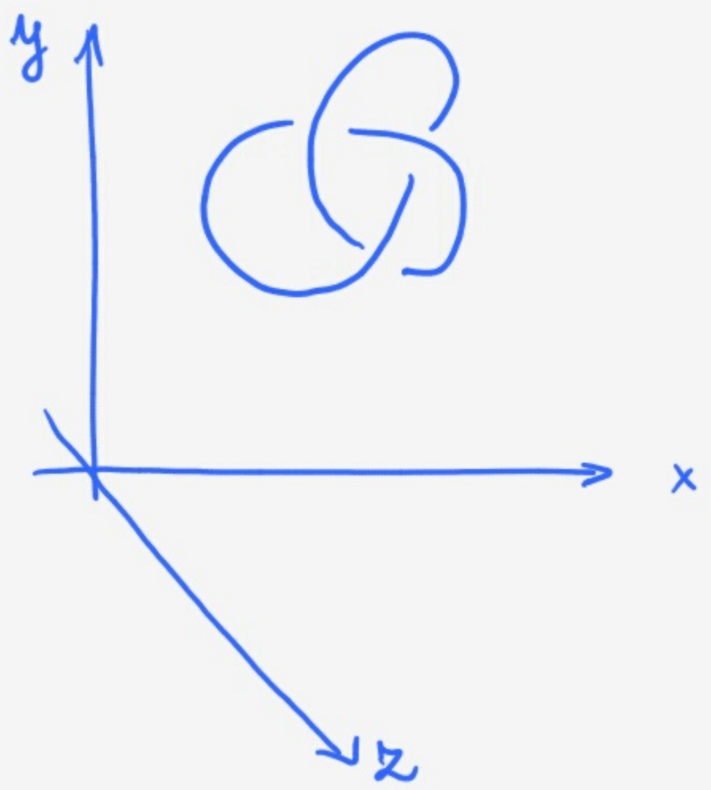
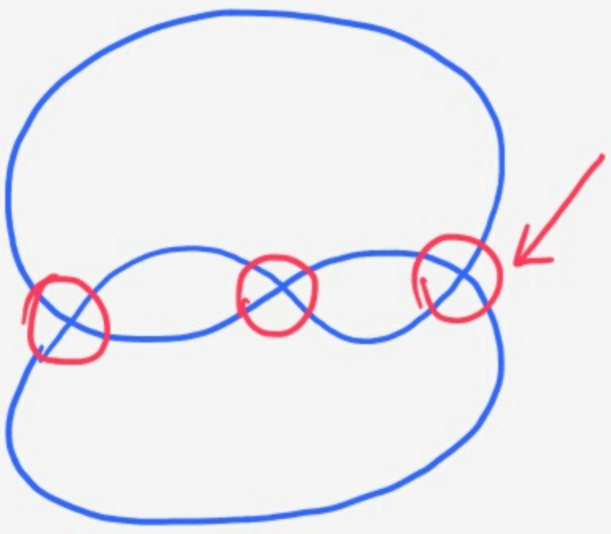
KNOTS in S^3



projection



under - overcrossing ?!

declare ⊗ = X

planefield (standard ctct structure)

$\left(\frac{\partial y}{\partial x} = z\right)$

- Legendrian Knots:

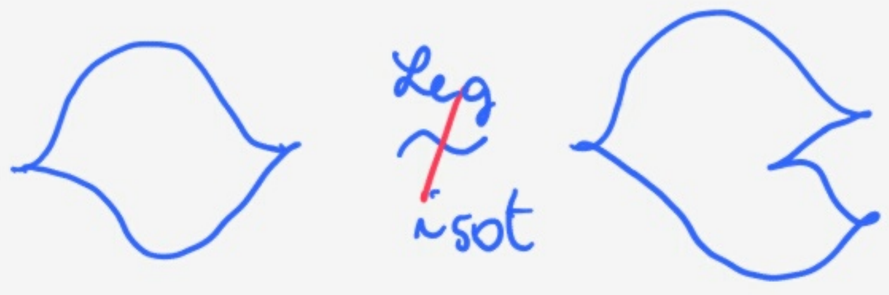
slope in $\mathbb{R}^2_{(x,y)}$ = height

$\Rightarrow$ $\rightsquigarrow$

(Q1): L & L' Legendrian &nots

$L \overset{isot}{\sim} L' \overset{?}{\Longleftrightarrow} L \overset{Leg}{\underset{isot}{\sim}} L'$

$\Leftarrow$ ✓

$\Rightarrow$ X simplest example : 

How do we distinguish Legendrian &nots?

• <u>Invariants</u>: - Thurston-Bennequin #

$$L \longmapsto tb(L) \in \mathbb{Z}$$

- rotation #

$$L \longmapsto r(L) \in \mathbb{Z}$$

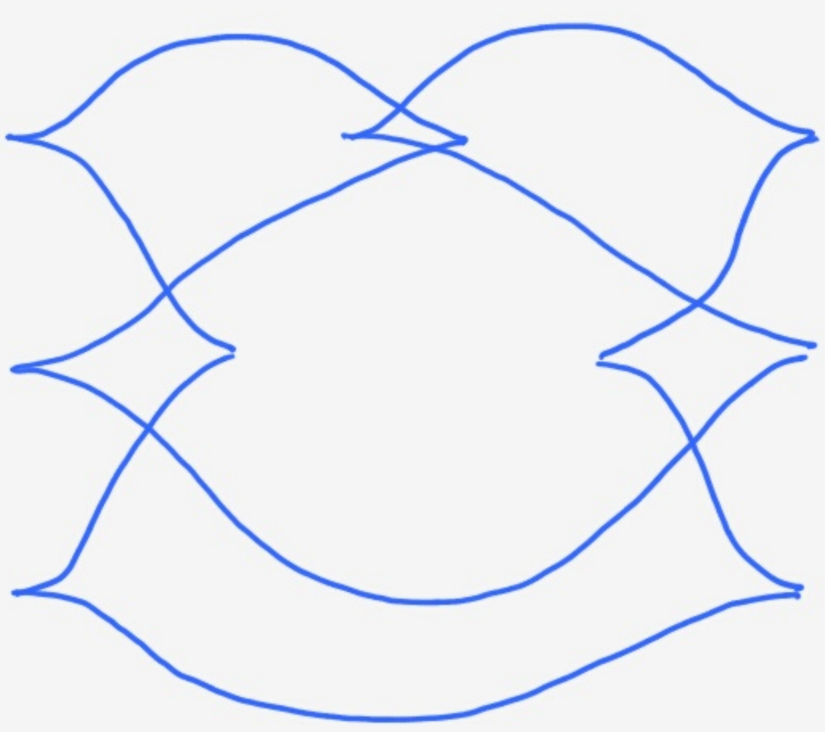
(Q2) L, L' Leg knot $L \overset{isot}{\sim} L'$

$tb(L) = tb(L')$? $\Rightarrow$ $L \overset{Leg}{\underset{isot}{\sim}} L'$

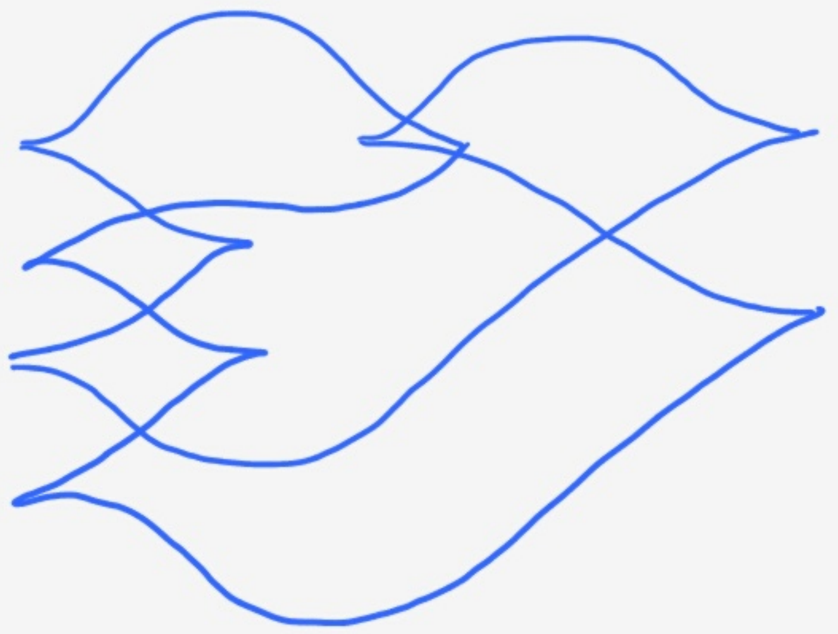
$r(L) = r(L')$

- unknot (Eliashberg) ✓
- figure eight & torus knots (Etnyre-Honda) ✓

But! not all knots! simplest example: $(tb=1, r=0)$



twist knots (Epstein-Fuchs-Meier)



<u>RL</u>: (Etnyre-Ng-V) complete classification of all twist knots

• <u>more invariants</u>:
- Legendrian contact homology
- <u>Heegaard Floer homology</u>

K knot $\xrightarrow{\text{Ozsváth-Szabó, Rasmussen}}$ $\widehat{HFK}(K)$ bigraded $\mathbb{Z}_2$-vectorspace

$\| \qquad \|$

L~K Legendrian knot $\xrightarrow[\text{Thurston}]{\text{Ozsváth-Szabó}}$ $\lambda(L) = \mathcal{L}(L)$

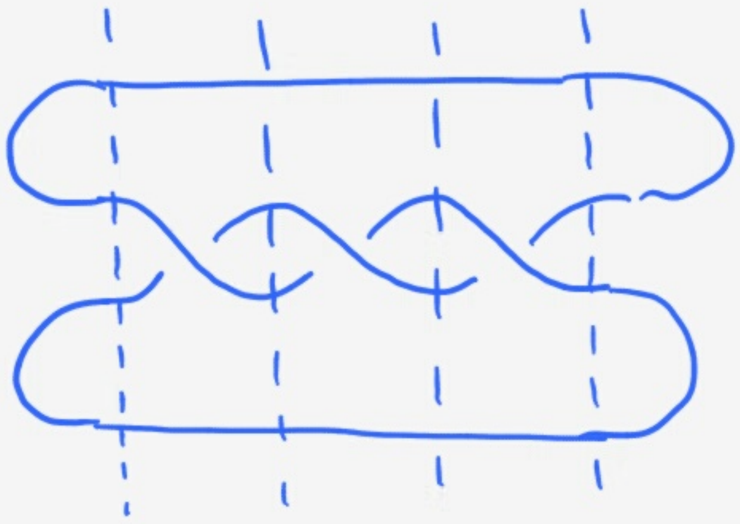
Lisca-Ozsváth Stipsicz-Szabó

Baldwin

Vela-Vicke, V

- generalises Alexander polynomial
- detects knot-genus (Ozsváth-Szabó)
- fiberedness (Ghiggini, Ni)
- bounds slice-genus (Ozsváth, Szabó)

Computing: (defined using PDE)



PetRava-V: computing HFK from pieces