Elemental numerical methods

S1

Numerical resolution of differential equations

1. We consider the following system of differential equations

$$\begin{aligned} x' &= ax - bxy \\ y' &= cxy - dy \end{aligned}$$

   where a, b, c, d are positive parameters. This system is a simple model for the evolution of prey-predator (x, y) populations. Justify this model and give realistic values to the four parameters.

2. Give the general equation of the solution curves in the (x, y) phase space. Verify that these curves are closed and that turn around the point $x = d/c, y = a/b$.

3. Recall the principles of the explicit and implicit Euler methods. Write a program that solves the system of differential equations by the explicit method. What time step Δt do you need to obtain an acceptable precision?

4. Plot x as a function of t. Check that if you increase the time step, then the scheme becomes unstable. Could you guess the value of the limit time step?

5. Write a program that solves the system of differential equations by the implicit method. Check numerically that it is unconditionally stable.

6. Write a program that solves the system of differential equations by the explicit improved Euler method. Compare the precision and the stability of this new scheme with the previous one.

7. Write a program that solves the system of differential equations by the RK4 method. Compare the precision and the stability of this new scheme with the previous one.

8. We slightly change the differential system with a small parameter $\varepsilon > 0$

$$\begin{aligned} x' &= ax - bxy - \varepsilon x^2 \\ y' &= cxy - dy \end{aligned}$$

   Give the value of ε that insures that the point $(x = 1000, y = 0)$ is a fixed point of the system. Plot the phase portrait of the modified system. Conclusion?