Plateau transition in Hall effect: Matrix model for this class of problems

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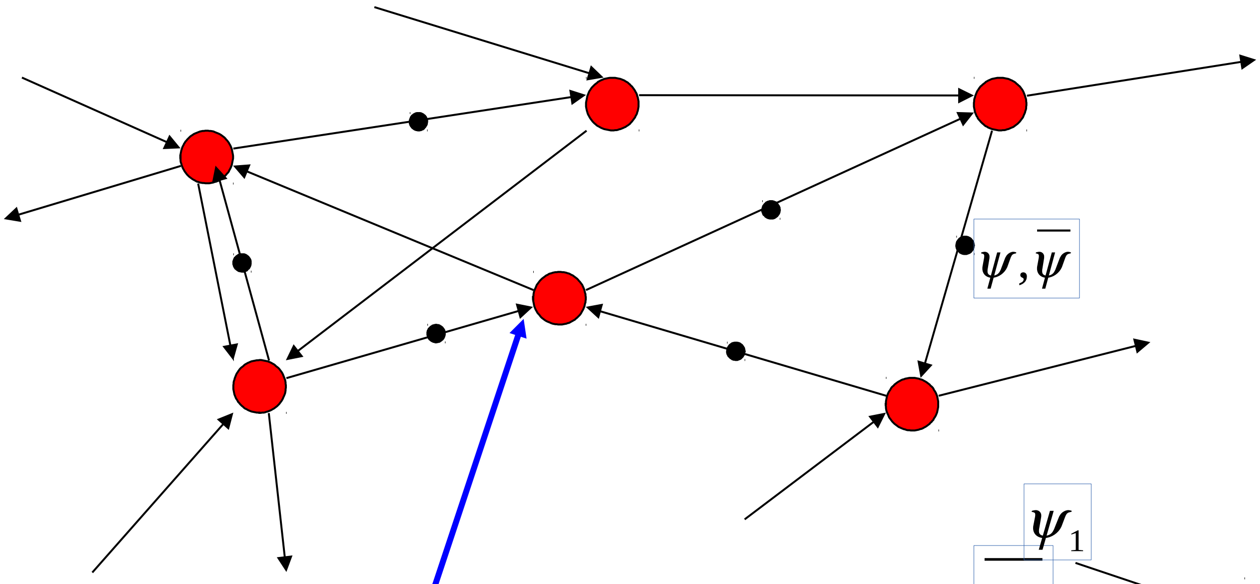
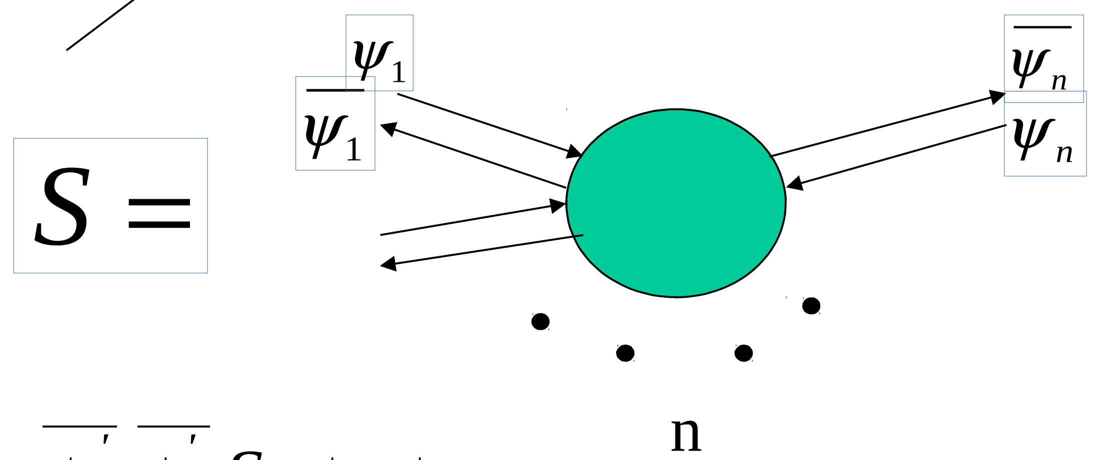
1. Random Networks and their connection to gravity

   a) Plateau transition in Hall effect
   b) Integrable models on random surfaces:
      Non-critical strings
   c) Sign-factor in 3D Ising model

3. Matrix Models for random networks

For any random network consisting of different n-channel S-matrices (n-incoming and n-outgoing waves)





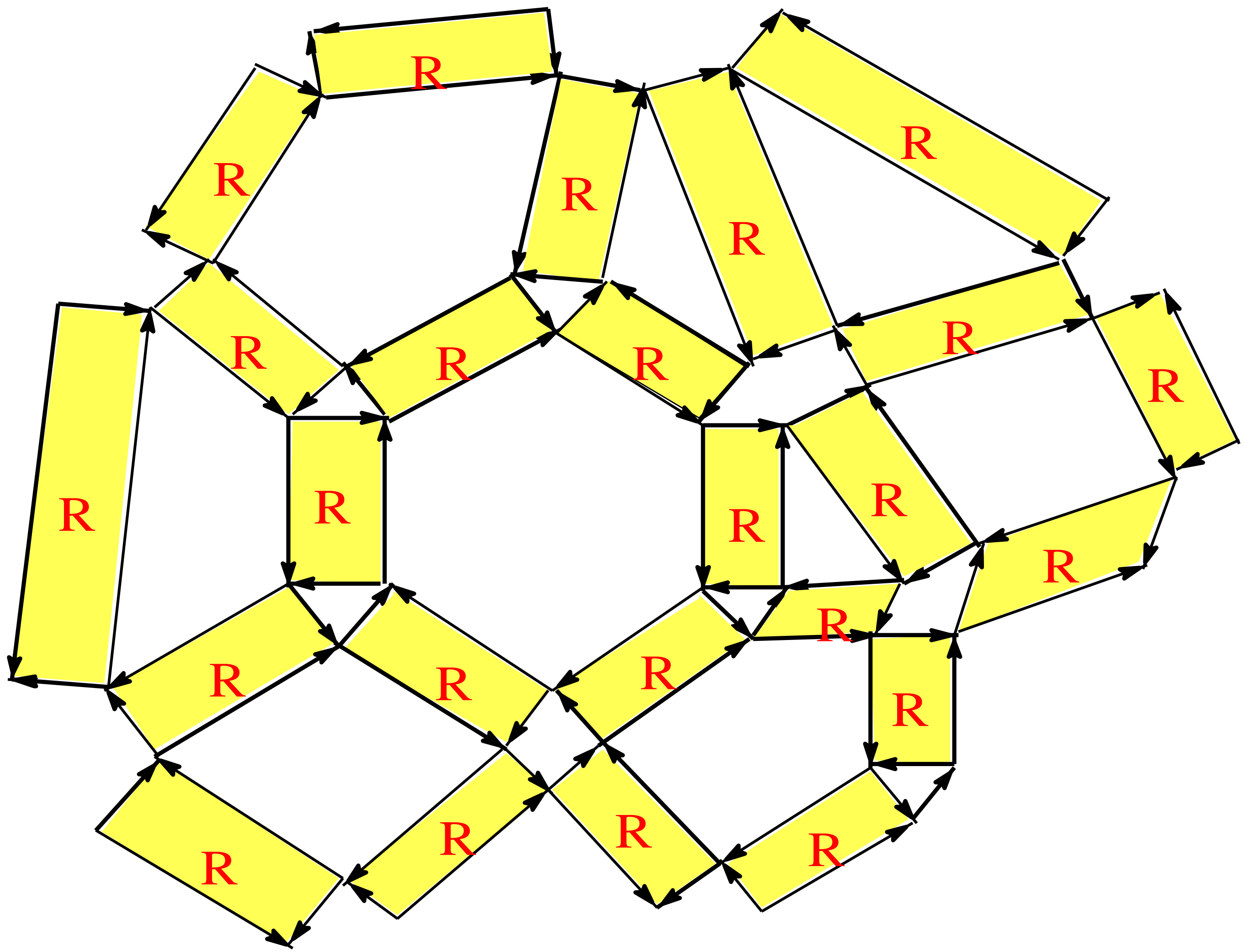
Define

$$R^{\bar{\psi}_1' \psi_2}_{\psi_1 \bar{\psi}_2'} = e^{\bar{\psi}_i' S_{ij} \psi_j + \bar{\psi}_2' \bar{\psi}_1' S_0 \psi_1 \psi_2}$$

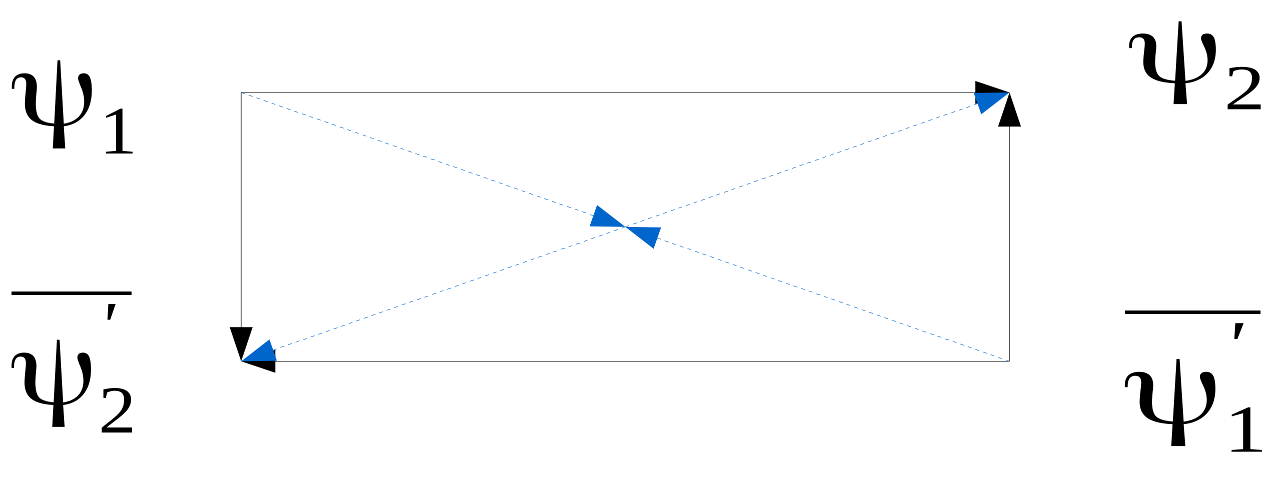
and plug into network

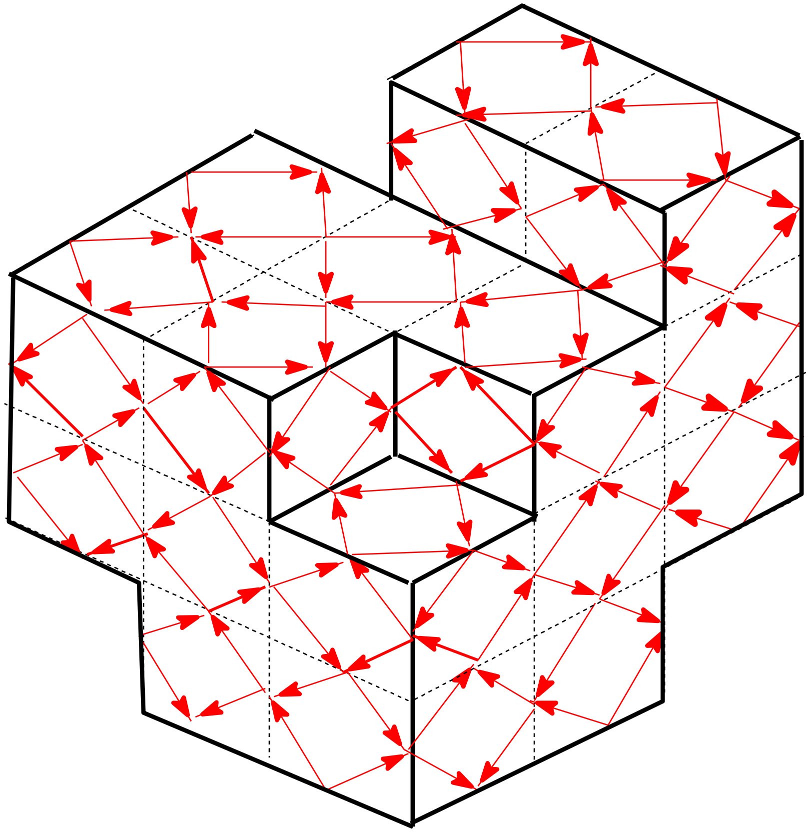
Khachatryan, Sedrakyan, Sorba- Nucl.Phys. B 825 (2009) 444
Khachatryan , Schrader, Sedrakyan -J. Phys. A 42 (2009)304019

ML



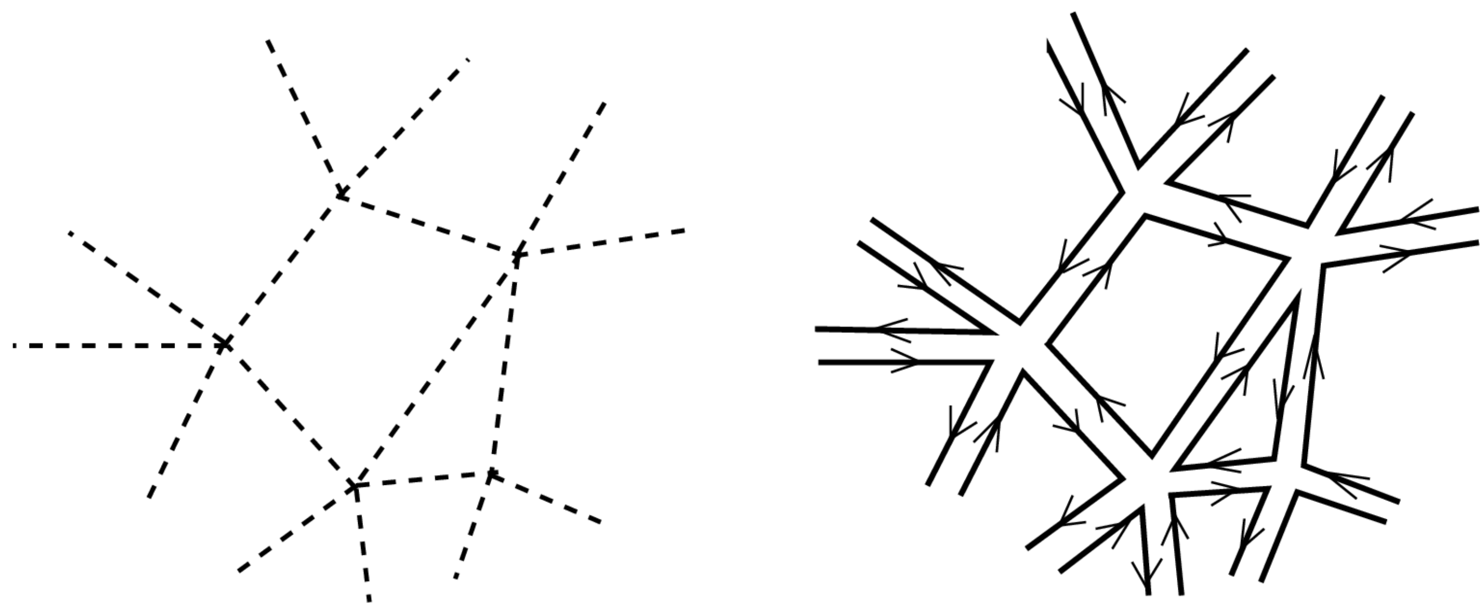
$$R^{\overline{\psi_1'}\,\psi_2}_{\psi_1\,\overline{\psi_2'}} =$$





Feynman diagrams of the gauge field

$$A_\mu^a \Gamma^a \in su(N)$$



(a) (b)

$$Z(\Omega) = \prod_{R \in \Omega} \check{R}(\lambda),$$

$$Z = \int \prod_r d\psi_r \prod_{plaquetts} R^{\overline{\psi_1'}\psi_2}_{\psi_1 \overline{\psi_2'}}$$

$$= \int \prod_r d\psi_r e^{\sum_{r,\mu} \overline{\psi_r'} S_{r,r+\mu} \psi_{r+\mu} + \sum_{plaquettes} \overline{\psi_2'} \overline{\psi_{r+\mu_x}'} S_0 \psi_{r+\mu_x+\mu_y} \psi_{r+\mu_y}}$$

This can be done for all quantum spin chain models with fixed R-matrix. All chain models with local Hamiltonian have R-matrix.

What we will have if network is random?

Example: XX-model with phases, equivalent to CC model

Inclusion of phase factors

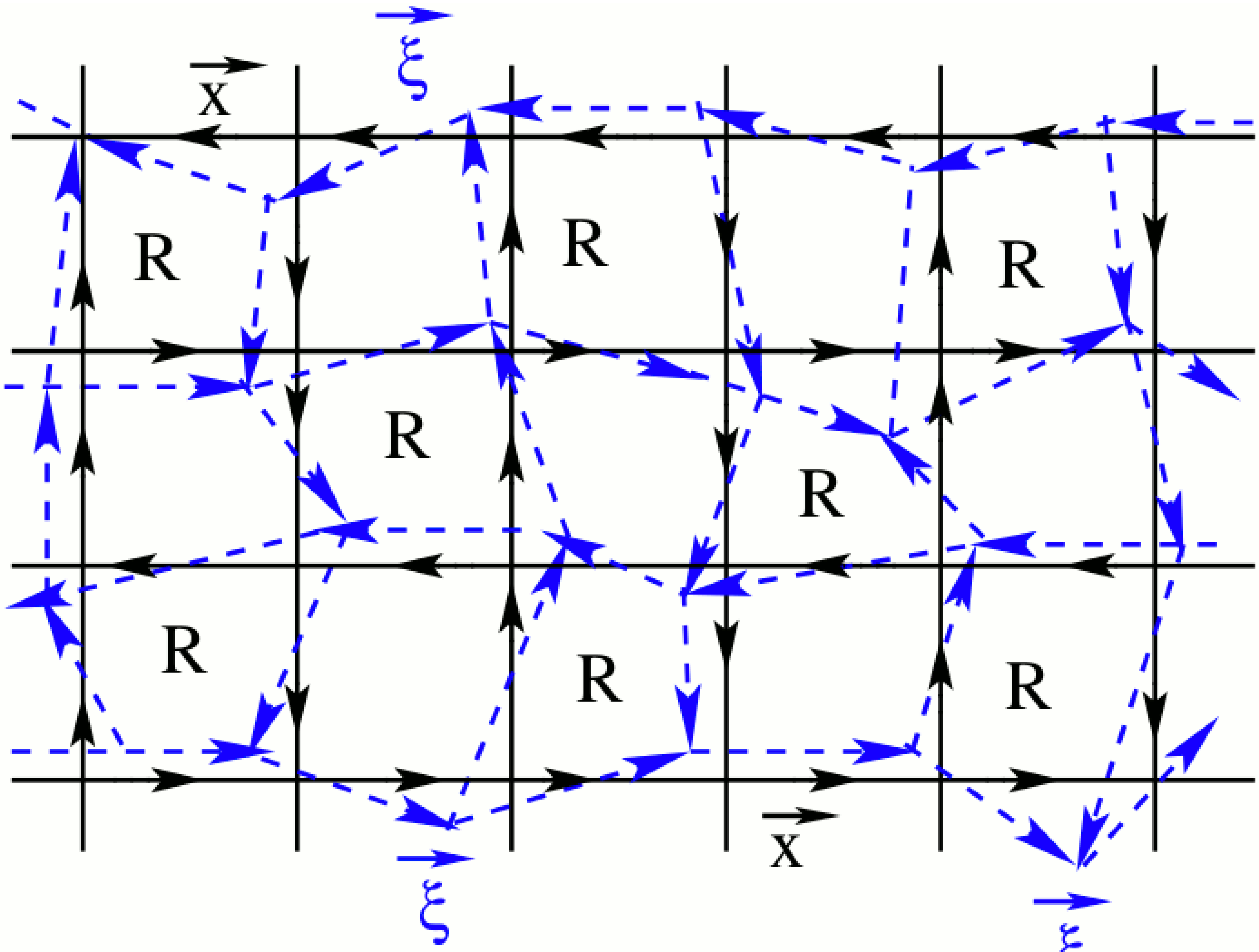
$$^{XX}R^{\bar{\psi}'_{2j},\,\psi'_{2j+1}}_{\psi_{2j},\,\bar{\psi}_{2j+1}} = \exp\Big\{ a e^{i\alpha}\,\bar{\psi}'_{2j}\,\psi_{2j} + a e^{i\alpha'}\,\bar{\psi}_{2j+1}\,\psi'_{2j+1} - b e^{i\alpha}\,\bar{\psi}_{2j+1}\,\psi_{2j} + b e^{i\alpha'}\,\bar{\psi}'_{2j}\,\psi'_{2j+1} \Big\} =$$

$$= \quad \mathrm{Exp}\{-S(\bar{\psi}'_{2j},\psi'_{2j+1}\;\; \psi_{2j},\bar{\psi}_{2j+1}\;\; e^{i\alpha})\}$$

Putting R_{XX} into partition function Z we get full action of the fermionic part

$$S\left(\{\psi_n\},\{\overline{\psi_n}\},e^{i\alpha_n}\right) = \sum_n S(\psi_n,\overline{\psi}_n,e^{i\alpha_n}) + \sum_n \psi_n\overline{\psi_n}$$

$$S[A_a, F] = \frac{i}{2}\bar{\psi}\sigma^a[\partial_a - \partial_a + A_a] - \bar{\psi}(m + F\sigma_3)\psi$$



The coordinate systems on random ML and regular ML can be connected via tetrads

$$\vec{\partial}_a = e_a^\alpha \vec{\partial}_\alpha$$

$$dx_\alpha = e_\alpha^a dx_a$$

curved regular

$$S[\psi, A_a, F] = \int d\xi\, e \frac{i}{2} \overline{\psi} \sigma^a e^a_\alpha [\overrightarrow{\partial}_\alpha - \overleftarrow{\partial}_\alpha + A_\alpha] - \overline{\psi}(e\, m + F \sigma_3)\psi$$

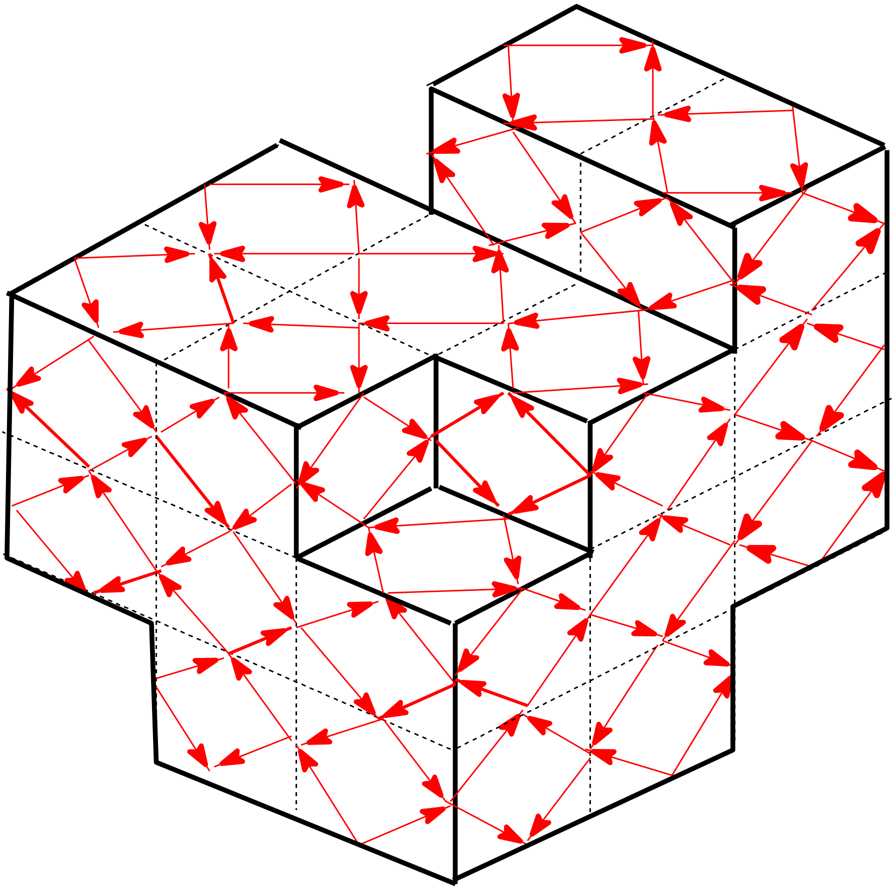
We got fermions interacting with U(1) gauge field and gravity

For the averaging over U(1) disorder in supersymmetric approach (Efetov) we need to introduce two fermionic fields, $\psi_\downarrow$ and $\psi_\uparrow$ interacting with U(1) gauge field with opposite charges together with two complex bosonic fields $\varphi_{\downarrow,\uparrow}$, which have the same action as fermions

$$S[\psi_{\downarrow,\uparrow}, \varphi_{\downarrow,\uparrow}, A_\alpha, F]_{total} = S[\psi_\downarrow, A_\alpha, F] + S[\psi_\uparrow, -A_\alpha, -F]$$

$$+ S[\varphi_\downarrow, A_\alpha, F] + S[\varphi_\uparrow, -A_\alpha, -F]$$

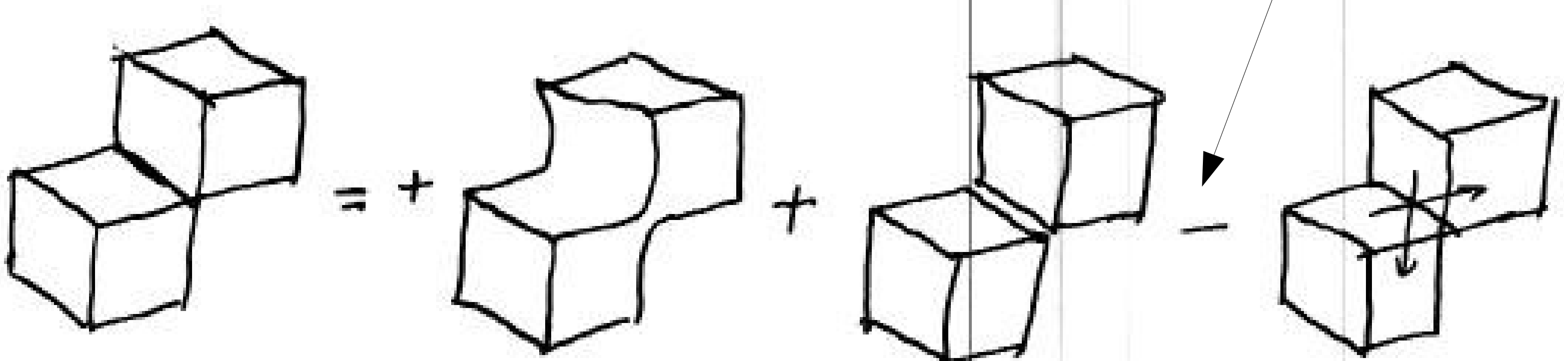
W. Nudding, I. Gruzberg, A. Kluemper, A. S.-article in preparation



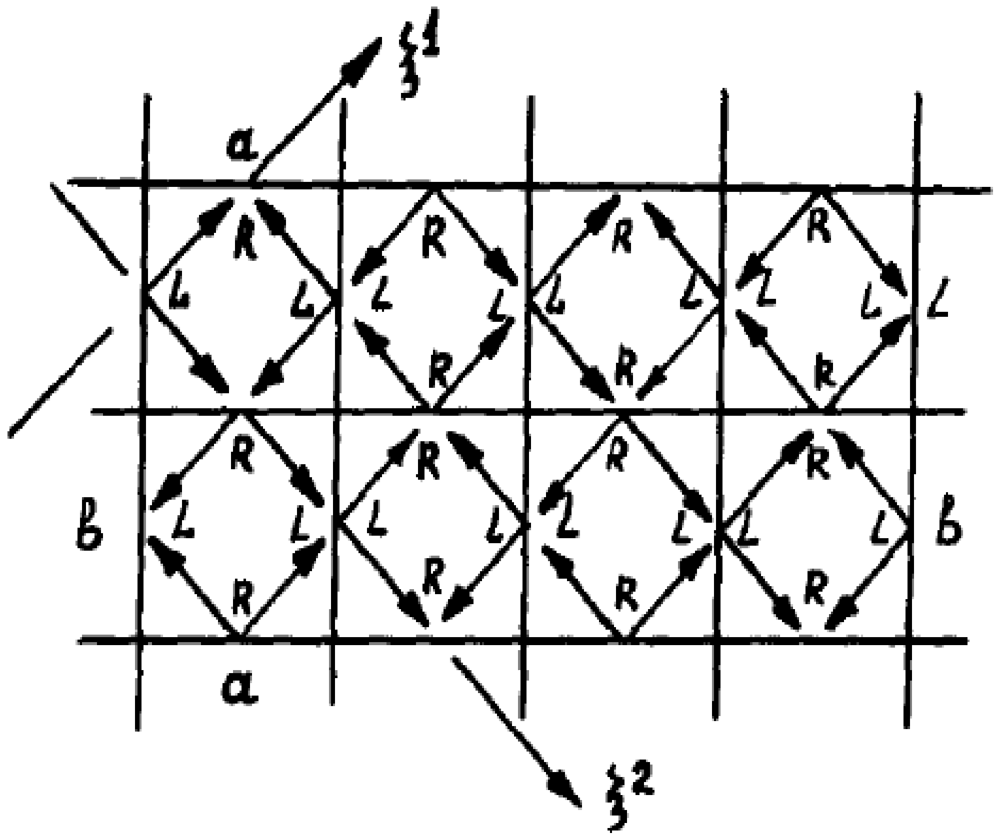
3D Ising model, fermionic string representation. Polyakov-1979

Sign factor exhibits Pauli principle for strings. Fermionic string

$$Z = \sum_{\vec{x}(\xi)} e^{\alpha\, Area}\, \Phi\left[\vec{x}(\xi)\right]$$



Sign-factor in 3D Ising model
Kavalov, Sedrakyan—Preprint ERPHI-695(10)-1984
Nucl. Phys. B285[FS19](1987)



$$S = \sum_{n,\mu} \bar{\psi}_n \Omega_{n,n+\mu} \psi_{n+\mu} t_{n,n+\mu}$$

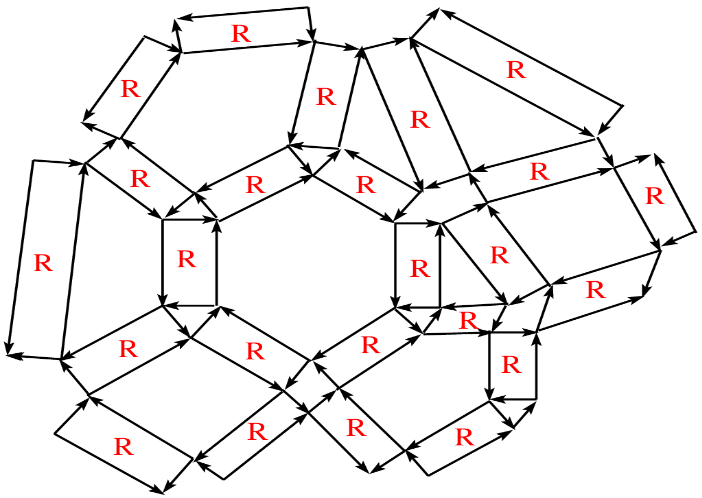
$$\Omega_{n,n+\mu} \in SU(2)$$

$$\Phi[\vec{x}(\xi), \psi] = \int d\psi\, e^{-S} = \prod_C \Omega_{n,n+\mu} = (-1)^N$$

N= # of fluxes

J. Ambjorn, A. Sedrakyan - Nucl.Phys.B874, 877(2013)
J. Ambjorn, Sh.Khachatryan, A.Sedrakyan - PRD92, 026002(2015)

Matrix Model for Random Network



$$consider \quad M_{\alpha j} \in SU(2N)$$

$$R-matrix \quad R_{\alpha i;\beta j}^{\alpha' i';\beta' j'} = R_{\alpha;\beta}^{\alpha';\beta'} \delta_i^{i'} \delta_j^{j'}$$

Action

$$S[M] = M^{+}_{\alpha' i';\beta' j'} R_{\alpha i;\beta j}^{\alpha' i';\beta' j'} M^{\alpha i;\beta j} + \sum_n Tr[M^n] + \sum_n Tr[M^{+n}]$$

$$Partition \quad Function$$

$$Z = \int DM \, e^{-S[M]}$$

$$M = U\, m\, U^{+}.$$

Consider adjoined representation

$$\Lambda = Tr[\tau_a U \tau_b U] \qquad \tau_a \in su((2s+1)N)$$

Action become

$$S[m,\Lambda] = m^b \Lambda_b^a R_a^c \Lambda_c^u m_u - \sum_a V(m_a)$$

Action is quadratic. Gaussian integration may be used if Λ_b^a is parametrized by free coordinates

$$\Lambda \in F(1,1,\ldots 1) = \frac{U((2s+1)N)}{U(1)^{(2s+1)N}} \simeq \prod_{1}^{(2s+1)N} CP^{i}$$

Flag Manifold

$$DM = DU\,dm = D\Lambda\,dm$$

$$D\Lambda = \prod_{k=1}^{(2s+1)N-1} D[CP^{k}] = \prod_{k=1}^{(2s+1)N-1} D\left[\frac{S^{k}}{S^{1}}\right]$$

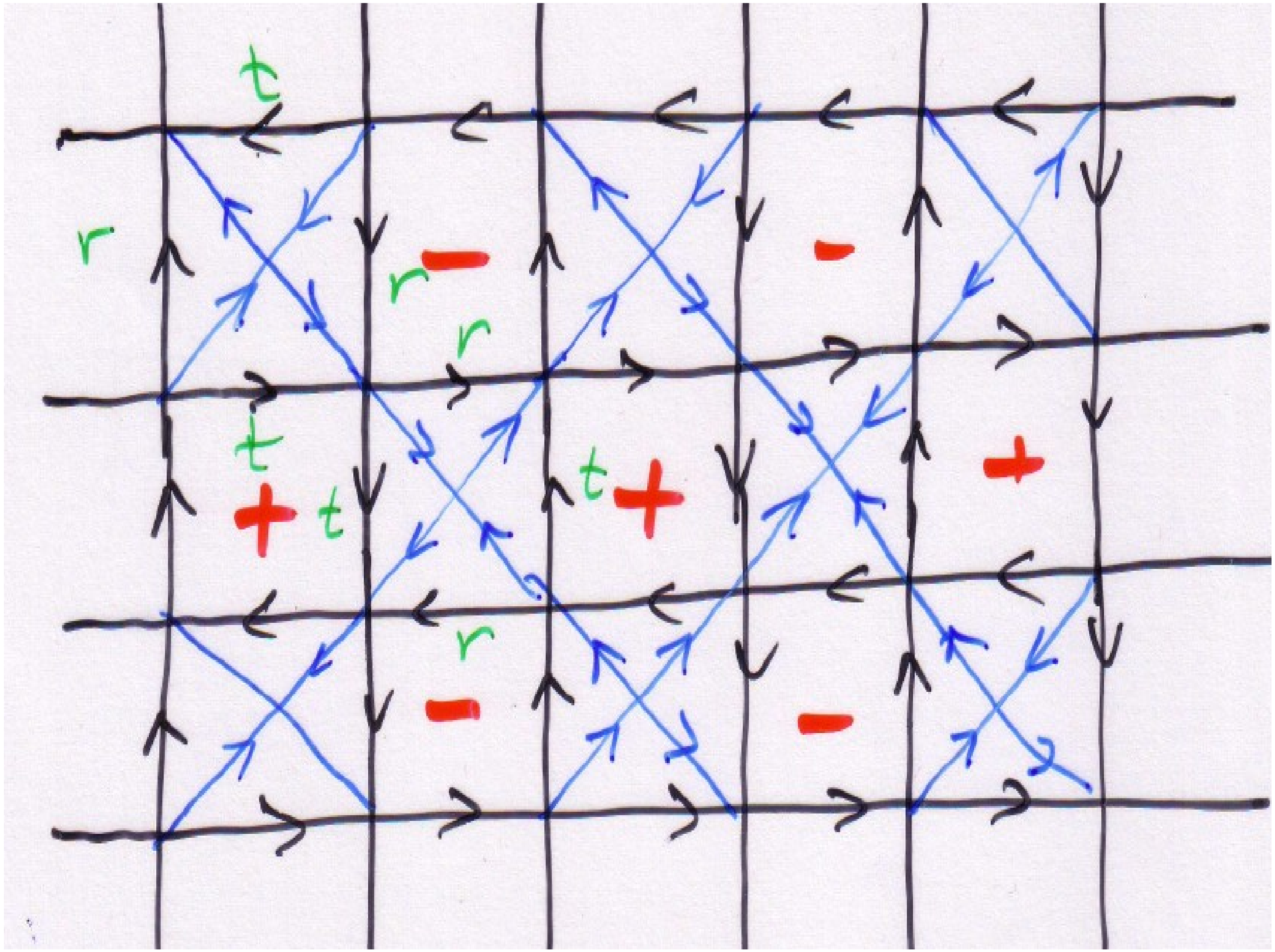
$$. \simeq \prod_{k=1}^{(2s+1)N-1} D[S^{k}]$$

$$D[S^{k}] = \delta\left(\sum_{1}^{k} |z_i|^2 - 1\right) \prod dz_i$$

$$Z = \int \prod dm_a \, d\lambda_a \, e^{-W(m_a, \lambda_a)}$$

Summary

Random network problems and strings in real space have the same physical background.
They contain matter and gauge fields interacting with gravity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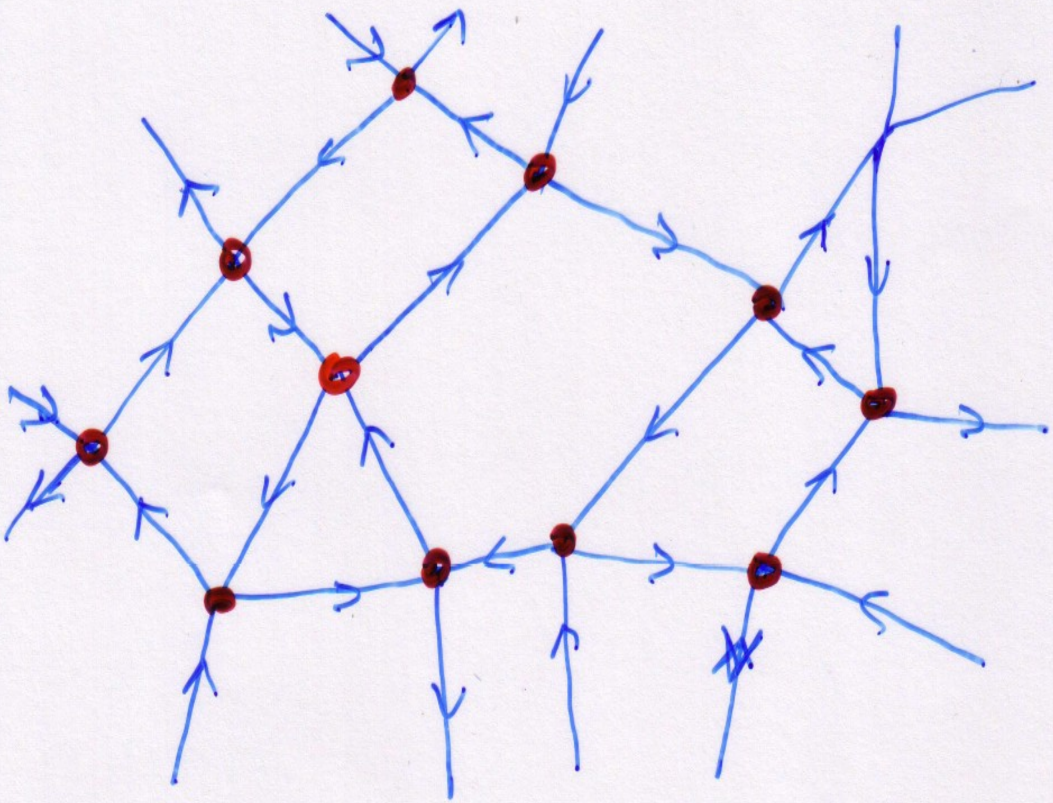
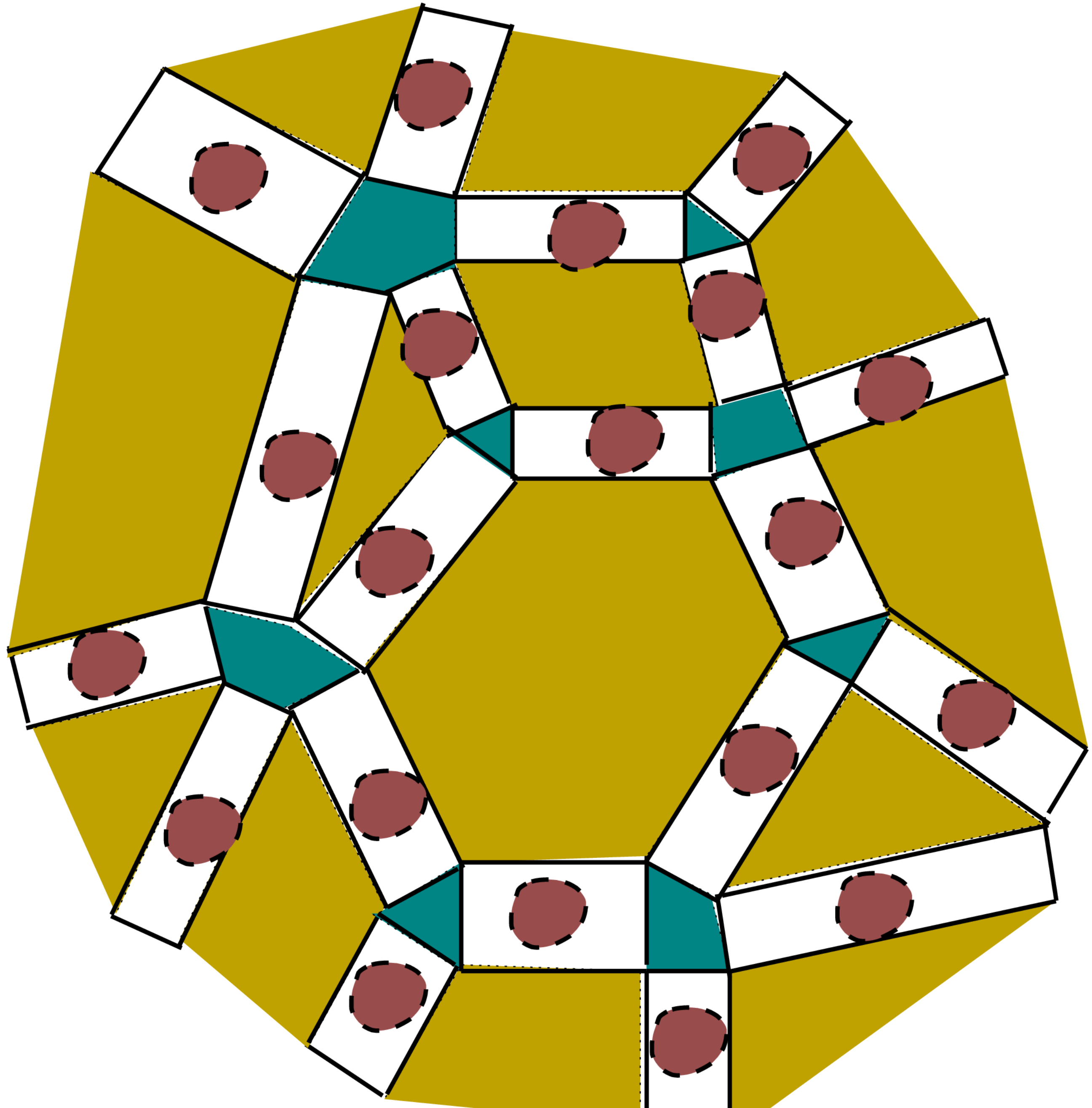
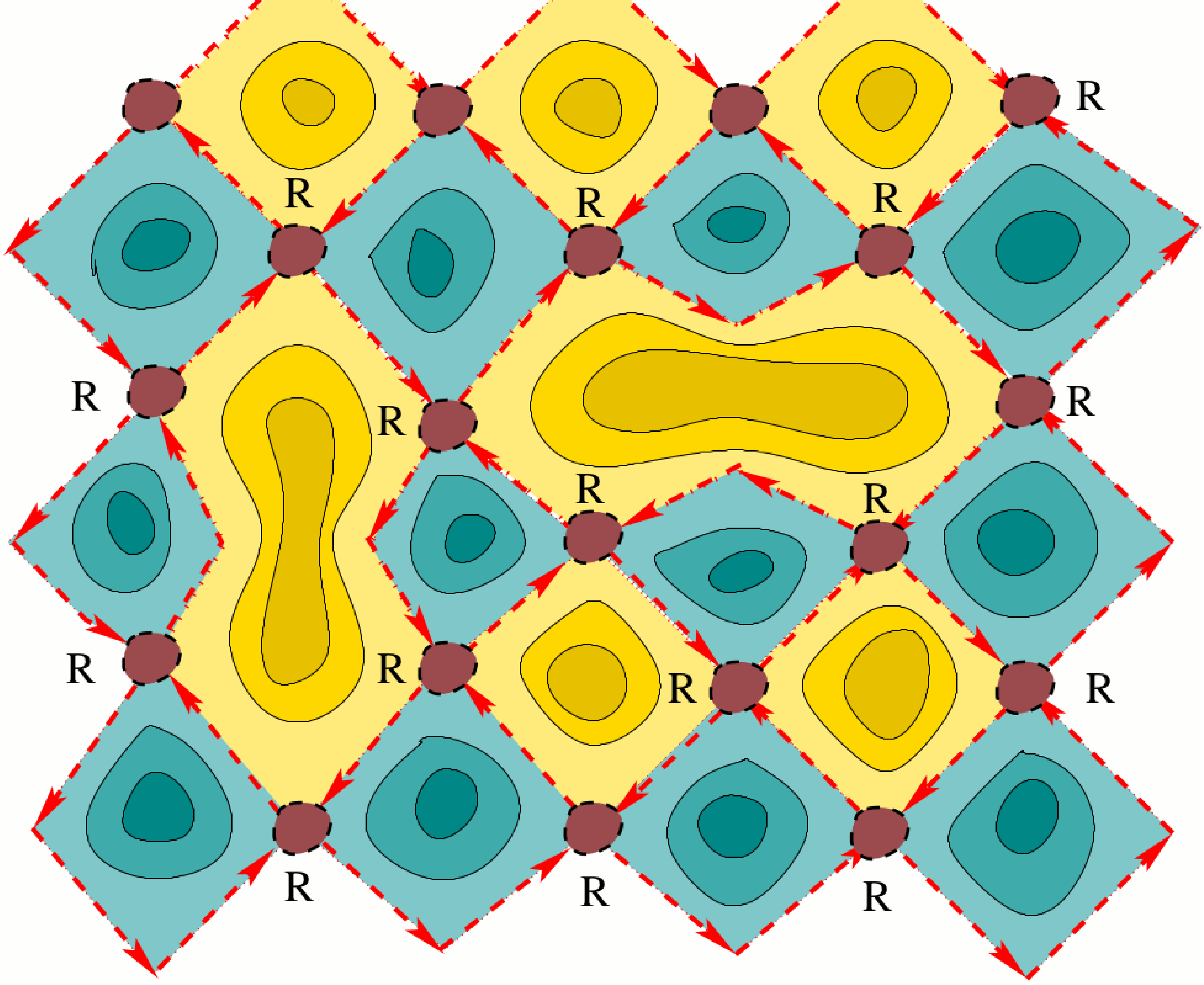
Instead of regular lattice we will have now



with random reflection/rotation
- r and transmission/tunnelling
- t parameters on the saddle points ●

Random network

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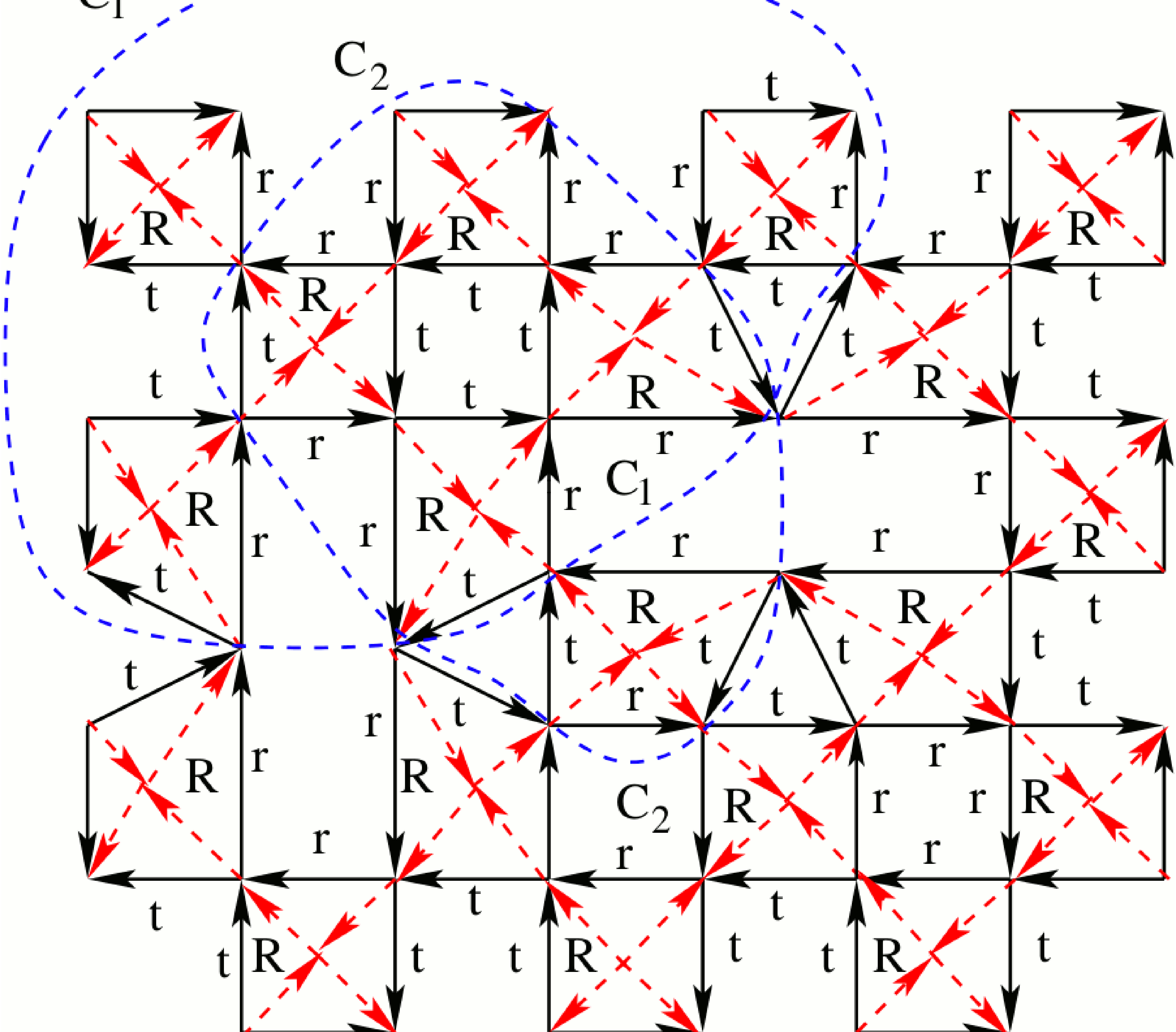
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