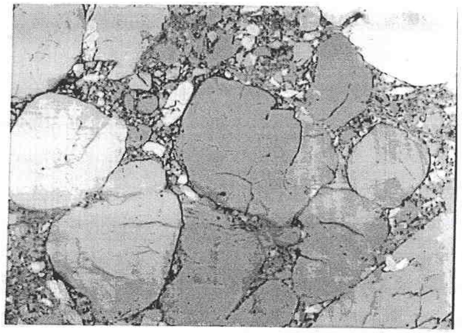
TRAVELING WAVE ANALYSIS OF TWO-PHASE DISSIPATIVE MODELS

Hervé Guillard,

INRIA Projet Smash, B.P. 93, 06902 Sophia-Antipolis Cedex, France,
Herve.Guillard@sophia.inria.fr

Joint work with :
- Mathieu Labois, Smash and CEA Cadarache
- Marc Grandotto, CEA Cadarache
- Vincent Perrier, IMB, Bordeaux

Two-phase flows



Granular medium : microphotography of HMX

Homogenized models : Averaged values of mass, momentum, energy of the two phases

Mixing zone where volume fraction $\in [0,1]$

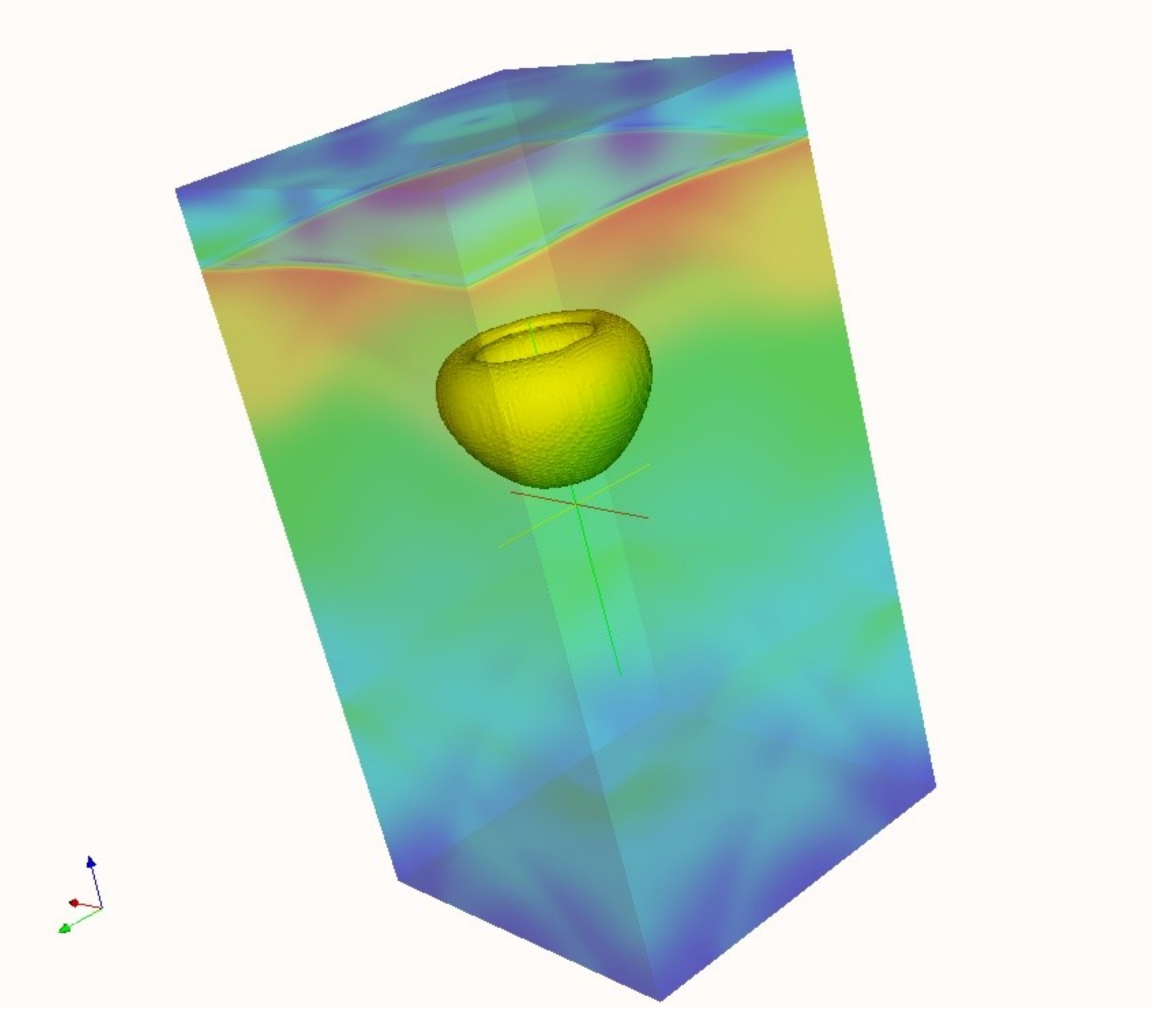
Multi-fluid problems :

--Accurate description of the interface between the two fluids

--No mixing zone

Y=0,1

--Pure fluids



However we require that the same models can be used for multiphase flows and multifluid problems

OUTLINES

- short review of two-phase flow models
    - entropy equation
    - relaxation towards equilibrium
    - hierarchy of two-phase models
- 5 equation one-velocity, one-pressure model
    - mathematical properties of this model
    - drawbacks
    - introduction of dissipative terms
    - Analysis of traveling waves (isothermal case)

Averaging procedure (Drew-Passman, Ishii textbooks) :

Statistical, volume, time averages :

$$Q_k(x,t)=\int_{\omega} 1_k\, q(x,t)\, d\omega$$

where 1_k is the characteristic function of the phase k

applying this procedure to mass, momentum and energy

of the two phases :

THE NON-EQUILIBRIUM TWO FLUID MODEL

Two mass conservation equations

$$\frac{\partial \alpha_k \rho_k}{\partial t} + div \alpha_k \rho_k \boldsymbol{u}_k = \Gamma_k \quad \text{for} \quad k = 1, 2$$

Two momentum equations

$$\frac{\partial \alpha_k \rho_k \boldsymbol{u}_k}{\partial t} + div(\alpha_k \rho_k \boldsymbol{u}_k \otimes \boldsymbol{u}_k) + \nabla(\alpha_k p_k) = p_I \nabla \alpha_k + \boldsymbol{u}^\Gamma \Gamma_k + \boldsymbol{M}_k^d$$

Two energy conservation equations

$$\frac{\partial \alpha_k \rho_k e_k}{\partial t} + div \alpha_k (\rho_k e_k + p_k) \boldsymbol{u}_k = p_I \frac{\partial \alpha_k}{\partial t} + h_k^\Gamma \Gamma_k + \boldsymbol{u}_I . \boldsymbol{M}_k^d + Q_I$$

Models for : $\Gamma_k, p_I, u^\Gamma, M_k^d, u_I, Q_I$

$$e.g\; M_k^d = \frac{\lambda}{\epsilon}(u_1 - u_2)$$

How to construct these models ?

Use the entropy equation :

$$
\begin{aligned}
\frac{\partial \rho s}{\partial t} + div(\alpha_1\rho_1 s_1 \boldsymbol{u}_1 + \alpha_2\rho_2 s_2 \boldsymbol{u}_2) \;\; &= \frac{p_1 - p_I}{T_1}\frac{D_1\alpha_1}{Dt} + \frac{p_2 - p_I}{T_2}\frac{D_2\alpha_2}{Dt} \\
&+[(\frac{1}{T_1} - \frac{1}{T_2})h_1^\Gamma - (\frac{\boldsymbol{u}_1}{T_1} - \frac{\boldsymbol{u}_2}{T_2})\boldsymbol{u}_\Gamma \\
&+(\frac{|\boldsymbol{u}_1|^2}{2T_1} - \frac{|\boldsymbol{u}_2|^2}{2T_2})]\Gamma_1 \\
&+[\frac{\boldsymbol{u}_I - \boldsymbol{u}_1}{T_1} - \frac{\boldsymbol{u}_I - \boldsymbol{u}_2}{T_2}]M_1^d \\
&+[\frac{1}{T_1} - \frac{1}{T_2}]Q_1 \\
&+[\frac{g_1}{T_1} - \frac{g_2}{T_2}]\Gamma_1
\end{aligned}
$$

Assume :

$$\frac{\partial \alpha_k}{\partial t} + u_\alpha \nabla \alpha_k = \dot{\alpha_k}$$

Then first line :

$$\left(\frac{p_1 - p_I}{T_1} - \frac{p_2 - p_I}{T_2}\right)\dot{\alpha_1} + \left(\frac{p_1 - p_I}{T_1}(u_1 - u_\alpha) - \frac{p_2 - p_I}{T_2}(u_2 - u_\alpha)\right)\nabla \alpha_1$$

<u>One important remark (Coquel, Gallouet,Herard, Seguin) </u>:

The two-fluid system + volume fraction equation is (always) hyperbolic

but

the field associated with the eigenvalue u_α

is linearly degenerate if and only if

$$u_\alpha \in \{u_1, u_2, u = Y_1 u_1 + Y_2 u_2\}$$

Final form of the entropy equation :

$$\frac{\partial \rho s}{\partial t} + div(\alpha_1 \rho_1 s_1 \underline{u}_1 + \alpha_2 \rho_2 s_2 \underline{u}_2) = (\frac{p_1 - p_I}{T_1} - \frac{p_2 - p_I}{T_2})\dot{\alpha}_1$$

$$+[\frac{\underline{u}_I - \underline{u}_1}{T_1} - \frac{\underline{u}_I - \underline{u}_2}{T_2}] M_1^d$$

$$+[\frac{1}{T_1} - \frac{1}{T_2}] Q_1$$

$$+[\frac{g_1}{T_1} - \frac{g_2}{T_2}] \Gamma_1$$

Simplest form ensuring positive entropy production :

$$\dot{\alpha}_1 = \lambda_p \frac{p_1 - p_2)}{\varepsilon_p} \qquad (15.1)$$

$$M_1^d = \lambda_u \frac{(\underline{u}_2 - \underline{u}_1)}{\varepsilon_u} \qquad (15.2)$$

$$Q_1 = \lambda_T \frac{T_2 - T_1}{\varepsilon_T} \qquad (15.3)$$

$$\Gamma_1 = \frac{\lambda_g}{\varepsilon_g}\left[\frac{g_2}{T_2} - \frac{g_1}{T_1}\right] \qquad (15.4)$$

A little Summary

- Two fluid system + volume fraction eq = hyperbolic system
  the entropy production terms are positive

- This system evolves to a state characterized by
  - pressure equality
  - velocity equality
  - temperature equality
  - chemical potential equality

⇒ Deduce from this system, several reduced systems characterized by <u>instantaneous equilibrium</u> between
- pressure
- pressure + velocity
- pressure + velocity + temperature
- pressure + velocity + temperature + chemical potential

Assume pressure equilibrium : Classical two-fluid model

(widely used in nuclear industry : Cathare, RELAPS, etc...)

$$\left\{\begin{array}{lcl}
\dfrac{\partial \alpha_1\rho_1}{\partial t} + \operatorname{div}\left(\alpha_1\rho_1\boldsymbol{u}_1\right) & = & 0 \\[2ex]
\dfrac{\partial \alpha_2\rho_2}{\partial t} + \operatorname{div}\left(\alpha_2\rho_2\boldsymbol{u}_2\right) & = & 0 \\[2ex]
\dfrac{\partial \alpha_1\rho_1\boldsymbol{u}_1}{\partial t} + \operatorname{div}\left(\alpha_1\rho_1\boldsymbol{u}_1 \otimes \boldsymbol{u}_1\right) + \alpha_1\nabla p & = & 0 \\[2ex]
\dfrac{\partial \alpha_2\rho_2\boldsymbol{u}_2}{\partial t} + \operatorname{div}\left(\alpha_2\rho_2\boldsymbol{u}_2 \otimes \boldsymbol{u}_2\right) + \alpha_2\nabla p & = & 0 \\[2ex]
\dfrac{\partial \alpha_1\rho_1 e_1}{\partial t} + \operatorname{div}\alpha_1\left(\rho_1 e_1 + p\right)\boldsymbol{u}_1 & = & 0 \\[2ex]
\dfrac{\partial \alpha_2\rho_2 e_2}{\partial t} + \operatorname{div}\alpha_2\left(\rho_2 e_2 + p\right)\boldsymbol{u}_2 & = & 0
\end{array}\right.$$

eos : solve p1 = p2 for the volume fraction

Assume - pressure equilibrium
- velocity equilibrium

(Stewart-Wendroff 1984, Kapila et al 2001, Murrone-Guillard 2005)

$$\left\{\begin{array}{rcl}
\dfrac{\partial \alpha_1\rho_1}{\partial t} + \mathrm{div}(\alpha_1\rho_1\boldsymbol{u}) & = & 0 \\[2ex]
\dfrac{\partial \alpha_2\rho_2}{\partial t} + \mathrm{div}(\alpha_2\rho_2\boldsymbol{u}) & = & 0 \\[2ex]
\dfrac{\partial \rho\boldsymbol{u}}{\partial t} + \mathrm{div}(\rho\boldsymbol{u}\otimes\boldsymbol{u}) + \nabla p & = & 0 \\[2ex]
\dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\boldsymbol{u} & = & 0 \\[2ex]
\dfrac{\partial \alpha_2}{\partial t} + \boldsymbol{u}.\nabla\alpha_2 & = & \alpha_1\alpha_2\dfrac{\rho_1 a_1^2 - \rho_2 a_2^2}{\sum_{k=1}^{2}\alpha_{k'}\rho_k a_k^2}\mathrm{div}\boldsymbol{u}
\end{array}\right.$$

Assume - pressure equilibrium

- velocity equilibrium
- temperature equilibrium

Multi-component Euler equations :

$$\left\{\begin{array}{lcl}
\dfrac{\partial \alpha_1 \rho_1}{\partial t} + \mathrm{div}(\alpha_1 \rho_1 \boldsymbol{u}) & = & 0 \\[2ex]
\dfrac{\partial \alpha_2 \rho_2}{\partial t} + \mathrm{div}(\alpha_2 \rho_2 \boldsymbol{u}) & = & 0 \\[2ex]
\dfrac{\partial \rho \boldsymbol{u}}{\partial t} + \mathrm{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p & = & 0 \\[2ex]
\dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\boldsymbol{u} & = & 0
\end{array}\right.$$

eos : solve : p1 = p2, T1=T2

Assume - pressure equilibrium

- velocity equilibrium
- temperature equilibrium
- chemical potential equilibrium

Euler equations (Homogeneous equilibrium model)

$$\partial_t \rho + \partial_x \rho u = 0$$

$$\partial_t \rho u + \partial_x \rho u^2 + p = 0$$

$$\partial_t \rho e + \partial_x (\rho e + p) u = 0$$

eos : solve p1 = p2, T1=T2, g1=g2

OUTLINES

- short review of two-phase flow models
    - entropy equation
    - relaxation towards equilibrium
    - hierarchy of two-phase models
- 5 equation one-velocity, one-pressure model
    - mathematical properties of this model
    - drawbacks
    - introduction of dissipative terms
    - Analysis of traveling waves (isothermal case)

The iso-pressure iso-velocity model :

$$\left\{\begin{array}{rcl}
\dfrac{\partial \alpha_1\rho_1}{\partial t} + \mathrm{div}(\alpha_1\rho_1\boldsymbol{u}) & = & 0\\[2ex]
\dfrac{\partial \alpha_2\rho_2}{\partial t} + \mathrm{div}(\alpha_2\rho_2\boldsymbol{u}) & = & 0\\[2ex]
\dfrac{\partial \rho\boldsymbol{u}}{\partial t} + \mathrm{div}(\rho\boldsymbol{u}\otimes\boldsymbol{u}) + \nabla p & = & 0\\[2ex]
\dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\boldsymbol{u} & = & 0\\[2ex]
\dfrac{\partial \alpha_2}{\partial t} + \boldsymbol{u}.\nabla\alpha_2 & = & \alpha_1\alpha_2\dfrac{\rho_1 a_1^2 - \rho_2 a_2^2}{\sum_{k=1}^2 \alpha_{k'}\rho_k a_k^2}\mathrm{div}\boldsymbol{u}
\end{array}\right.$$

Mathematical properties of the one-pressure, one velocity model (5 eq model)

Hyperbolic system

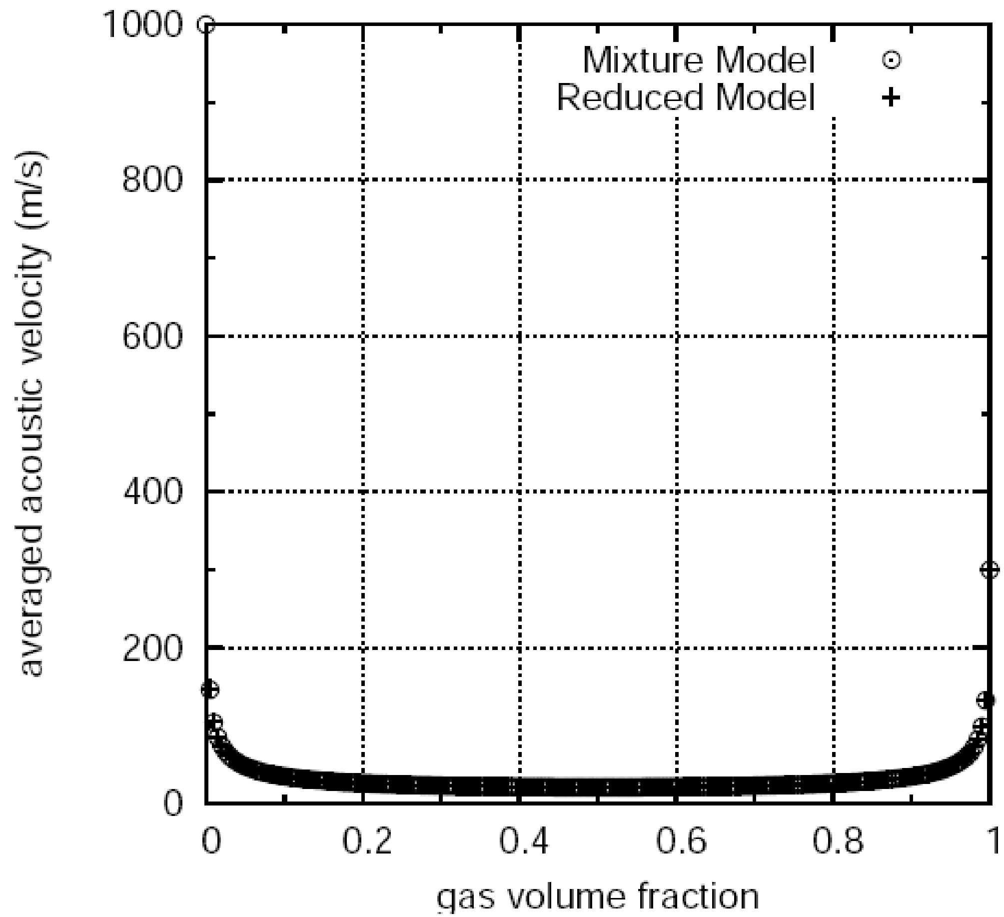
$$u-a\ ,u,u,u,u+a$$

$$with\ \frac{1}{\rho a^2}=\frac{\alpha_1}{\rho_1 a_1^2}+\frac{\alpha_2}{\rho_2 a_2^2}$$

$u-a$ gnl field

$u+a$ gnl field,

u ld field



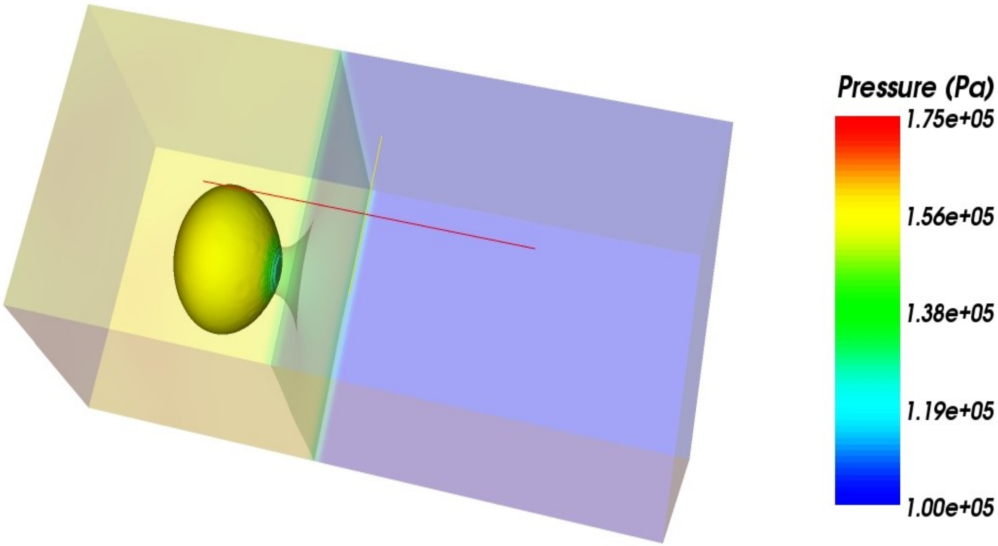
Mathematical properties of the one-pressure, one velocity model (5 eq model)

Entropy properties : 2 independent entropies that verify

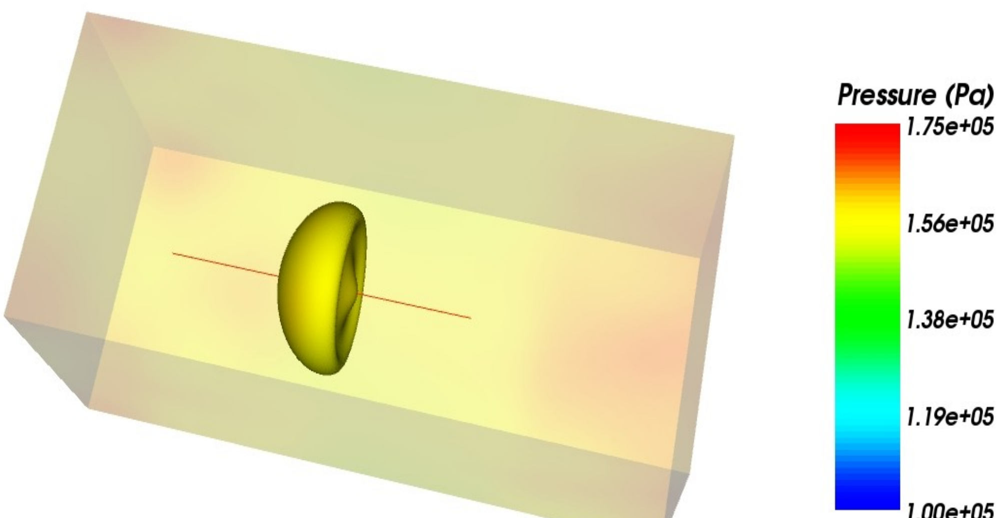
$$\partial_t s_1 + u.\nabla s_1 = 0$$

$$\partial_t s_2 + u.\nabla s_2 = 0$$

Example of multifluid application of the 5 equation model : Shock-bubble interaction at Ma =2



200 ms

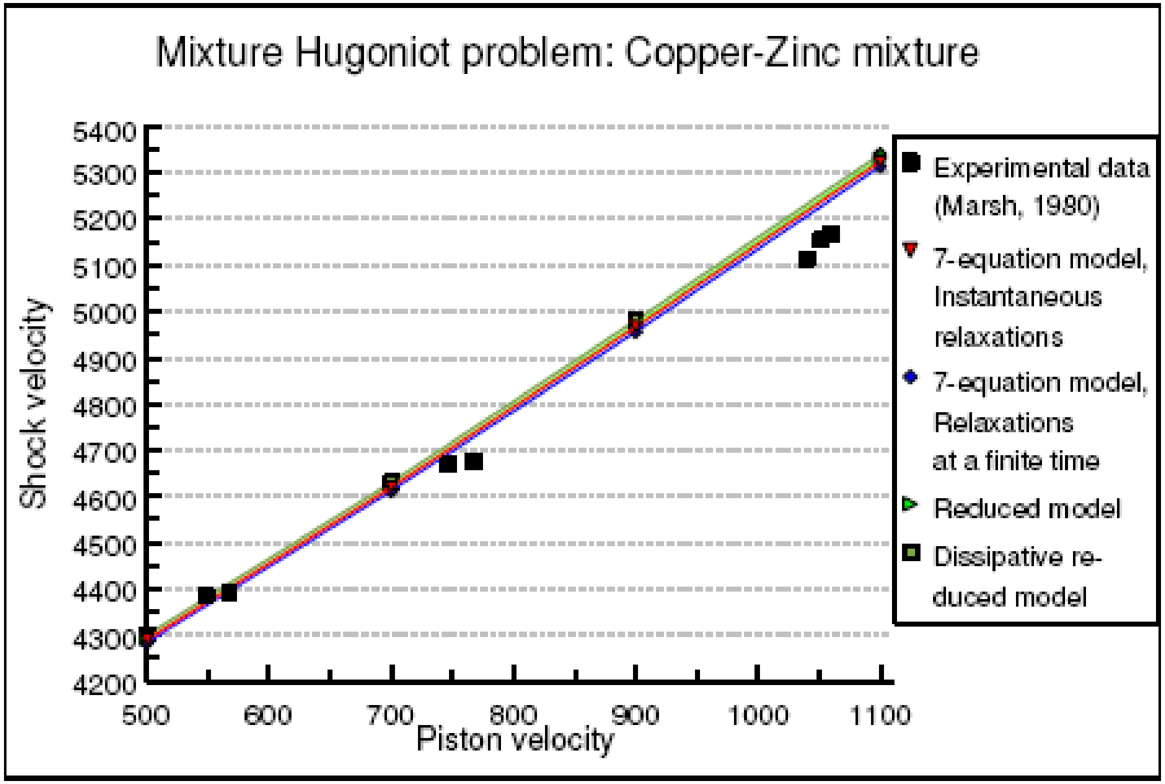


700 ms

Mesh 18 M nodes
128 procs

Example of two-phase application of the 5 equation model

Piston impact on two-phase mixture



Weak points of the iso-pressure iso-velocity model :

► non conservative equation : Shock solution are not defined

► cannot take into account velocity disequilibrium

AMOVI experiment (CEA Saclay)

pressure equilibrium time $10^{-8} -- 10^{-7}\, s$

velocity equilibrium time $10^{-3} -- 4.10^{-2}\, s$

bubble rising time $4.5\, 10^{-2} -- 3.3\, 10^{-1}\, s$

► Introduction of dissipative effects in the model

First weak point of the 5 eq model: Non conservative form

Shock solutions are not defined

One answer : LeFloch, Raviart-Sainsaulieu

change
$$\frac{\partial Q}{\partial t} + A(Q)\frac{\partial Q}{\partial x} = 0$$

into
$$\frac{\partial Q}{\partial t} + A(Q)\frac{\partial Q}{\partial x} = \epsilon\frac{\partial}{\partial x}(D(Q)\frac{\partial Q}{\partial x})$$

Define the shock solutions as limits of TW solutions of the regularized dissipative system for $\epsilon \longrightarrow 0$

Drawback of the approach : the limit solution depends on the dissipative tensor $D(Q)$

One simple example : 2-fluid shallow water system (Castro et al)

$$\partial_t u^\epsilon + u^\epsilon \partial_x (u^\epsilon + v^\epsilon) = \epsilon \delta_1 \partial_{xx} (u^\epsilon + v^\epsilon) , \delta_1 > 0$$

$$\partial_t v^\epsilon + v^\epsilon \partial_x (u^\epsilon + v^\epsilon) = \epsilon \delta_2 \partial_{xx} (u^\epsilon + v^\epsilon) , \delta_2 > 0$$

Travelling wave solutions

$$\vec{w}_\epsilon (x, t) = \bar{w}(x - st) \quad avec \quad \lim \xi \to \pm\infty \, \bar{w}(\xi) = w_\pm$$

$$\bar{w}_\epsilon (\xi) = \bar{w}(\xi / \epsilon) \to \vec{w}_0 \in L^1_{loc}$$

$$\vec{w}_0 (x, t) = \left\{ \begin{array}{l} w_- \;\; if \; x < st \\ w_+ \;\; if \; x > sts \end{array} \right\}$$

$$Let \quad w_- \;\; , \;\; s \quad be\, given\, then$$

$$v_+ = \frac{\delta_2}{\delta_2 + \delta_1} (2\mathrm{s} - u_- - v_-) + \frac{\delta_1 v_- \delta_2 u_-}{\delta_2 + \delta_1} \, \mathrm{e}^{(2 - 2(u_- + v_-)/s)}$$

$$u_+ = 2s - u_- - (v_- + v_+)$$

How to be sure that the dissipative tensor $D(Q)$ encode the right physical informations ?

In many works <u>it is assumed</u> that the dissipative tensor is of viscous type :

$$\partial_t \rho u + \partial_x (\rho u^2 + p) = \partial_x (\mu \partial_x u)$$

but why ?

In two-phase flows, relaxation toward mechanical (and thermodynamical) equilibrium is the most important phenomenon ---> dissipative tensor must include this effect

Equilibrium (one-pressure,one-velocity) model =
limit model for pressure and velocity relaxation times = 0

Now consider pressure and velocity relaxation times
are small but non zero

then

Consider instead of the zero-order expansion,
the first order Chapman-Enskog asymptotic expansion
of the non-equilibrium model

CHAPMAN-ENSKOG EXPANSION

$$\frac{\partial \boldsymbol{U}}{\partial t} + A(\boldsymbol{U})\frac{\partial \boldsymbol{U}}{\partial x} = \frac{R(\boldsymbol{U})}{\varepsilon}$$

when $\varepsilon \to 0$ we expect $\boldsymbol{U}$ to be close to

$$\mathcal{E} = \{\boldsymbol{U} \in I\!R^N; R(\boldsymbol{U}) = 0\}$$

Assumption 1 : The set of equations $R(\boldsymbol{U}) = 0$ defines a smooth manifold of dimension n. Moreover, for any $\boldsymbol{U} \in \mathcal{E}$ we *explicitly* know a parametrization M from ω an open subset of $I\!R^n$ on V a neighborhood of $\boldsymbol{U}$ in $\mathcal{E}$.

CHAPMAN-ENSKOG EXPANSION

$$\boldsymbol{U} = M(\mathbf{u}) + \varepsilon \boldsymbol{V}$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + A(\boldsymbol{U})\frac{\partial \boldsymbol{U}}{\partial x} = \frac{R(\boldsymbol{U})}{\varepsilon}$$

becomes :

$$\frac{\partial M(\mathbf{u})}{\partial t} + A(M(\mathbf{u}))\frac{\partial M(\mathbf{u})}{\partial x} - R'(M(\mathbf{u})).\boldsymbol{V}$$

$$+\varepsilon[\frac{\partial \boldsymbol{V}}{\partial t} + A(M(\mathbf{u}))\frac{\partial \boldsymbol{V}}{\partial x} + [\frac{\partial A}{\partial \boldsymbol{U}_i}\boldsymbol{V}_i]\frac{\partial M(\mathbf{u})}{\partial x} - \frac{1}{2}R''(M(\mathbf{u}))(\boldsymbol{V},\boldsymbol{V})] = \mathcal{O}(\varepsilon^2)$$

$$\boxed{\boldsymbol{V} \in \mathrm{Rng}(\mathrm{R}'(\mathrm{M}(\boldsymbol{v})))}$$

Let $I\!P$ and $Q\!\!\!\!Q$ be respectively the projection on $\ker(R'(M(\boldsymbol{q})))$ in the direction of $\mathrm{Rng}(R'(M(\boldsymbol{q})))$ and the projection on $\mathrm{Rng}(R'(M(\boldsymbol{q})))$ in the direction of $\ker(R'(M(\boldsymbol{q})))$.

$$\frac{\partial \boldsymbol{v}}{\partial t} + I\!P A(M(\boldsymbol{v}))\frac{\partial M(\boldsymbol{v})}{\partial x}$$

$$+\varepsilon I\!P[\frac{\partial \boldsymbol{V}}{\partial t} + A(M(\boldsymbol{v}))\frac{\partial \boldsymbol{V}}{\partial x} + [\frac{\partial A}{\partial \mathbf{Q}_i}\boldsymbol{V}_i]\frac{\partial M(\boldsymbol{v})}{\partial x} - \frac{1}{2}R''(M(\boldsymbol{v}))(\boldsymbol{V},\boldsymbol{V})] = \mathcal{O}(\varepsilon^2) \tag{14}$$

$$Q\!\!\!\!Q R'(M(\boldsymbol{v})).\boldsymbol{V} = Q\!\!\!\!Q A(M(\boldsymbol{v})).\frac{dM(\boldsymbol{v})}{d\boldsymbol{v}}\frac{\partial \boldsymbol{v}}{\partial x} + \mathcal{O}(\varepsilon) \Longrightarrow \boldsymbol{V} = \mathcal{D}(v)\frac{\partial \boldsymbol{v}}{\partial x} + \mathcal{O}(\varepsilon)$$

$$\boxed{\begin{aligned}&\frac{\partial \boldsymbol{v}}{\partial t} + I\!P A(M(\boldsymbol{v}))\frac{\partial M(\boldsymbol{v})}{\partial x} = -\varepsilon I\!P[A(M(\boldsymbol{v}))\frac{\partial}{\partial x}(\mathcal{D}(v)\frac{\partial \boldsymbol{v}}{\partial x})\\ &+[\frac{\partial A}{\partial \mathbf{Q}_i}(\mathcal{D}(v)\frac{\partial \boldsymbol{v}}{\partial x})_i]\frac{\partial M(\boldsymbol{v})}{\partial x} - \frac{1}{2}R''(M(\boldsymbol{v}))(\mathcal{D}(v)\frac{\partial \boldsymbol{v}}{\partial x},\mathcal{D}(v)\frac{\partial \boldsymbol{v}}{\partial x}))]\end{aligned}}$$

Dissipative iso-pressure iso-velocity model

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}\rho u &= 0 \\
\frac{\partial}{\partial t}(\rho Y_2) + \frac{\partial}{\partial x}(\rho Y_2 u) - \frac{\partial}{\partial x}J_2 &= 0 \\
\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho (u)^2 + p) &= 0 \\
\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x}(\rho e + p)u - \frac{\partial}{\partial x}(h_1 J_1 + h_2 J_2) &= 0 \\
\frac{D\alpha_2}{Dt} - \alpha_1\alpha_2 \frac{C_1 - C_2}{\alpha_1 C_2 + \alpha_2 C_1}\frac{\partial u}{\partial x} &= \cancel{0}\ \mathrm{A}
\end{aligned}
$$

$$J_k = \chi(\tau_k - \tau_k')\frac{\partial p}{\partial x} \quad \text{with} \quad \chi = \frac{(\rho Y_1 Y_2)^2}{\lambda} > 0$$

$$
\begin{aligned}
\mathrm{A} = {} & ((Y_1\alpha_1 C_2 - Y_2\alpha_2 C_1)u_r . \nabla\alpha_1)/(\alpha_1 C_2 + \alpha_2 C_1) \\
& -(\alpha_1\alpha_2(C_1 \, div(Y_2 u_r) + C_2 \, div(Y_1 u_r))/(\alpha_1 C_2 + \alpha_2) C_1) \\
& -\alpha_1\alpha_2(\ldots) u_r . \nabla p/(\alpha_1 C_2 + \alpha_2 C_1)
\end{aligned}
$$

$$u_r = u_2 - u_1 = \epsilon_u \frac{Y_1 - \alpha_1}{\rho}\nabla p$$

Darcy law for velocity disequilibrium

Dissipative iso pressure iso velocity model

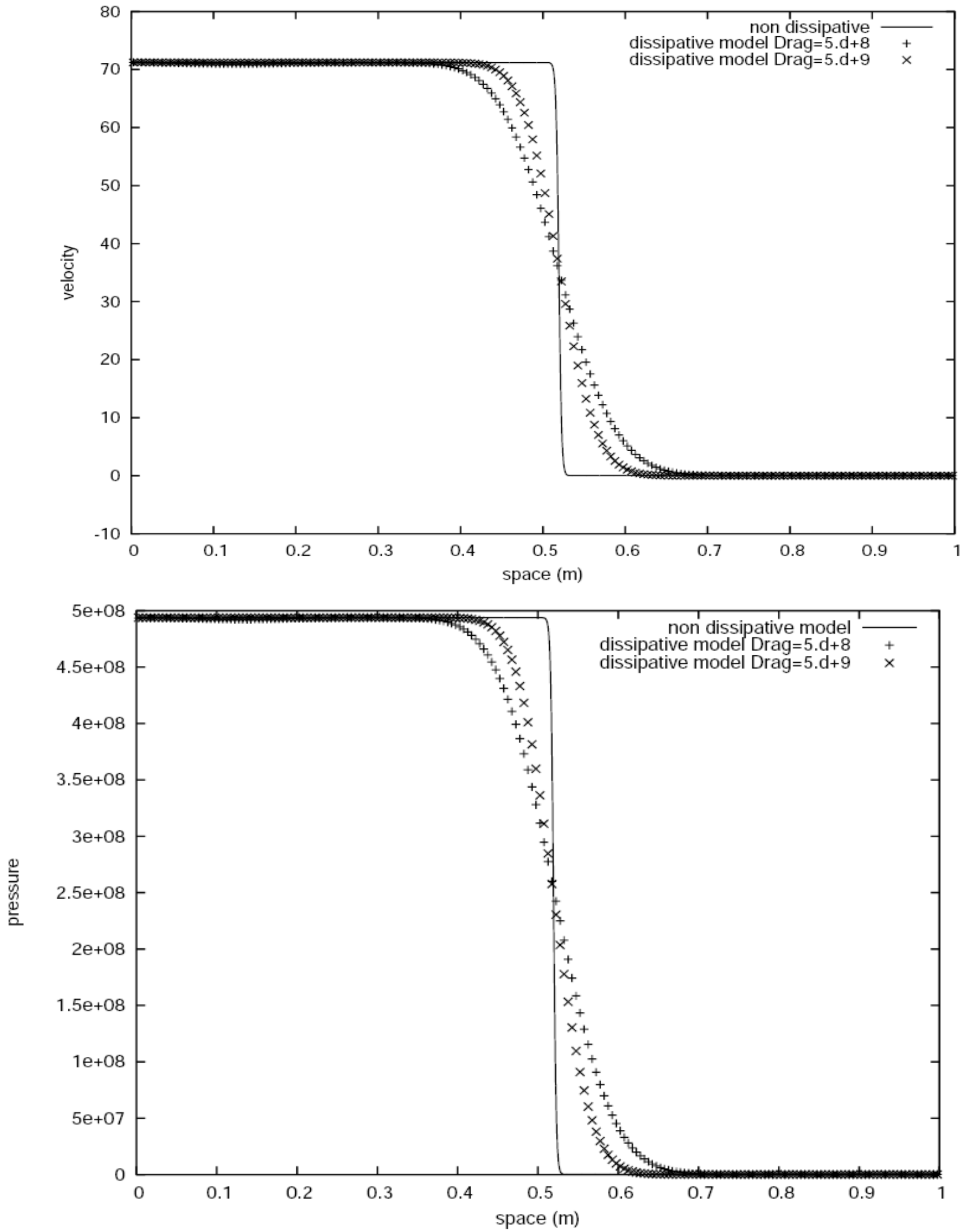
-- form of the dissipative tensor (Guillard and Duval JCP 2007)

Darcy Law for the drift velocity

$$u_r = u_2^1 - u_1^1 = \rho Y_1 Y_2 (\alpha_2 - Y_2) \nabla p$$

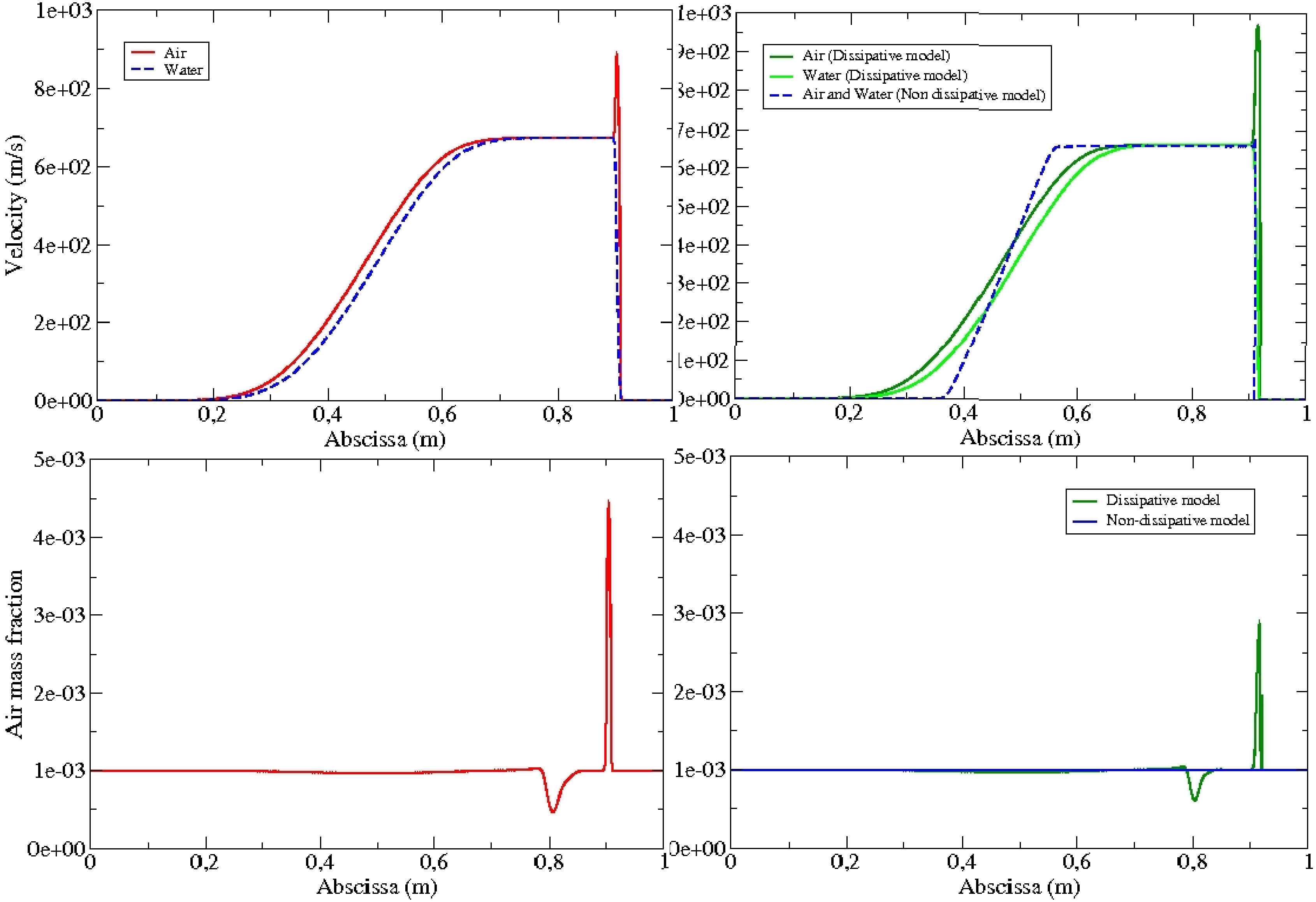
--Not of viscous type ! (different from Navier-Stokes regularization)

--For hydrostatic pressure field, recover known drift formula (e.g Stokes)



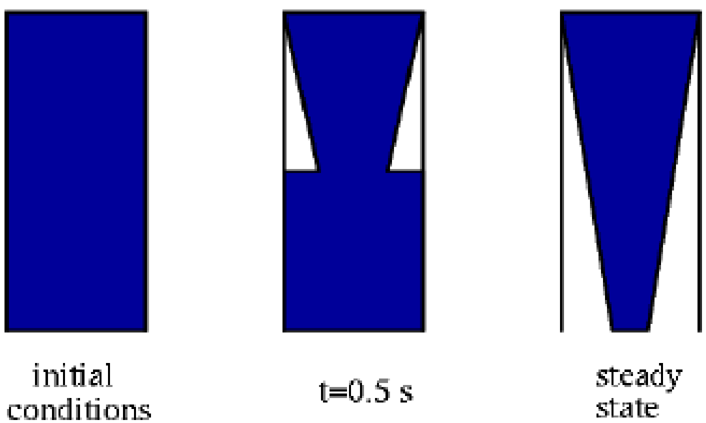
Convergence of travelling waves solutions of the 5eqs dissipative model toward shock solutions

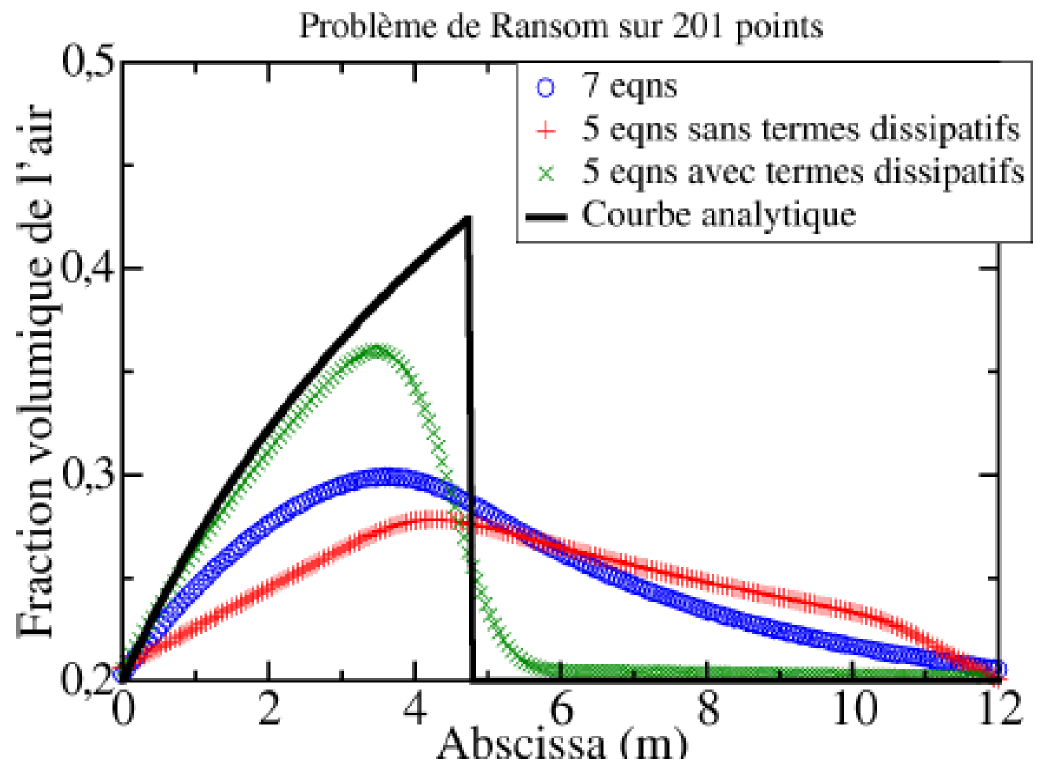
Non-equilibrium model (Baer-Nunziato) Equilibrium model



Ransom water faucet test case

velocity of air $\neq$ velocity of water

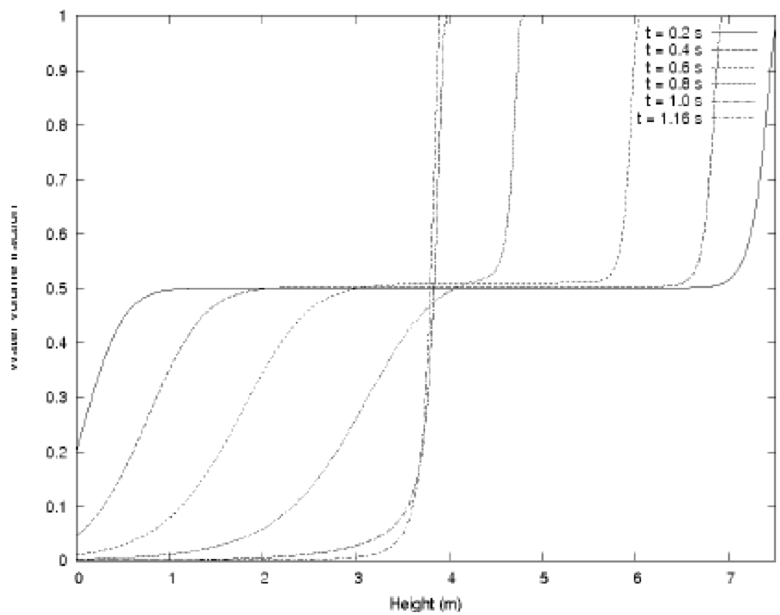




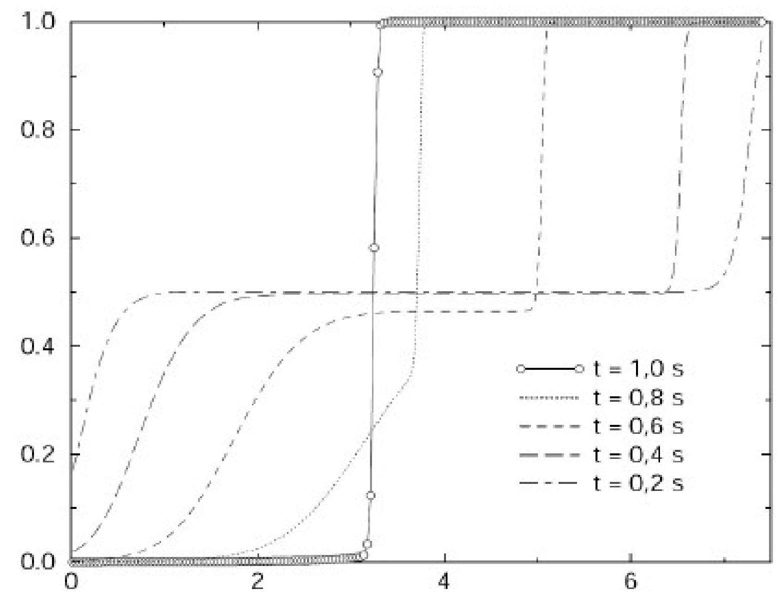
On the same mesh : better results than for the non-equilibrium model

Sedimentation test case

counter current



Dissipative model



Non-equilibrium model

(Gallouët, Hérard & Seguin, 2004)

OUTLINES

- short review of two-phase flow models
  - entropy equation
  - relaxation towards equilibrium
  - hierarchy of two-phase models
- 5 equation one-velocity, one-pressure model
  - mathematical properties of this model
  - drawbacks
  - introduction of dissipative terms
  - Analysis of traveling waves (isothermal case)

ISOTHERMAL MODEL with DARCY-LIKE DRIFT LAW

$$\frac{\partial \rho_m}{\partial t} + div \rho_m \boldsymbol{u}_m = 0$$

$$\frac{\partial \rho_m Y_2}{\partial t} + div \rho_m Y_2 \boldsymbol{u}_m = -div \rho_m Y_2 Y_1 \boldsymbol{u}_r$$

$$\frac{\partial \rho_m \boldsymbol{u}_m}{\partial t} + div[\rho_m \boldsymbol{u}_m \otimes \boldsymbol{u}_m] + \nabla p = -\rho_m g \boldsymbol{k}$$

$$\lambda \boldsymbol{u}_r = \alpha_1 \alpha_2 \frac{(\rho_2 - \rho_1)}{\rho_m} \nabla p$$

ISOTHERMAL MODEL with DARCY-LIKE DRIFT LAW

$$U_t + f(U)_x = \epsilon (D(U) U_x)_x$$

The system is hyperbolic

$$u-a,\ u\ , u+a \quad \text{with} \quad \frac{1}{\rho a^2} = \frac{\alpha_1}{\rho_1 a_1^2} + \frac{\alpha_2}{\rho_2 a_2^2} \quad (\textit{Wood sound speed})$$

u+a, u-a, gnl, u ld

Entropy : $\eta = \frac{\rho v^2}{2} + \sum \alpha_k \rho_k f(\rho_k) \ \text{with} \ f'(\rho_k) = p_k / \rho_k^2$

Diffusion tensor D is strictly dissipative for the entropy

$$\partial_t \eta + \partial_x G(U) - \partial_x H(U, U') = -\alpha_1 \alpha_2 \frac{(\rho_2 - \rho_1)^2}{\rho^2} (p')^2 < 0$$

ISOTHERMAL MODEL with DARCY-LIKE DRIFT LAW

$$U_t + f(U)_x = \epsilon (D(U) U_x)_x$$

Rankine-Hugoniot Relations :

$$[\rho u] = s[\rho]$$

$$[\rho u^2 + p] = s[\rho u]$$

$$[\rho Y u] = s[\rho Y]$$

$$if \quad M = \rho(u - s) \neq 0 \quad (shock) \quad then \quad [Y] = 0$$

TRAVELLING WAVE SOLUTIONS

$$\frac{\partial \rho_m}{\partial t} + div \rho_m \boldsymbol{u}_m = 0$$

$$\frac{\partial \rho_m Y_2}{\partial t} + div \rho_m Y_2 \boldsymbol{u}_m = -div \rho_m Y_2 Y_1 \boldsymbol{u}_r$$

$$\frac{\partial \rho_m \boldsymbol{u}_m}{\partial t} + div[\rho_m \boldsymbol{u}_m \otimes \boldsymbol{u}_m] + \nabla p = -\rho_m g \boldsymbol{k}$$

Travelling waves : $U = \tilde{U}(x - st)$

Ode system :

$$(\rho(u-s))' = 0$$

$$(\rho u(u-s) + p)' = 0$$

$$(\rho Y(u-s))' = (\rho Y(1-Y)(\alpha - Y)p')'$$

TRAVELLING WAVES II

Integrating the mass and momentum eqs :

$$\rho(u-s)=M= \; constant$$

$$M^2=\frac{-(p-p_R)}{(\tau-\tau_R)} \; = \; \frac{z}{(\tau-\tau_R)}$$

Compressive waves : Lax condition for a right going wave

$$u_R+a_R<s<u_L+a_L$$
$$u_L<s$$

gives

$$M=\rho(u-s)<0$$
$$\tau_R M^2>\rho_R a_R^2=p_R$$

Moreover using (1) &(2) & eos gives

$$Y=Y_R+\frac{z(M^2\tau_R-p_R-z)}{M^2(a_2^2-a_1^2)}=Y_R+\frac{z(z_L-z)}{M^2(a_2^2-a_1^2)}$$

TRAVELLING WAVES III

Now, integrate the mass fraction equation :

$$\rho Y(1-Y)(\alpha(Y)-Y)z'=\frac{z(z_L-z)}{M(a_2^2-a_1^2)}$$

2 equilibrium points : z = 0 and z = z_L

Stability of the equilibrium

$$z=0 \qquad z'=\frac{z_L}{M(a_2^2-a_1^2)f(0)}z \qquad \textit{Stable}$$

$$z=z_L \qquad z'=\frac{-z_L}{M(a_2^2-a_1^2)f(z_L)}z \qquad \textit{Unstable}$$

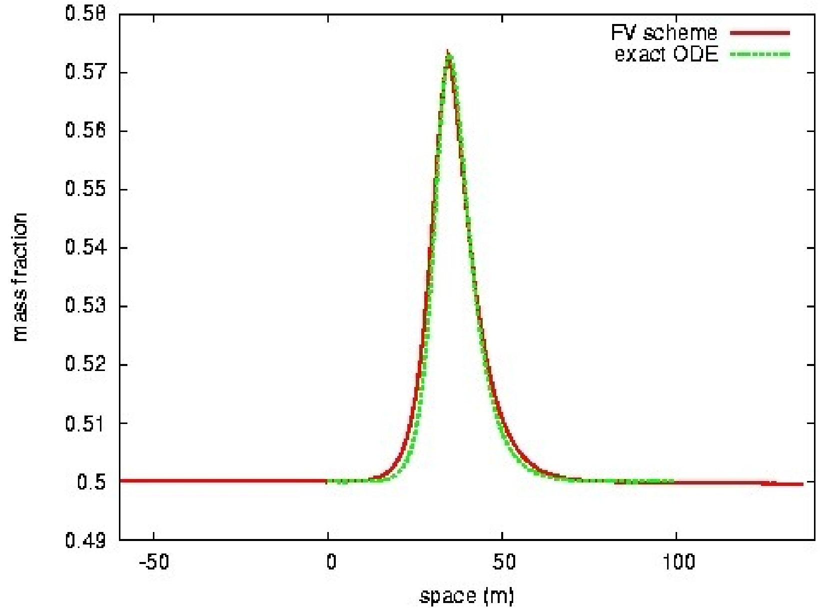
TRAVELLING WAVES IV

First case : weak shock

$$if \quad M^2 < M^2_{crit}$$

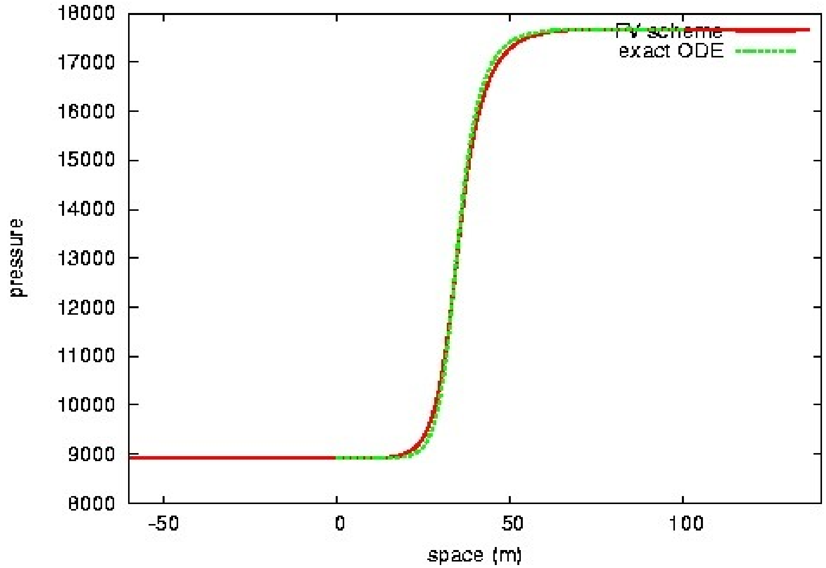
$$Y = Y_R + \frac{z(z_L - z)}{M^2(a_2^2 - a_1^2)} \quad \in [Y_R, 1]$$

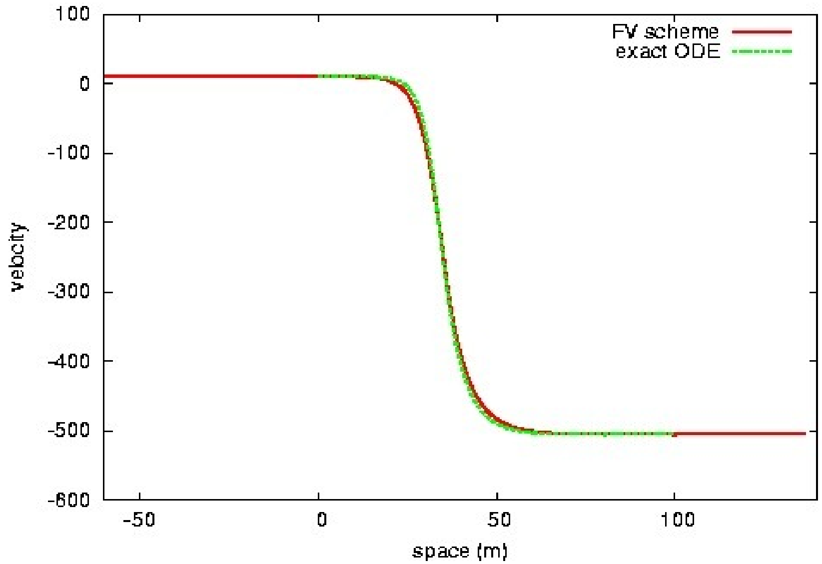
The ode is never singular and it exists a unique C1 solution connecting z = 0 and z = z_L



Mass fraction is <u>not</u> constant in a shock !

Effect not predicted by viscous regularization





TRAVELLING WAVES V

2nd case : strong shock

$$if \quad M^2 > M^2_{crit} \quad then$$

$$\exists z^0_R, \; z^0_L \;\; with \;\; 0 < z^0_R < z^0_L < z_L \quad such \;\; that$$

$$Y(z^0_R) = z^0_L = 1$$

The ode becomes singular

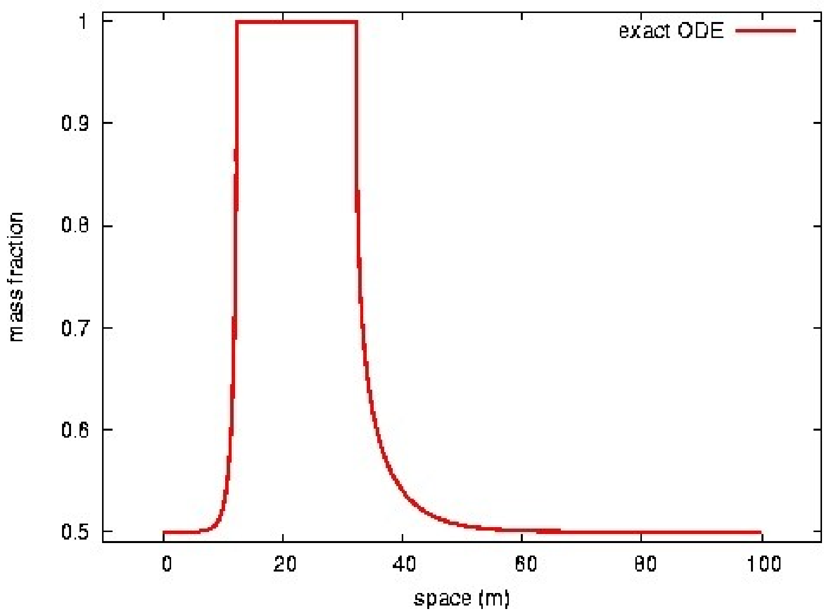
However

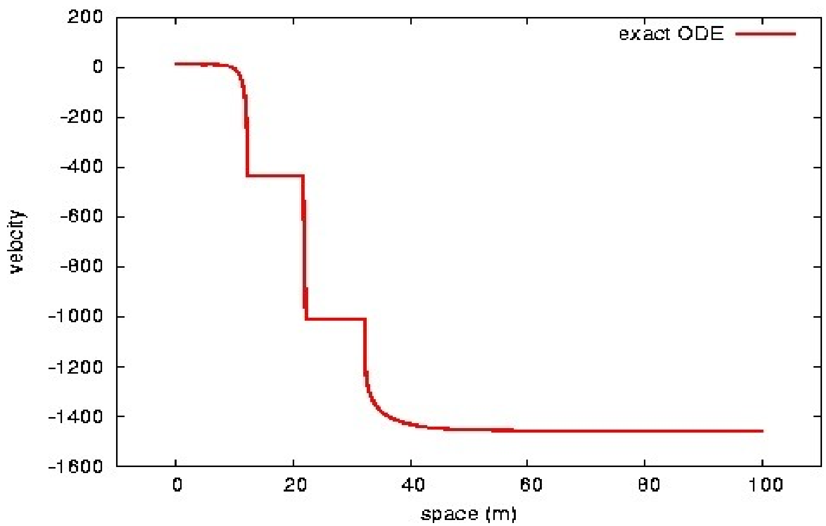
$$The\ two\ couples \;\; (\tau(z^0_R), p(z^0_R)), (\tau(z^0_L), p(z^0_L))$$

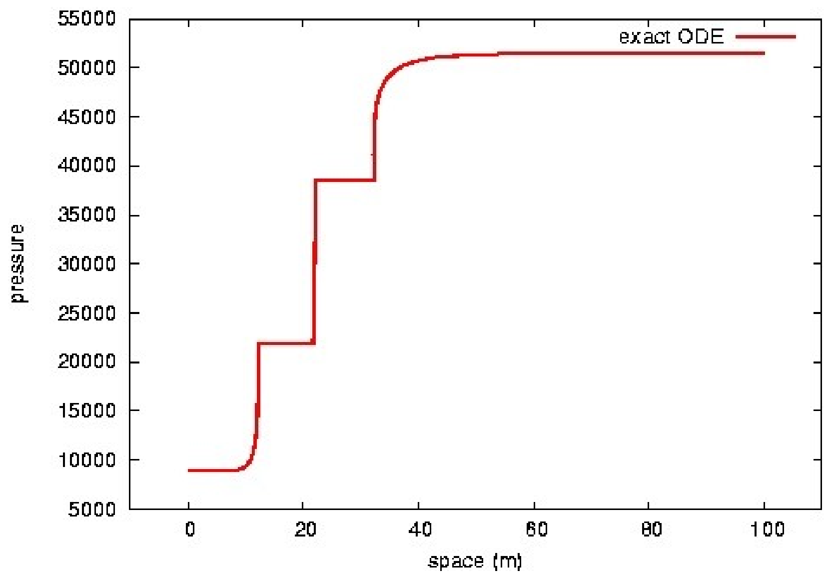
Satisfies the Rankine-Hugoniot conditions :

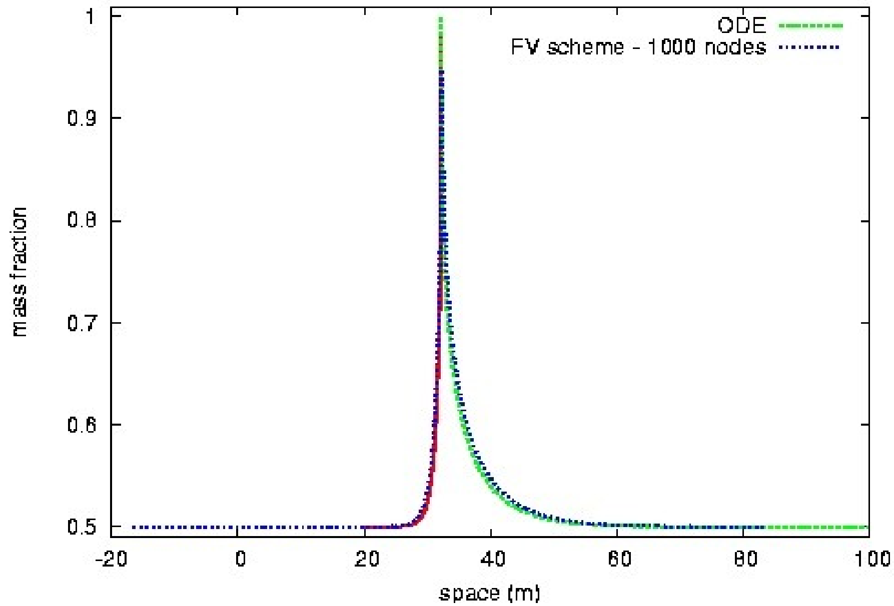
They can be connected by a one-phase shock

SHOCK STRUCTURE IN THE STRONG SHOCK CASE

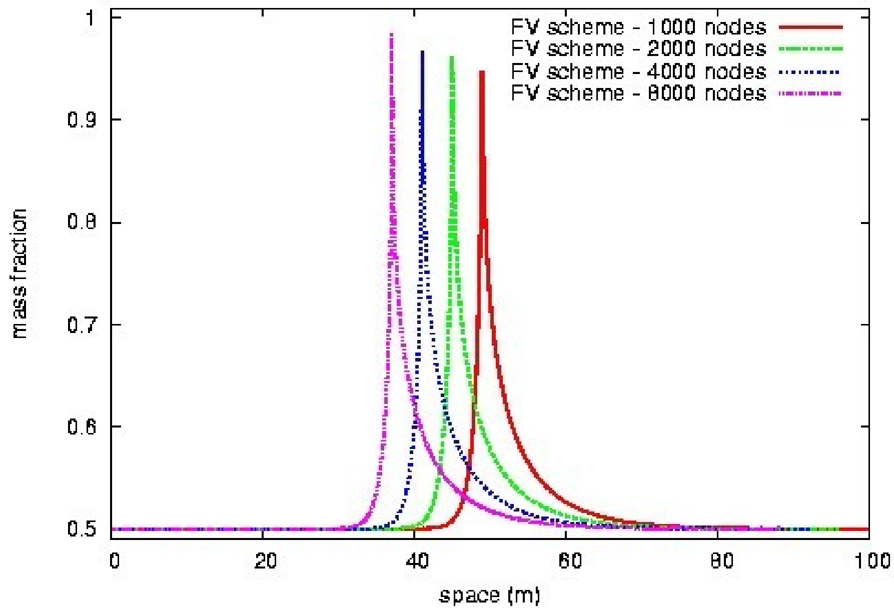








Comparison
FV- ODE integration



Influence of number of grid points

CONCLUSIONS

Definition of shock wave solutions in two-phase averaged models :

Traveling waves analysis using a non-standard dissipative tensor coming from a first-order Chapman-Enskog expansion of non-equilibrium two phase model.

Isothermal case :

- Existence of TW solutions
- Mass fraction is not a constant in the shock

Non-Isothermal case (iso pressure, iso velocity model)

- numerical experiments confirm the results of Isothermal case
- model can take into account velocity disequilibrium
- on-going work to define shock solutions for this 5 eq model as limit of TW of a dissipative system characterized by a dissipative tensor that retain physical informations on velocity disequilibrium