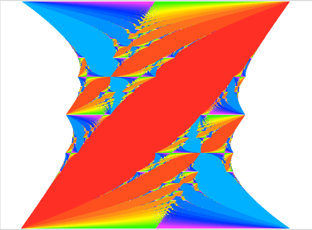
Adiabatic response of open systems

Yosi Avron **Martin Fraas** Gian Michele Graf

December 17, 2015

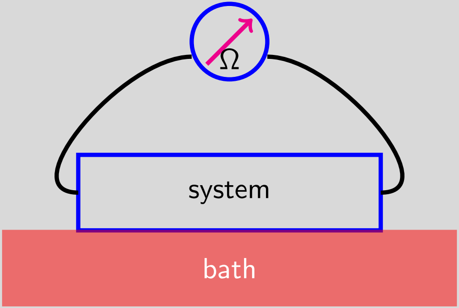


Question

Are topological phase protected form contact with the world?

Choosing:

Observables and states$\Longrightarrow$ immunity



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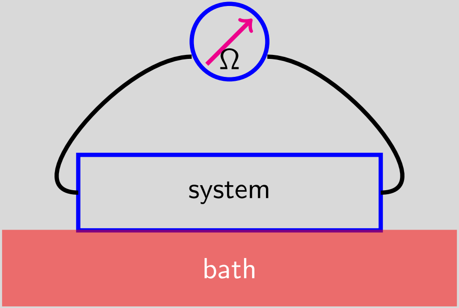
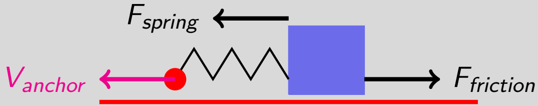


Ambiguity: Which transport coefficients?

Open vs. isolated systems



F as response to V.
Which F?

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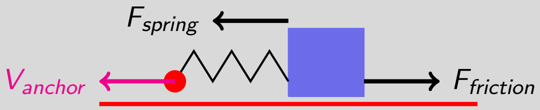


F as response to V.
Which F?

Open systems

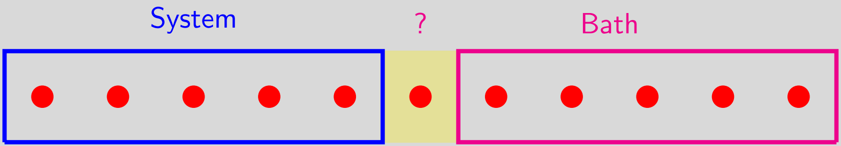
- $F_{friction} = -\nu\ V$
- $F_{spring} = \nu\ V$
- $F_{Total} = \dot{P} = 0\ V$

Isolated system

- $F_{friction} = 0$
- $F_{spring} = F_{Total}$

Sources of ambiguities

Tensor product & Observables



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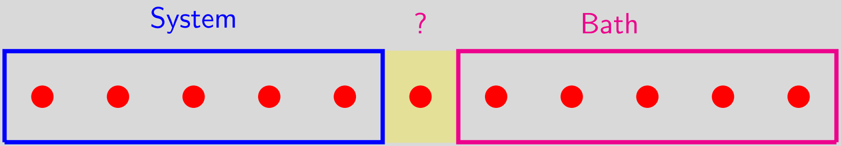

Ambiguous tensor decomposition

$$\mathcal{H}_{system} \otimes \mathcal{H}_{bath} = \mathcal{H}'_{system} \otimes \mathcal{H}'_{bath}$$

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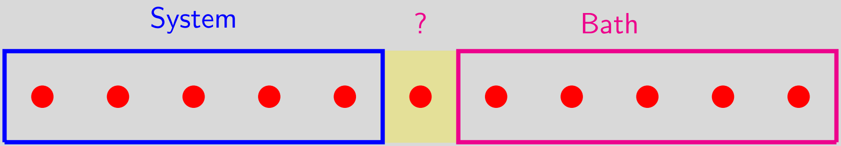

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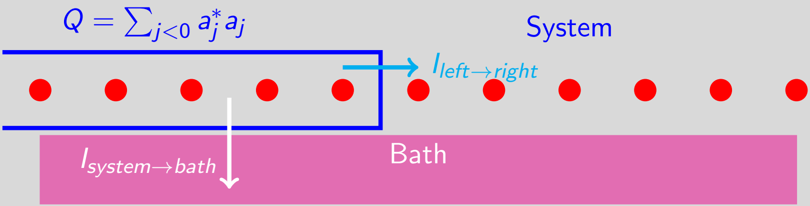
Ambiguous observables of sub-system

$$H = H_s \otimes \mathbb{1} + H_{interaction} + \mathbb{1} \otimes H_b$$

Ambiguity in fluxes (aka rates aka currents)



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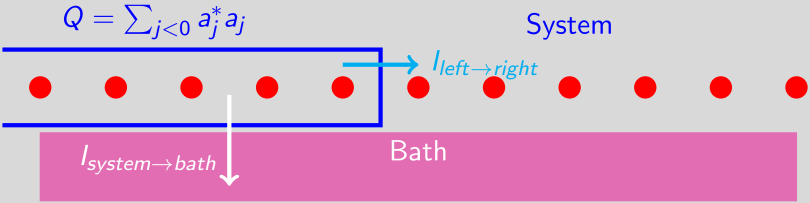


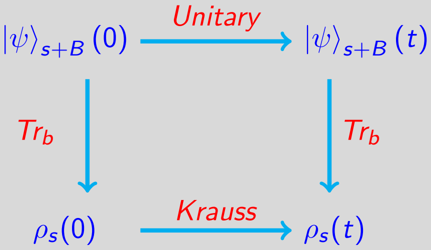
$Q \otimes \mathbb{1}$

$$I_{left \to right} = i[H_s, Q]$$

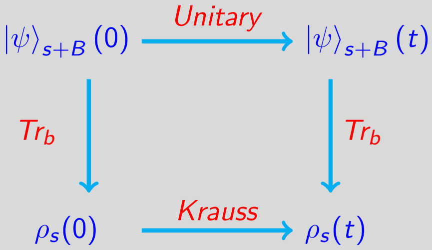
$$I_{Total} = i\,[H_{s+b}, Q \otimes \mathbb{1}]$$

Formulate an I_{Total} as a property of the system

Krauss maps


$|\psi\rangle_{s+B}(0)$
Unitary
$|\psi\rangle_{s+B}(t)$
Tr_b
Tr_b
$\rho_s(0)$
Krauss
$\rho_s(t)$

Krauss maps



Kraus: Positivity and Trace preserving

$$\rho \mapsto \sum K_j \rho K_j^\dagger, \qquad \sum K_j^\dagger K_j = \mathbb{1}$$

$$K\rho K^\dagger = (K\sqrt{\rho})(K\sqrt{\rho})^\dagger \geq 0$$

$$Tr\,\rho \mapsto \sum Tr\, K_j \rho K_j^\dagger = Tr\left(\sum K_j^\dagger K_j\right)\rho$$

Generators of Krauss maps

$$\rho \mapsto \rho + \underbrace{\delta t\, \mathcal{L}\rho}_{\textit{generator}} \qquad \rho \mapsto \sum \underbrace{K_j \rho K_j^\dagger}_{\textit{positive}}$$

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trace preserving: $\Gamma_0 + \Gamma_0^\dagger + \sum \Gamma_j^\dagger \Gamma_j = 0$

Solve for Γ_0: $\Gamma_0 = -iH - \frac{1}{2}\sum \Gamma_j^\dagger \Gamma_j$

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Lindbladians

$$\mathcal{L}\,\rho = -i[H, \rho] + \mathcal{D}\rho$$

$$\mathcal{D}\rho = \sum 2\Gamma_j \rho \Gamma_j^\dagger - \Gamma_j^\dagger \Gamma_j \rho - \rho \Gamma_j^\dagger \Gamma_j$$

Schrodinger and Heisenberg

States and Observables

Adjoint (Banach space)

$$Tr\,(X\mathcal{A}\rho) = Tr((\mathcal{A}^*X)\rho)$$

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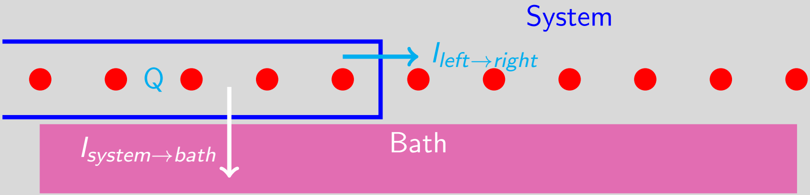
Schrodinger and Heisenberg

$$\dot{\rho} = \mathcal{L}\rho, \quad \dot{X} = \mathcal{L}^* X$$

Fluxes (aka rates) $\mathcal{L}^* Q$

Two notions of currents in open systems



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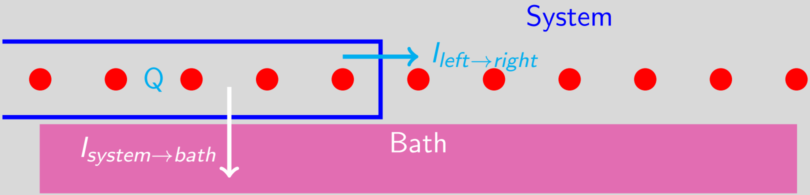

Left-right current

$$I_{left \to right} = i[H, Q] = i\,Ad(H)Q$$

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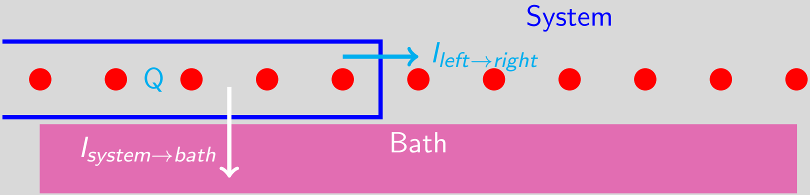

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Flux: Total current

$$I_{Total} = \dot{Q} = \mathcal{L}^* Q$$

Loop currents are not rates

Fluxes vanish in stationary states:

$$I_{loop} = -\frac{\partial}{\partial\phi} H \neq \mathcal{L}^* Q$$

Stationary states of $\mathcal{L}$

Towards adiabatic theory of fluxes

$$\mathcal{L}^*\mathbb{1} = 0 \Longrightarrow 0 \in \text{Spectrum}(\mathcal{L})$$

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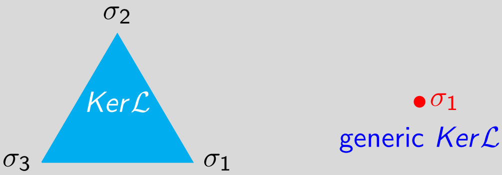


If $dim \mathcal{H} < \infty$ dim Ker $\mathcal{L}$=1 generically

Fluxes vanish in stationary states

Fluxes vanish in stationary states:

$$Tr\ \dot{Q}\,\sigma = 0$$

$$Tr\ \dot{Q}\sigma = Tr\ (\mathcal{L}^* Q)\sigma = Tr\ Q\mathcal{L}\sigma = 0$$

Adiabatic expansion for fluxes

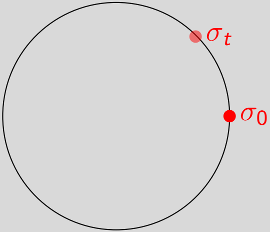
Slow manifold

Adiabatic evolutions

$$\epsilon\dot{\rho} = \mathcal{L}_t \rho, \quad \epsilon \ll 1$$

Adiabatic expansion

$$\rho_t = \sigma_t + \epsilon \mathcal{L}^{-1} \dot{\sigma}_t + \dots$$



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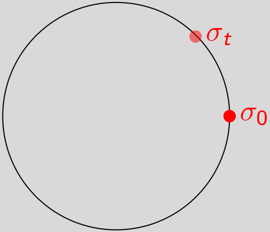

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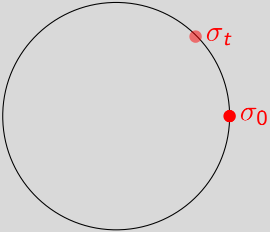

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$$\epsilon^0 : \qquad \mathcal{L}_t \rho_0 = 0 \Longrightarrow \rho_0 = \sigma_t$$

σ_t instantaneous stationary state

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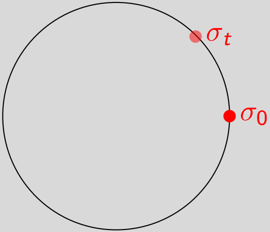

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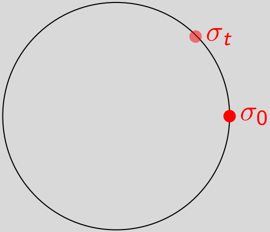

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$\dot{Q}$ in adiabatically evolving state

No need to invert $\mathcal{L}$

The magic

$$Tr\,\dot{Q}\,\rho_t \approx Tr\;Q\dot{\sigma}_t$$

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$$\rho = \sigma_t + \epsilon\mathcal{L}^{-1}\dot{\sigma}_t + \dots$$

$$\mathcal{L}_t\rho_t = \epsilon\mathcal{L}\mathcal{L}^{-1}\dot{\sigma} + \dots \approx \epsilon\dot{\sigma}_t$$

$$Tr\ \dot{Q}\rho_t = \frac{1}{\epsilon} Tr\ \mathcal{L}^* Q\rho_t = Tr\ Q\mathcal{L}_t\rho_t \approx Tr\ Q\dot{\sigma}_t$$

The irrelevant dynamics

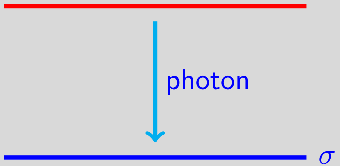
Adiabatic fluxes oblivious to $\mathcal{L}$. Only care about $\dot{\sigma}$

Sharing stationary states.

Sharing instantaneous stationary states

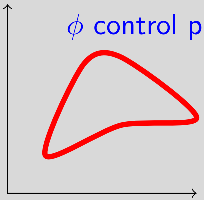
$$Ker\mathcal{L}_t = Ker(Ad\ H_t)$$

Example: $\mathcal{D} = \gamma Ad^2(H)$



Geometry of transport

Transport coefficients



$$\dot\sigma = (\partial_\mu \sigma)\,\dot\phi_\mu$$

$$\underbrace{Tr\,\dot Q \rho_t}_{response} \approx Tr\,(Q\partial_\mu\sigma)\;\underbrace{\dot\phi_\mu}_{driving}$$

Transport coefficient

$$F_\mu = Tr\,(Q\partial_\mu\sigma)$$

Oblivious to $\mathcal{L}$, cares bout $\dot\sigma$

Transport coefficients & Adiabatic curvature

$$`U(\phi) = e^{iQ_\mu \phi_\mu}$$

$$\sigma(\phi) = U(\phi)\sigma U^*(\phi)$$

$$\partial_\mu \sigma = i[Q_\mu, \sigma]$$

Transport coefficients

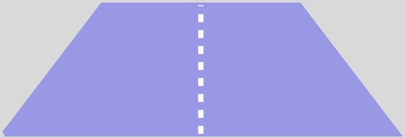
$$F_{\mu\nu} = iTr\,[Q_\mu, Q_\nu]\sigma$$

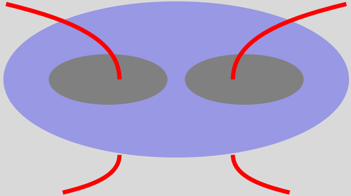
If σ projections

$$F_{mu\nu} = i\; Tr\,\sigma[\partial_\mu \sigma, \partial_\nu \sigma]$$

Conclusions and Overview

Views of the QHE





Macroscopic

$$\mathcal{L}^* Q = \frac{\partial}{\partial A} H$$

Index=Chern

Gap condition?

Multiply connected

$$\frac{\partial}{\partial A} H$$

Chern

Dissipation: Kahler geometry

A. Fraas and Graf, JSP (2012) `arxiv1202.5750`

A. Fraas, Kenneth and Graf, NJP (2011) `arxiv1008.4079`