Elemental numerical methods, 1h30

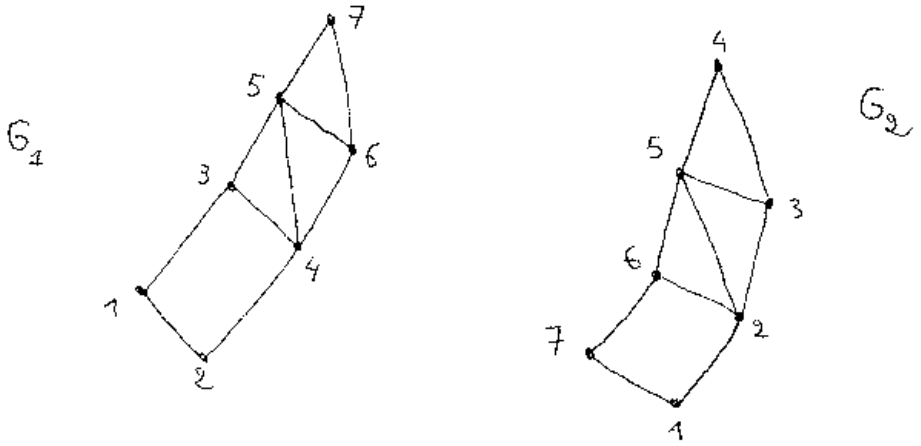
Exercise 1

Compute the LU decomposition of

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

Exercise 2

We consider the following graphs G_1 and G_2



To each graph, we associate a matrix $A^{(1)}$ or $A^{(2)}$ in the following way

$$A^{(k)}_{ii} = 2, \quad i = 1 \cdots 7,$$

and

$$A^{(k)}_{ij} = -1$$

if an edge joins the vertices i and j. The other terms are zero.

1. Construct the matrices $A^{(1)}$ and $A^{(2)}$.
2. How many memory values do you need for each matrix with the skyline storage ?
3. Conclusion ?

Exercise 3

We consider the following scheme for solving the differential equation $x'(t) = f(x(t))$:

$$x_{n+1} = x_n + \frac{\Delta t}{2} \left(f(x_n) + f(x_n + \Delta t f(x_n)) \right).$$

Prove that this scheme is of order 2.

Exercise 4

Prove that the Simpson's numerical integration method is exact for third order polynomial but not for fourth order polynomial.