

Fano varieties, Hyper-Kähler varieties and algebraic cycles

January 9-11, 2023, Strasbourg, France

Titles and Abstracts

Mini-course:

Paolo Stellari: *Stability conditions: from curves to hyperkähler manifolds.*

Abstract: In these three lectures we will review the basic material about stability conditions and focus on examples. We will start reviewing the simplest example given by algebraic curves and illustrate how this allows us to move to higher dimensions passing through the case of noncommutative surfaces. The goal is to illustrate how to construct stability conditions on special hyperkähler manifolds which are Hilbert schemes of points on special K3 surfaces. The new results are a joint work in progress with Chunyi Li, Emanuele Macrì and Xiaolei Zhao.

Research talks:

Giuseppe Ancona: Lefschetz standard conjecture for some lagrangian fibrations

Abstract: The Lefschetz standard conjecture predicts the existence of some specific algebraic classes on the square of an algebraic variety, namely the inverse of the Lefschetz operator should be induced by an algebraic correspondence. We will show this conjecture for the hyper-kähler varieties constructed by Laza-Saccà-Voisin. This will be a special case of a general criterion which will tell that hyper-kähler varieties admitting sufficiently "nice" lagrangian fibrations satisfy this conjecture.

I will start by recalling the conjecture and the known results. Then I will study how the conjecture behaves under fibration and explain why several problems appear. Finally I show that one gets indeed a good behaviour in the setting of lagrangian fibrations. This is a joint work with Mattia Cavicchi, Robert Laterveer and Giulia Saccà. The talk will be related but independent with Cavicchi's one.

Pieter Belmans: *Hochschild cohomology of Hilbert schemes of points.*

Abstract: I will present a formula describing the Hochschild cohomology of symmetric quotient stacks, computing the Hochschild-Kostant-Rosenberg decomposition of this orbifold, and thus in the case of I will illustrate this in some interesting examples, that explain how this invariant behaves differently from say Betti or Hodge numbers, which have been studied intensively in the past 30 years. This is joint work with Lie Fu and Andreas Krug.

Olivier Benoist: *Smooth subvarieties of Jacobians.*

Abstract: I will present new examples of algebraic cohomology classes on smooth projective complex varieties that are not integral linear combinations of classes of smooth subvarieties. Some of these examples have dimension 6, the lowest possible. More precisely, I will consider the case of minimal cohomology classes on Jacobians of very general curves. This is joint work with Olivier Debarre.

Mattia Cavicchi: Relative Lefschetz operators, Ngo fibrations and LSV tenfolds.

Abstract: Consider a smooth complex projective variety X , equipped with a fibration $f : X \rightarrow S$ towards a smooth base and an f -ample class η . Then, one knows that cup product with η provides a "relative hard Lefschetz isomorphism" between suitable pieces of $H^*(X)$. Suppose that on the (smooth) generic fiber of f , there exists a correspondence C , inducing the inverse of the cup product with the restriction of η ; for instance, this is the case when the generic fiber is an abelian variety. Can one extend C to a correspondence on the whole of X , which induces the inverse of the relative hard Lefschetz isomorphism? The aim of this talk will be to show that the answer is positive when f is a so-called Ngô fibration with irreducible fibers, and that a fibration with such properties exists when X is one of the hyper-kähler varieties constructed by Laza-Saccà-Voisin (LSV tenfolds). These results are part of a joint work with Giuseppe Ancona, Robert Laterveer and Giulia Saccà, where they are applied to show the Lefschetz standard conjecture for LSV tenfolds. The talk will be related but independent with Ancona's one.

Ana-Maria Castravet: Higher Fano manifolds

Abstract: Fano manifolds are complex projective manifolds having positive first Chern class. The positivity condition on the first Chern class has far reaching geometric and arithmetic implications. For instance, Fano manifolds are covered by rational curves, and families of Fano manifolds over one dimensional bases always admit holomorphic sections. In recent years, there has been great effort towards defining suitable higher analogues of the Fano condition. Higher Fano manifolds are expected to enjoy stronger versions of several of the nice properties of Fano manifolds. For instance, they should be covered by higher dimensional rational varieties, and families of higher Fano manifolds over higher dimensional bases should admit meromorphic sections (modulo Brauer obstruction).

In this talk, I will discuss higher Fano manifolds which are defined in terms of positivity of higher Chern characters. After a brief survey of what is currently known, I will present recent joint work with Carolina Araujo, Roya Beheshti, Kelly Jabbusch, Svetlana Makarova, Enrica Mazon and Nivedita Viswanathan, regarding higher Fano manifolds which are toric varieties. More generally, we introduce a new invariant for smooth toric varieties, the minimal projective bundle dimension, capturing the minimal degree of a dominating family of rational curves. I will then explain a strategy towards proving that projective spaces are the only higher Fano manifolds among smooth projective toric varieties.

Salvatore Floccari: Sixfolds of generalized Kummer type and K3 surfaces

Abstract: I will present a construction which associates to any sixfold K of generalized Kummer type a hyper-Kähler manifold Y deformation equivalent to a Hilbert scheme of length 3 subschemes on a K3 surface, relating the most well studied deformation types of hyper-Kähler manifolds in dimension 6. Our construction is reminiscent of the classical construction of Kummer K3 surfaces, in the sense that Y is obtained as resolution of the quotient of K by a group of symplectic automorphisms. As a consequence we are able to show that any projective sixfold K as above has a

naturally associated K3 surface. As main application I will explain that the K3 surfaces so obtained satisfy the Kuga-Satake Hodge conjecture, producing infinitely many new families of K3 surfaces of general Picard rank 16 for which this conjecture holds.

Pablo Magni Derived equivalences of generalized Kummer varieties

Abstract: In this talk we will be concerned with derived equivalences of generalized Kummer varieties $\text{Kum}^n(A)$, which are one of the well-known families of Hyperkähler varieties of dimension $2n$; they are associated to an abelian surface A . By the derived McKay correspondence, the derived category of a generalized Kummer variety can be described as an equivariant derived category, i.e. a category of complexes of sheaves together with an equivariant structure (usually called a linearization when the sheaf is a line bundle).

We utilize these equivariant structures systematically and explain how to exhibit Fourier–Mukai kernels with an equivariant structure in the context above. This leads on the one hand to a large supply of non-trivial derived autoequivalences, and on the other hand to new derived equivalences between the generalized Kummer varieties associated to certain abelian surfaces and their duals.

Jørgen Rennemo: K-theoretic sheaf counting invariants on \mathbb{C}^4 .

Abstract: Oh and Thomas have recently defined a K-theoretic sheaf counting invariant for moduli spaces of sheaves on a Calabi-Yau 4-fold. One of the simplest examples of such a moduli scheme is the Hilbert scheme of n points on \mathbb{C}^4 . I will discuss these invariants and explain the proof of a certain formula for the generating functions of invariants of these Hilbert schemes, confirming a conjecture of Nekrasov (as well a generalisation to Quot schemes of \mathbb{C}^4 , conjectured by Nekrasov and Piazzalunga). This is joint work with Martijn Kool.

Mingmin Shen: Grothendieck period conjecture on a product of elliptic curves.

Abstract: The Grothendieck comparison theorem between algebraic de Rham cohomology and Betti cohomology gives rise to very interesting complex numbers, called periods, associated to an algebraic variety. The Grothendieck period conjecture is about the transcendental degree of the field generated by such numbers. In this talk, I will explain a framework in which the period conjecture can be decomposed into two steps. On a product of elliptic curves, we establish one of them. This is joint work with Charles Vial.

Charles Vial: The co-radical filtration on the Chow group of zero-cycles on hyper-Kähler varieties

Abstract: I will discuss a filtration on the Chow group of zero-cycles on a smooth projective variety obtained roughly by considering the successive kernels of the iterates of some modified diagonal embedding of the variety. This filtration is particularly relevant in the case of abelian varieties and of hyper-Kähler varieties, where it is expected to be opposite to the conjectural Bloch-Beilinson filtration. In the case of abelian varieties, it can in fact be described explicitly in terms of the Beauville decomposition, while in the case of hyper-Kähler varieties, I conjecture (and prove in some cases) that it coincides with a filtration introduced earlier by Claire Voisin. As a by-product we obtain a criterion for two effective zero-cycles on a moduli space of stable objects on a K3 surface to be rationally equivalent, generalising a result of Marian-Zhao.

Shizhuo Zhang: Categorical Torelli theorem for Fano varieties.

Abstract: It is well known that for a smooth Fano variety, bounded derived category of coherent sheaves determine their isomorphism classes. It is natural to ask whether it is possible to

reconstruct them with less information, say a non-trivial semi-orthogonal component, known as categorical Torelli problem. I will talk about recent progress on this problem, in particular for Fano threefolds and fourfolds. It is based on the work myself and the joint work with Zhiyu Liu, Augustinas Jacovskis. If time allows, I will also talk about an intrinsic proof for categorical Torelli theorem for del Pezzo threefolds, based on a recent joint work with Zhiyu Liu and Soheyla Feyzbakhsh.