Reading group: Specialization of Rationality

Spring 2026

The goal of this reading group is to study the behaviour of rationality of algebraic varieties unders specialization. The main references are the papers of Nicaise–Shinder [16], Kontsevich–Tschinkel [14] and Nicaise–Ottem [15]. If we have enough time, it will be interesting to study also the categorification in [9].

Week 1 (19/2 Lie Fu) The Grothendieck ring of varieties $K_0(\mathsf{Var})$ and examples of motivic measures. A survey over the pathological properties of $K_0(\mathsf{Var})$. If time permits, extension to varieties over a base.
[6, Chapter 2, §2, §3, §6.3–6.5].

Week 2 (5/3 Junhui Qin) Weak factorization theorem [2]. Bittner theorem and its dimensional generalization in [15] (if time allows). Relation with motives.
[4], [6, Chapter 2, §5].

Week 3 (12/3 Haohao Liu) Larsen–Lunts theorem. Applications.
[6, Chapter 2, §6].

Week 4 (19/3 Kenza Memlouk) Motivic integration. Application: Batyrev–Kontsevich theorem on invariance of Hodge numbers under K-equivalences.
[7] and [6, Chapter 7, §3.3 – 3.5].

Week 5 (26/3 Christopher Nicol) Denef–Loeser motivic nearby fiber (for a function $X \to \mathbb{A}^1$).
[6, Chapter 7, §3.3 and §4].

Week 6 (2/4 Ludovic Felder) Semi-stable reduction theorem [13]. Some details about toroidal embeddings. Present the argument for curves in Deligne–Mumford [8]. If time permits, more recent results for higher-dimensional bases, e.g. [1] and [3].

Week 7 (9/4 Yufei Qian) Log geometry. Toroidal embeddings. [11], [12], [5, §3].

Week 8 (16/4 Mauro Porta) Constructions of the key specialization map: the *motivic volume*. Precise formula in the semistable reduction case and in the strict toroidal case. [16, §3] (some arguments are in Appendix, the language of log geometry is used).

Week 9 (30/4 Giuseppe Ancona) Refined motivic volume of Nicaise–Ottem [15]. Ring of birational types (Burnside rings) [14, §3]. Specialization maps for birational types. Specialization theorem for stable rationality and rationality [16, §4.1 – 4.2], .

Week 10 (7/5 Robert Laterveer) Concrete geometric examples [16, §4.3]: Artin–Mumford example, degree-4 hypersurfaces in $\mathbb{P}^4$, and many more in [15, §4].

Week 11 (21/5 Thomas Agugliaro) Functorial stably birational invariants. Kahn–Sujatha category [10]. R-equivalence.

Week 12 (28/5 Arthur-César Le Bras) Categorification of the motivic volume [9]. Applications: obstructions to retract rationality, dual complex.

References

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