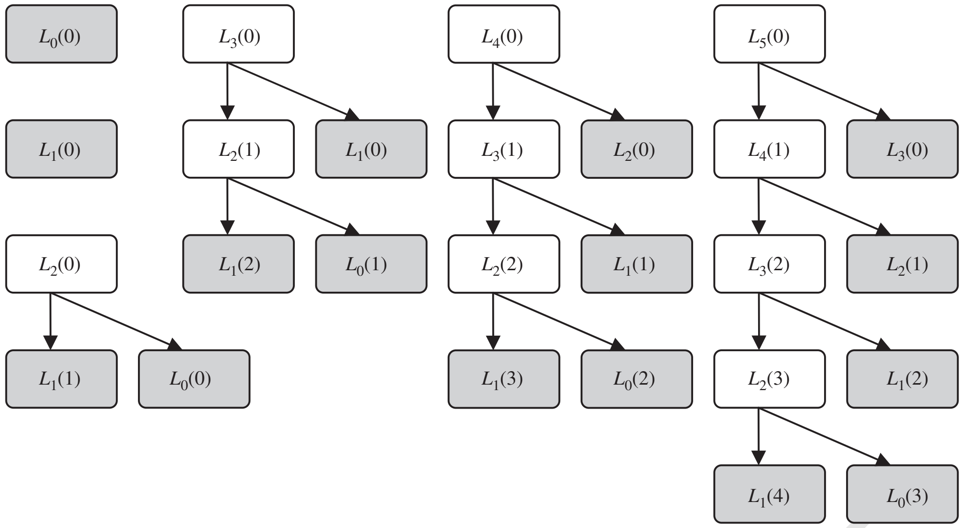

Fig. 1. Computation process of $L_p(0)$ with p from 0 to 5. Grey level boxes correspond to already computed coefficients and white boxes to coefficients that will be computed from those which appear in grey level boxes.

```
for i = 1 to N
        computing L0(q) (0 ≤ q ≤ M-2) and L1(q) (0 ≤ q ≤ M-1) using Eqs. (9) and (10)
    for q = 0 to M
        computing Yiq for each row i of the image using Eq. (8)
    endfor
endfor
for p = 0 to M
        computing L0(a) (0 ≤ a ≤ M-p-2) and L1(a) (0 ≤ a ≤ M-p-1) using Eqs. (9) and
        (10) from pre-calculated Yiq
    for q = 0 to M – p
        computing the 2D Legendre moments Lpq using recursive method
    endfor
endfor
```

Fig. 2. Algorithm for computing L_{pq}.

It can be seen from Eqs. (13) and (25) that we need to calculate the summation of the form

$$S_M(a) = \sum_{i=1}^{M} i^a g(x_i). \tag{28}$$

Note that $S_M(a)$ is the 1D geometric moment of order a of an arbitrary 1D signal. Since many algorithms are available in the literature to speed up the computation of Eq. (28), we decided to choose the method proposed by Chan et al. [9]. Their algorithm is able to efficiently compute the grey level image moments. It makes use of a systolic array for computing the moments in which no multiplication is required. We recently applied such a method to efficiently calculate the Zernike moments [10]. For a detailed description of this algorithm, please refer to Ref. [10].

Thus, the 1D Legendre moments $L_p(0)$, for $0 \leqslant p \leqslant M$ (M denotes the maximal order we want to calculate), can be efficiently obtained using the previously presented algorithm. Fig. 1 shows the computation order of $L_p(0)$ for p varying from 0 to 5.

Let us now describe the method for fast computation of the 2D Legendre moments L_{pq}. The double summation in Eq. (3) can be split into the following separate form:

$$\begin{aligned} L_{pq} &= \frac{(2p+1)(2q+1)}{(N-1)^2} \sum_{i=1}^{N} \sum_{j=1}^{N} P_p(x_i) P_q(y_j) f(x_i, y_j) \\ &= \frac{2p+1}{N-1} \sum_{i=1}^{N} P_p(x_i) \left(\frac{2q+1}{N-1} \sum_{j=1}^{N} P_q(y_j) f(x_i, y_j) \right) \\ &= \frac{2p+1}{N-1} \sum_{i=1}^{N} P_p(x_i) Y_{iq}, \end{aligned} \tag{29}$$

where Y_{iq} is the qth-order row moments of row i given by

$$Y_{iq} = \frac{2q+1}{N-1} \sum_{j=1}^{N} P_p(y_j) f(x_i, y_j). \tag{30}$$

These equations show that the computation of 2D Legendre moments of grey level images can be decomposed into two steps. First, the 1D Legendre moments Y_{iq}, for $1 \leqslant i \leqslant N$ and $0 \leqslant q \leqslant M$, are computed by using the algorithm described in Sections 2.1 and 2.2, according to the different image modalities of $f(x_i, y_j)$. Then, the row moments Y_{iq} are applied to compute the 2D Legendre moments L_{pq}. Thus, after the first step, the 2D Legendre moments L_{pq} can be calculated as 1D moments by setting the image intensity function $f(x_i, y_j)$ to the Y_{iq} previously computed. The algorithm for computing the 2D Legendre moments is depicted in Fig. 2. It should be pointed out that such a strategy can also be realized in parallel.

3. Computation complexity and experimental results

Let the image size be $N \times N$ pixels, and M be the maximum order of Legendre moments to calculate. The maximum order M is usually less than the image size N.

The direct computation of Eq. (3) requires approximately $O\left(M^2N^2\right)$ additions and multiplications, respectively.

3.1. Computational complexity of the proposed method for binary images

The computation of the geometrical moments up to the order M of a binary image with $N \times N$ pixels, requires approximately $4M$ power calculations, $2M^2$ multiplications, and M^2 additions (note that these numbers are not dependent on N) [8]. The computation of the 2D Legendre moments L_{pq}, by using the recursive algorithm, needs $O\left(NM^3\right)$ additions and $O\left(M^3\right)$ multiplications. Therefore, the algorithm is very efficient compared with the direct method.

3.2. Computational complexity of the proposed method for grey level images

The computational complexity of the method for grey level images takes into account the parity of N.

(1) For odd values of N:

Let us first consider the number of operations required in the computation of the ith row moments $Y_{iq} (0 \leqslant q \leqslant M)$. Note that the functions $g_1(x)$ and $g_2(x)$ defined by Eqs. (14) and (15) are used for odd values of N. To obtain the values of Y_{iq}, we must calculate $G_N(a)$ with Eq. (13) for $0 \leqslant a \leqslant M$. This step needs only $(M+1)^2(N/2-1)$ additions. The computation of Y_{iq} (for $0 \leqslant q \leqslant M$) from the pre-calculated $G_N(a)$, requires $M(M-1)/2$ additions and $2M(M-1)$ multiplications. Therefore, the computation of N rows of Y_{iq} (for $1 \leqslant i \leqslant N$) needs approximately $M^2\left(N^2+N\right)/2$ additions and $2M^2N$ multiplications.

When all Y_{iq}, for $1 \leqslant i \leqslant N$ and $0 \leqslant q \leqslant M$, are obtained, the 2D Legendre moments L_{pq}, for $0 \leqslant p+q \leqslant M$, can be calculated in a similar way. The corresponding additions and multiplications are $M^3N/12 + M^2N$ and $2M^3/3 + 2M^2$.

In conclusion, the overall computation makes use of $O\left(M^2N^2\right)$ additions and $O\left(M^2N\right)$ multiplications approximately for $M \leqslant N$.

(2) For even values of N:

The functions $h_1(x)$ and $h_2(x)$, which are defined by Eqs. (26) and (27), will be used in the computation of $G_N(a)$. The only difference between case (2) and case (1) is that Eq. (25) is adopted instead of Eq. (13). The computation of Eq. (25) requires additions twice more than that is needed in Eq. (13). Thus, the total computational complexity is approximately $O\left(M^2N^2\right)$ additions and $O\left(M^2N\right)$ multiplications.

3.3. Experimental results and comparison

The computational complexities of the proposed algorithm and the direct method are summarized in Table 1. From this table, we can see that the number of additions of the proposed method is in the worst case (N even and $M = N$) approximately twice of the direct method, but the number of multiplications is smaller, with a ratio of $3/N$ with regard to the direct method for $M \leqslant N$. For odd values of N, the number of additions of the proposed method is approximately the same as that of the direct method, but the number of multiplications decreases considerably. Table 2 shows the number of arithmetic operations and the CPU elapsed time of the two methods for some values of N and M (the program was implemented in C++ on PIII-M 1G, 384M). In order to further decrease the computation time for even values of N, the image can be zero-padded to achieve an odd N. Such a strategy was adopted by Yap et al. in the computation of Krawtchouk moments [11]. Fig. 3(a) shows the original grey level image of size 256×256. The reconstruction results with $M = 40$ of the original image and its zero-padded image of size 257×257 are depicted in Fig. 3(b) and (c), respectively. Fig. 3(d) shows the difference image, $\varepsilon(x, y)$, between the two reconstructed images where $\varepsilon(x, y)$ is defined as

$$\varepsilon(x, y) = \left| \widehat{f}_1(x, y) - \widehat{f}_2(x, y) \right|, \tag{31}$$

where $\widehat{f}_1(x, y)$ is the reconstructed result of original image and $\widehat{f}_2(x, y)$ is the reconstructed result of zero-padded image.

Note that in both cases, the reconstruction of the image is performed by using the following formula:

$$\hat{f}(x_i, y_j) = \sum_{p=0}^{M} \sum_{q=0}^{p} L_{p-q,q} P_{p-q}(x_i) P_q(y_j). \tag{32}$$

Table 1
Comparison of computational complexity for the two methods

		Addition	Multiplication
Direct method		$M^2N^2/2 \approx O(M^2N^2)$	$M^2N^2 \approx O(M^2N^2)$
Our method	N is even	$M^2N^2 + M^3N/6 \approx O(M^2N^2)$	$2M^2N + 2M^3/3 \approx O(M^2N)$
	N is odd	$M^2N^2/2 + M^3N/12 \approx O(M^2N^2)$	$2M^2N + 2M^3/3 \approx O(M^2N)$

Table 2
Comparison of computation time for the two methods

	Direct method			Our method		
	Addition	Multiplication	Time (ms)	Addition	Multiplication	Time (ms)
$N = 40$ $M = 40$	1 377 600	2 758 640	70	3 460 320	172 442	60
$N = 41$ $M = 40$	1 447 340	2 892 130	70	1 762 100	175 644	50
$N = 80$ $M = 40$	5 510 000	11 024 000	210	12 347 500	300 522	180
$N = 81$ $M = 40$	5 650 000	11 300 000	210	6 240 000	303 724	140
$N = 256$ $M = 40$	56 426 500	112 856 000	2113	115 256 000	864 074	1843
$N = 257$ $M = 40$	56 868 200	113 740 000	2103	57 891 400	867 276	1022

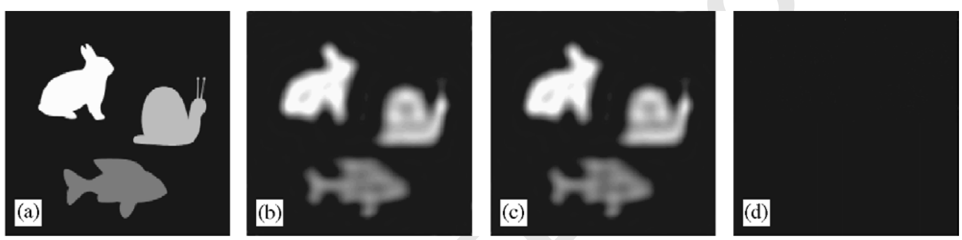

Fig. 3. Comparison of reconstruction results of the image with and without zero-padding ($M = 40$), (a) original image (256×256), (b) reconstruction result of original image, (c) reconstruction result of zero-padded image (257×257), and (d) error image $\varepsilon(x, y)$.

It can be seen from Fig. 3(d) that the two reconstructed images only have a slight difference, but the computation time required in the moment calculation process using the zero-padded strategy, which is 1022 ms (see Table. 2), is much shorter than that of the moment computation based on the original image, which is 1843 ms.

4. Conclusion

In this paper, a new fast algorithm for computing the 2D Legendre moments of grey level images has been presented. The proposed method has the following advantages:

(1) The 1D Legendre moments can be obtained by a recurrence relation. Moreover, the initial value used in the iterative method can be calculated with additions only.
(2) The 2D moment computation can be decomposed into two 1D moment calculations.
(3) It does not require as many multiplications as the direct method, thus leads to a better efficiency in terms of computational time.
(4) The algorithm can be implemented in parallel.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant no. 60272045 and Program for New Century Excellent Talents in University.

References

[1] M.K. Hu, Visual pattern recognition by moment invariants, IRE Trans. Inf. Theory 8 (1) (1962) 179–187.
[2] R.J. Prokop, A.P. Reeves, A survey of moment based techniques for unoccluded object representation, Graph. Models Image Process.—CVGIP 54 (5) (1992) 438–460.
[3] M.R. Teague, Image analysis via the general theory of moments, J. Opt. Soc. Am. 70 (8) (1980) 920–930.
[4] R. Mukundan, K.R. Ramakrishnan, Fast computation of Legendre and Zernike moments, Pattern Recognition 28 (9) (1995) 1433–1442.
[5] H.Z. Shu, L.M. Luo, W.X. Yu, Y. Fu, A new fast method for computing Legendre moments, Pattern Recognition 33 (2) (2000) 341–348.
[6] J.D. Zhou, H.Z. Shu, L.M. Luo, W.X. Yu, Two new algorithms for efficient computation of Legendre moments, Pattern Recognition 35 (5) (2000) 1143–1152.
[7] J. Martinez, F. Thomas, Efficient computation of local geometric moments, IEEE Trans. Image Process. 11 (9) (2002) 1102–1112.

[8] I.M. Spiliotis, B.G. Mertzios, Real-time computation of two-dimension moments on binary images by using image block representation, IEEE Trans. Image Process. 7 (11) (1998) 1609–1614.
[9] F.H.Y. Chan, F.K. Lam, An all adder systolic structure for fast computation of moments, J. VLSI Signal Process 12 (1996) 159–175.
[10] J. Gu, H.Z. Shu, C. Toumoulin, L.M. Luo, A novel algorithm for fast computation of Zernike moments, Pattern Recognition 35 (2002) 2905–2911.
[11] P.T. Yap, P. Raveendran, S.H. Ong, Image analysis by Krawtchouk moments, IEEE Trans. Image Process. 12 (11) (2003) 1367–1377.