Geometry and asymptotics in Quantum Hall states

Proposition de sujet de thèse

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The topic of this thesis is at the interface of analysis, probability, geometry and quantum physics. The main object of study is a quantum-mechanical wave function ("quantum Hall state") of N particles on a manifold M. When $M = \mathbb{C}$, the wave function is a holomorphic degree-d homogeneous polynomial in the polynomial ring in N complex variables, completely symmetric or anti-symmetric, which belongs to certain ideals in $\mathbb{C}[z_1, ..., z_N]$. The best known case is the so-called Laughlin wave function, when the wave function belongs to the diagonal ideal, but more general choices of ideals are also interesting. Such wave functions describe the Quantum Hall effect in condensed matter physics.

We would like to define Quantum Hall states for M being a compact Riemann surface or a compact complex manifold of dimension 2. How many Quantum Hall states are there on M for given N and d? This question arises in topological phases of matter, and there are a number of physics conjectures regarding this question.

In quantum mechanics the wave functions belong to a Hilbert space. We would like to study the asymptotics of the L^2 normalisation of the Quantum Hall states as the number of particles N tends to infinity. These asymptotics are of particular interest, as they allow to compute certain physical parameters, such as Hall conductance and Hall viscosity, and they are also related to questions in Coulomb gases, gaussian free fields and Liouville quantum gravity.

References

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